The BRITISH UNIVERSITY IN EGYPT

CENTRE FOR THEORETICAL PHYSICS

Lower than freeze-in portal couplings can still generate DM and be measured via GW

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08.02.2023

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CEICO

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research, Development and Education

MŠMT MINISTRY OF EDUCATION, YOUTH AND SPORTS

- Beyond freeze-in: Dark Matter via inverse phase transition and gravitational wave signal,
  e-Print: 2104.13722, PRD

- Gravitational shine of dark domain walls,
  e-Print: 2112.12608, JCAP

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Eugeny Babichev (IJCLab, Orsay)
Dmitry Gorbunov (INR and MIPT, Moscow)
Alexander Vikman (CEICO, FZU Prague)

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- Inverse phase transition allows to have domain wall network in the early universe, as this network melts away
- The domain wall network emits GWs defined by the properties of the (otherwise not directly detectable) DM. This GW signal can be observable in the near future observations
- Precise numerical simulations are needed to determine the spectral shape of the GW signal

PHYSICAL REVIEW D VOLUME 9, NUMBER 12 15 JUNE 1974

Gauge and global symmetries at high temperature*

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 19 February 1974)

It is shown how finite-temperature effects in a renormalizable quantum field theory can restore a symmetry which is broken at zero temperature. In general, for both gauge symmetries and ordinary symmetries, such effects occur only through a temperature-dependent change in the effective bare mass of the scalar bosons. The change in the boson bare mass is calculated for general field theories, and the results are used to derive the critical temperatures for a few special cases, including gauge and nongauge theories. In one case, it is found that a symmetry which is unbroken at low temperature can be broken by raising the temperature above a critical value. An appendix presents a general operator formalism for dealing with higher-order effects, and it is observed that the one-loop diagrams of field theory simply represent the contribution of zero-point energies to the free energy density. The cosmological implications of this work are briefly discussed.

Inverse Phase Transition

$$\mathscr{L} = \frac{(\partial\chi)^2}{2} - \frac{\left(M^2 - \mu^2(t, \mathbf{x})\right) \cdot \chi^2}{2} - \frac{\lambda\chi^4}{4}$$

Babichev, Gorbunov, Ramazanov(2020)

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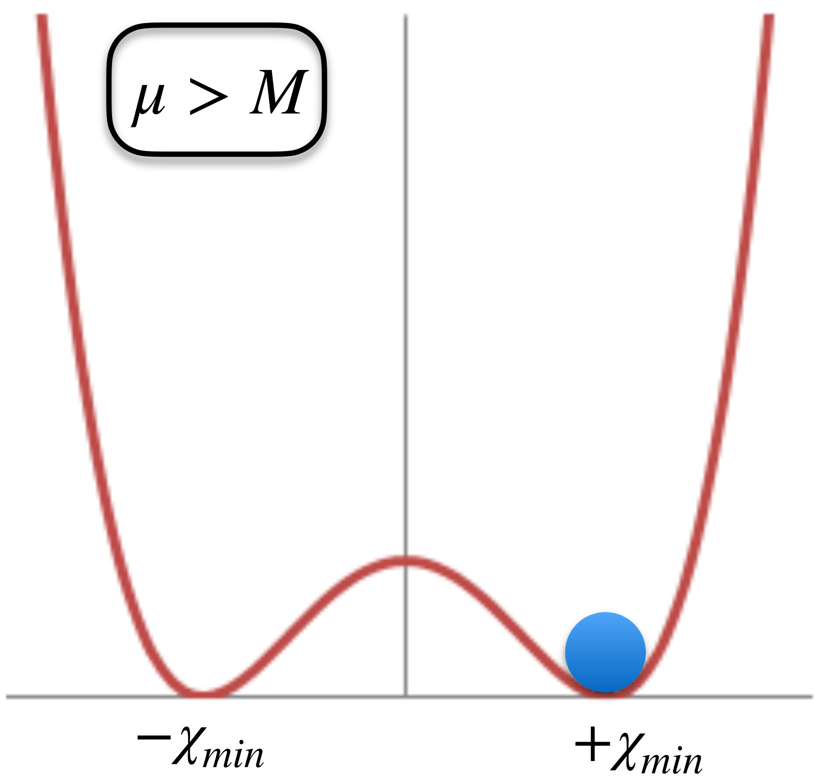

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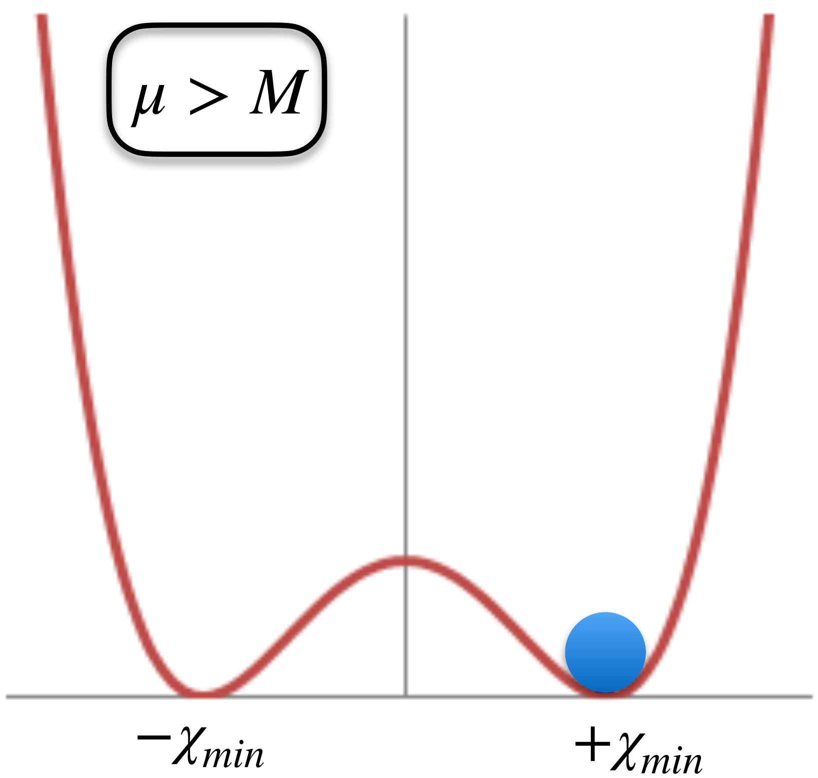

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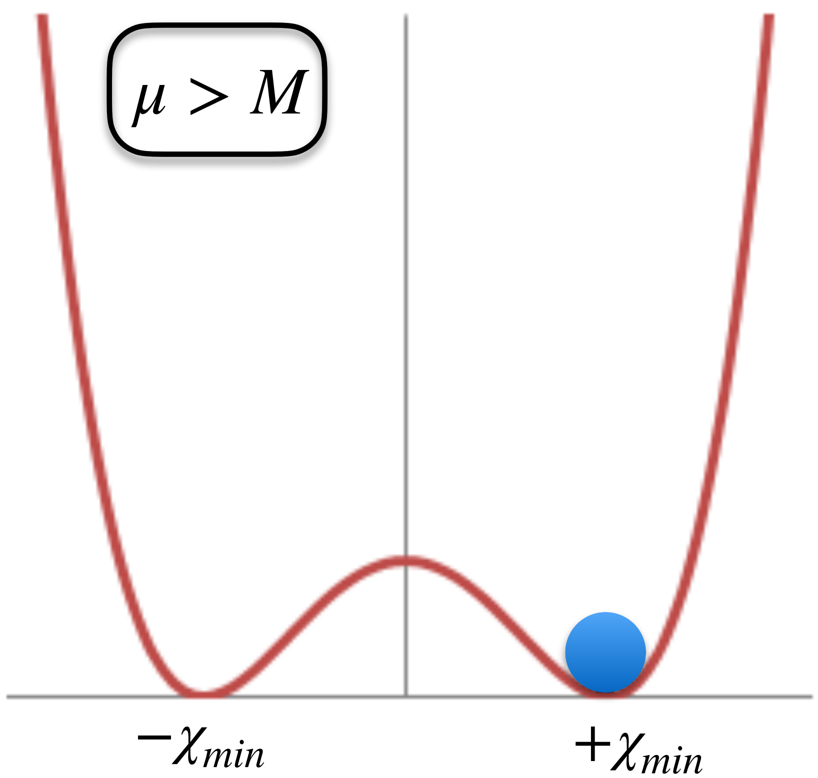

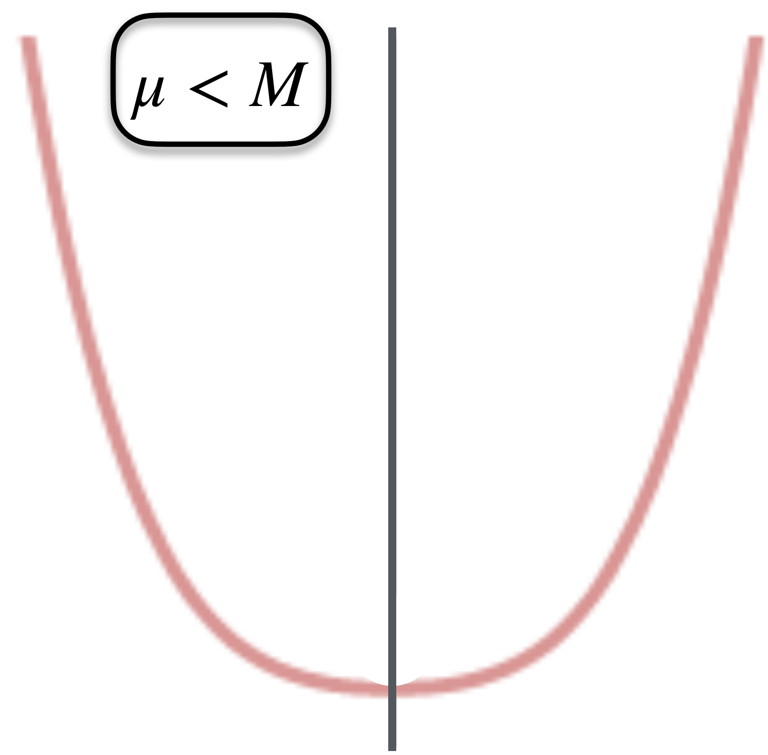

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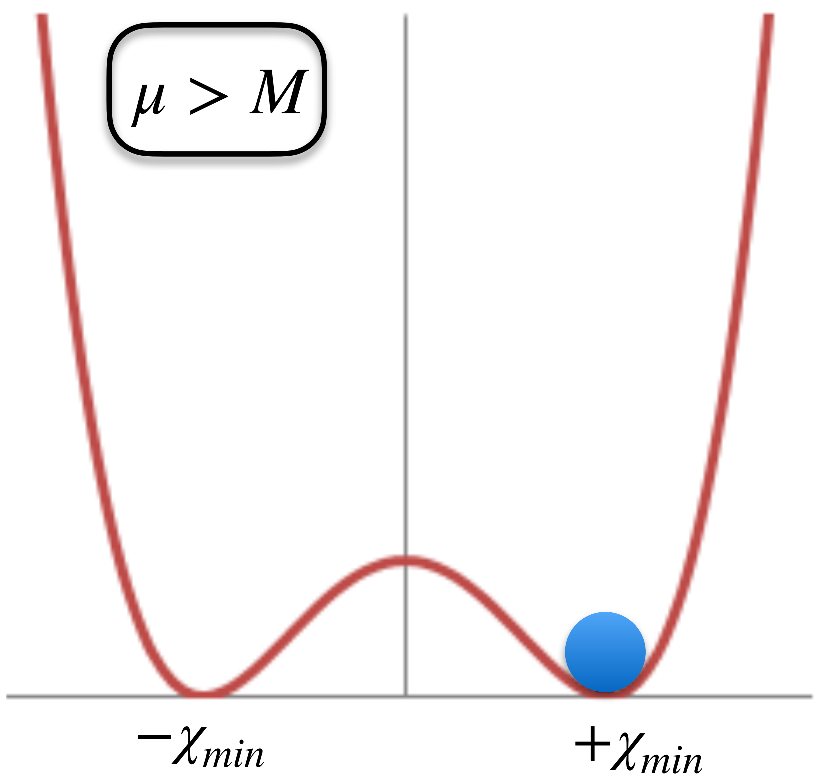
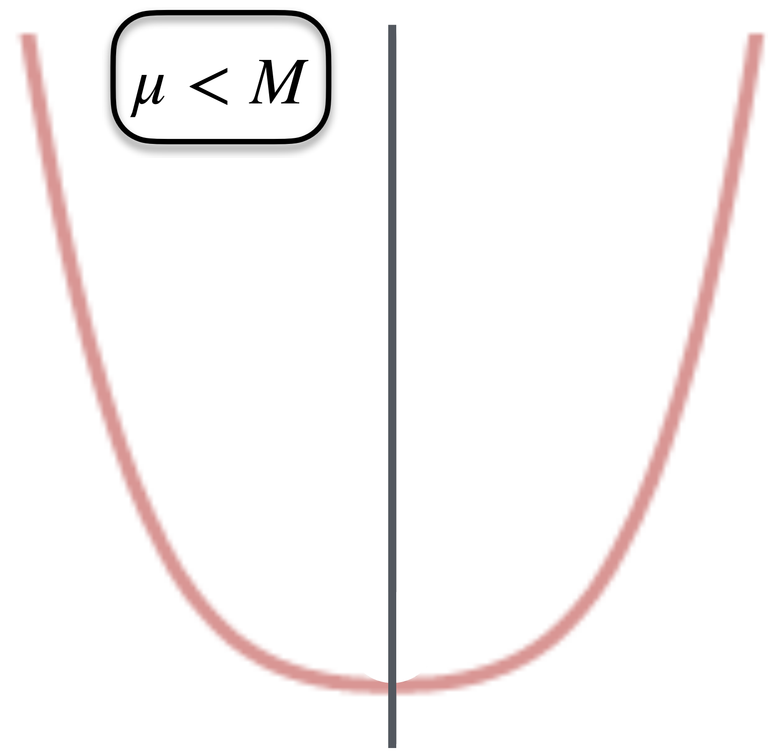
Early Universe
spontaneously Broken Phase





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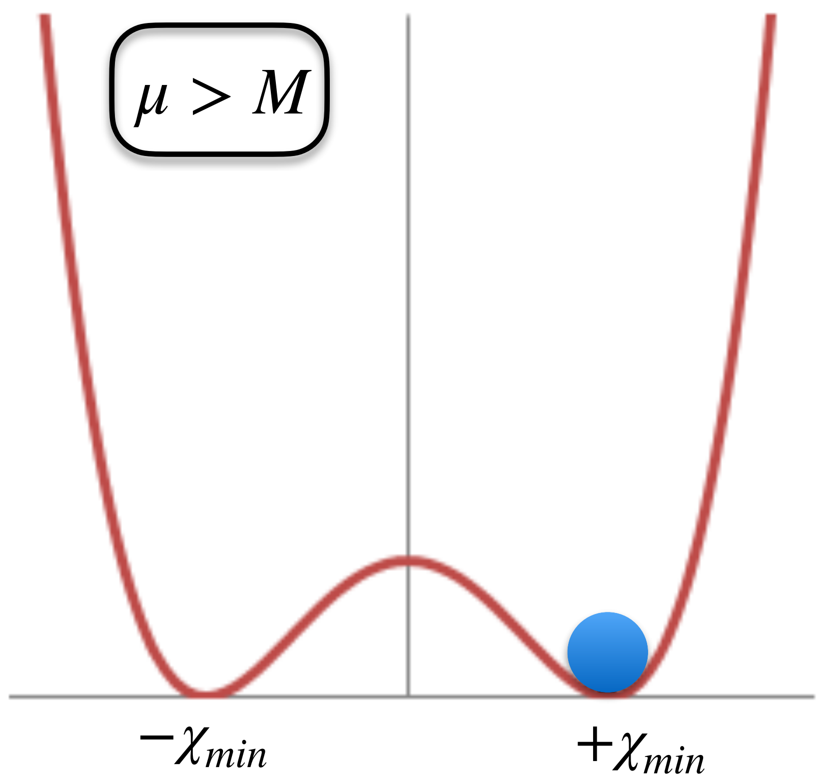

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Tachyonic mass $\mu(t)$
slowly decreases /
redshifts
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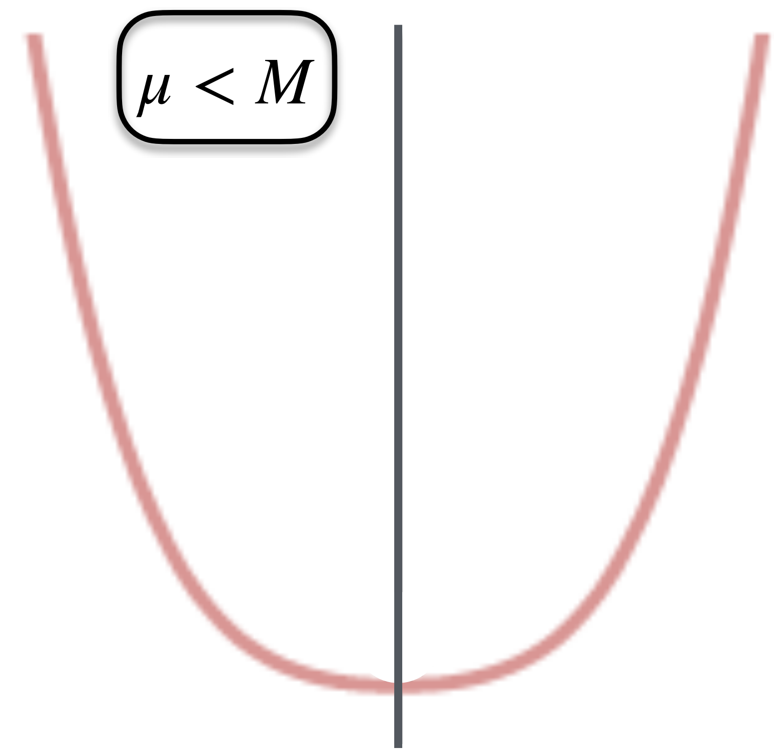
expansion



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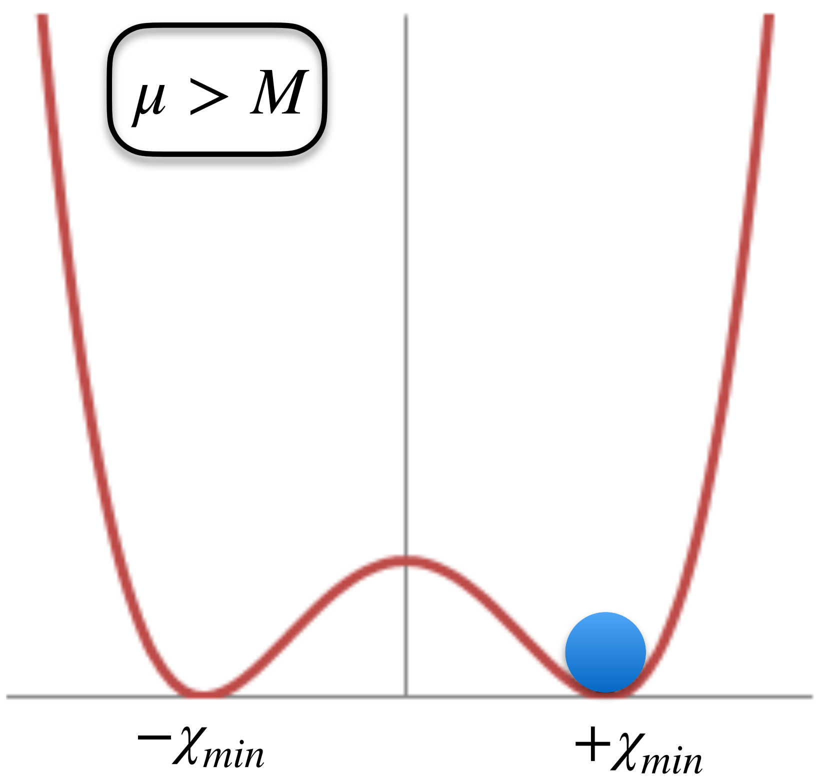

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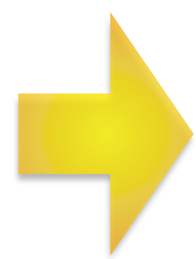

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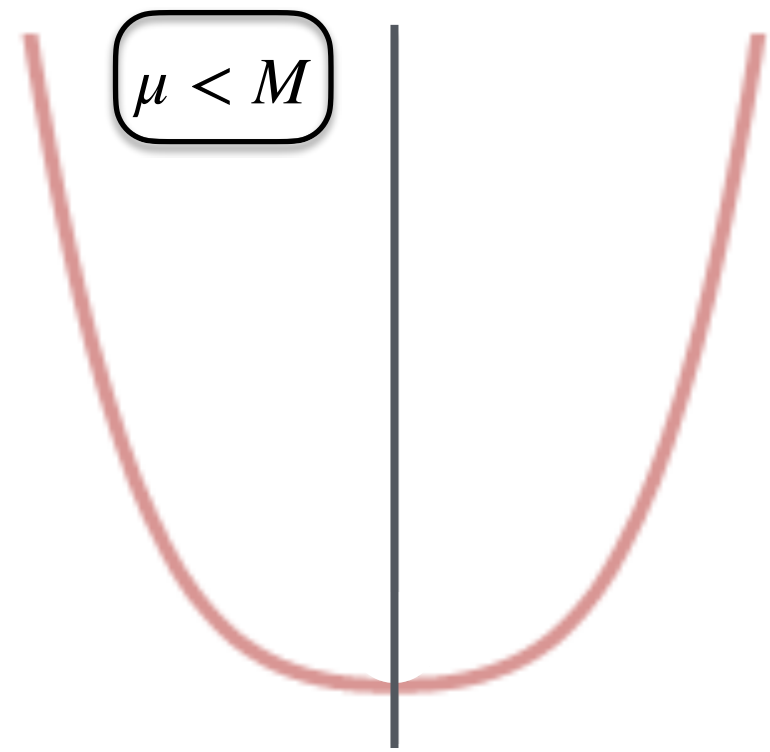
Late Universe



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Babichev, Gorbunov, Ramazanov(2020)

Early Universe
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flipped time order

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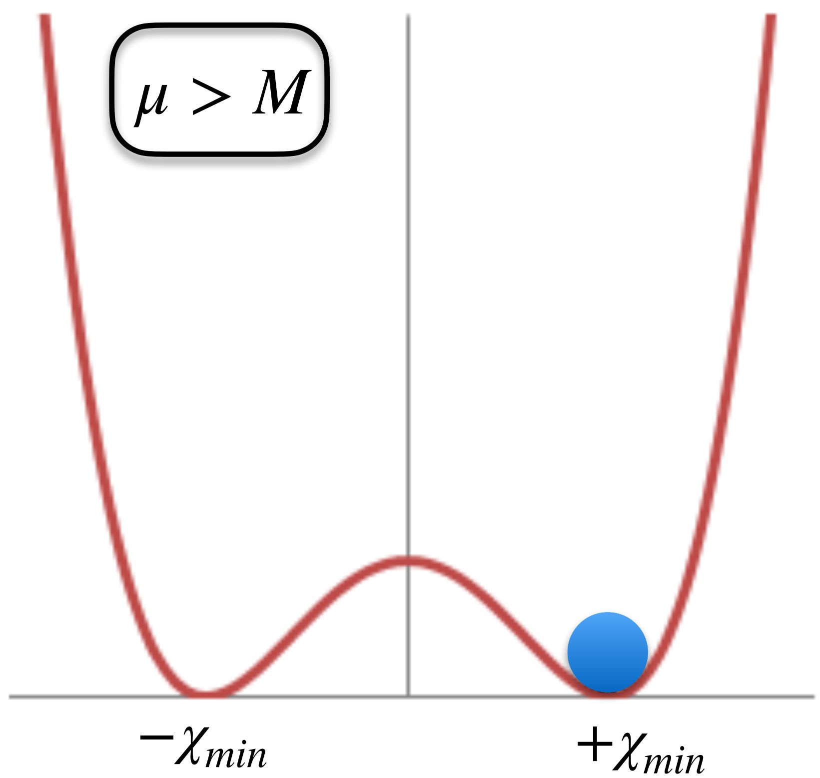

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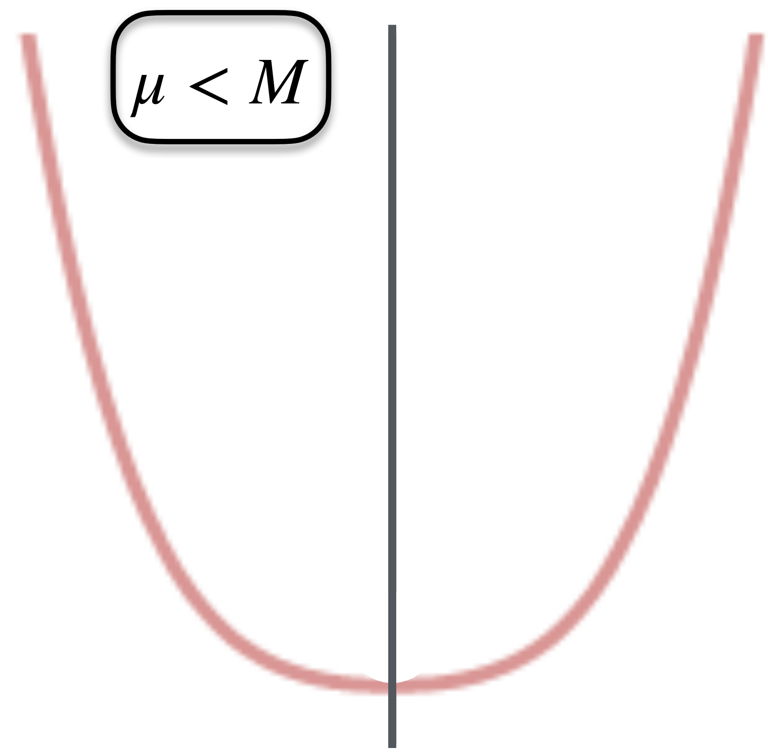

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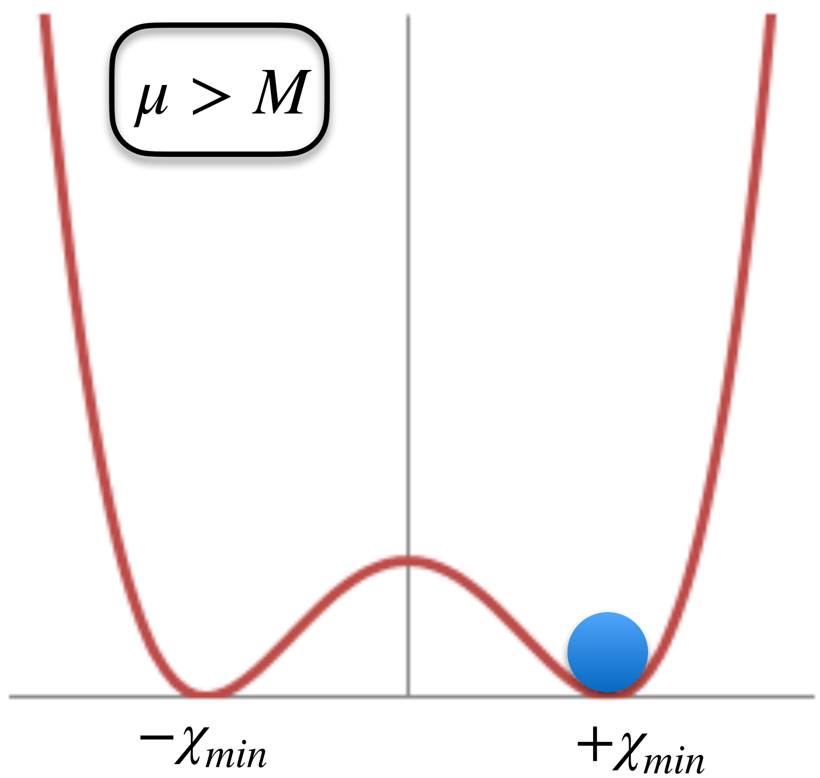

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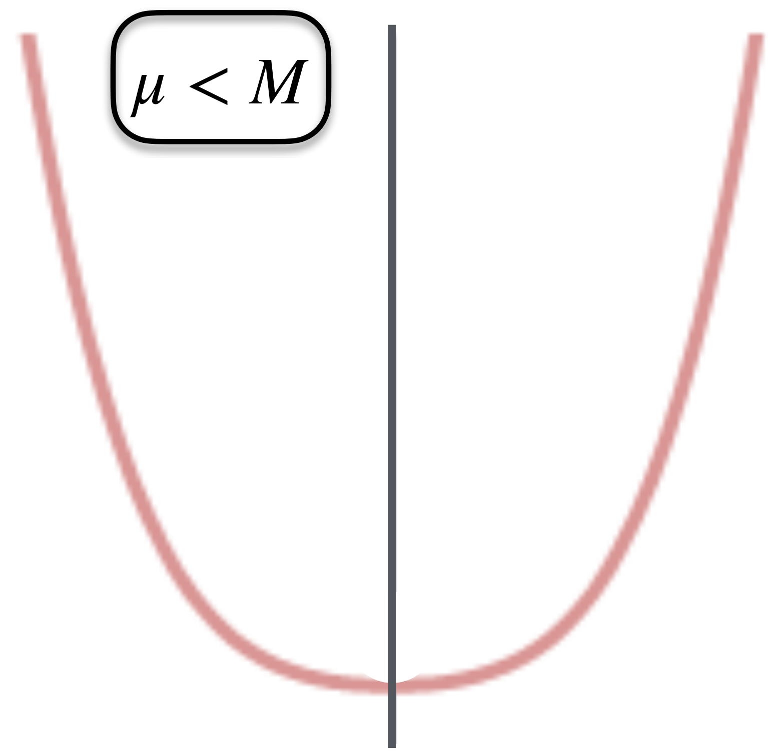

for Hubble parameter

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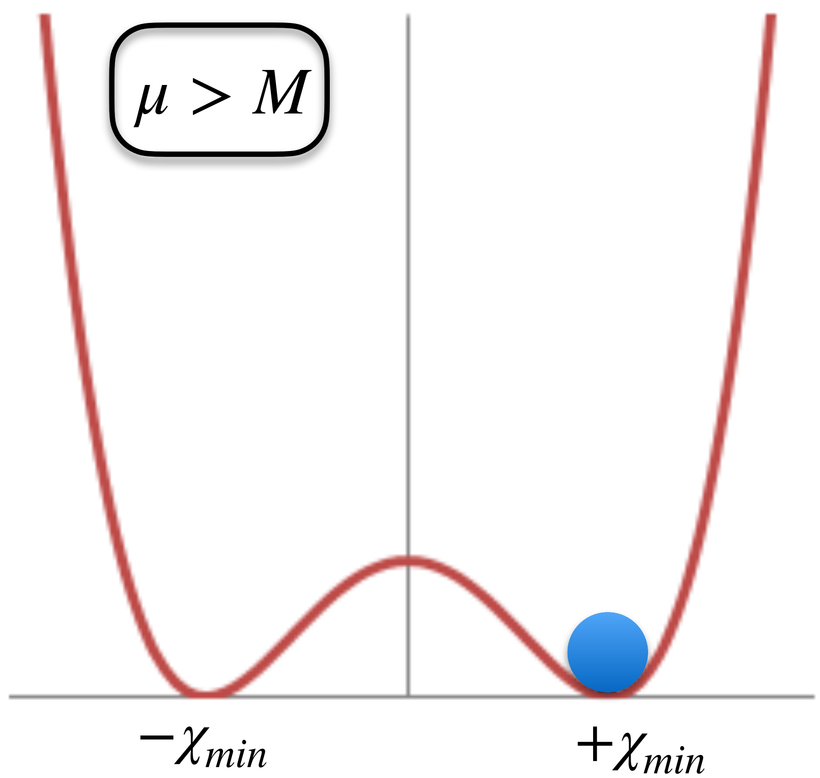
Late Universe
oscillations around restored symmetric vacuum



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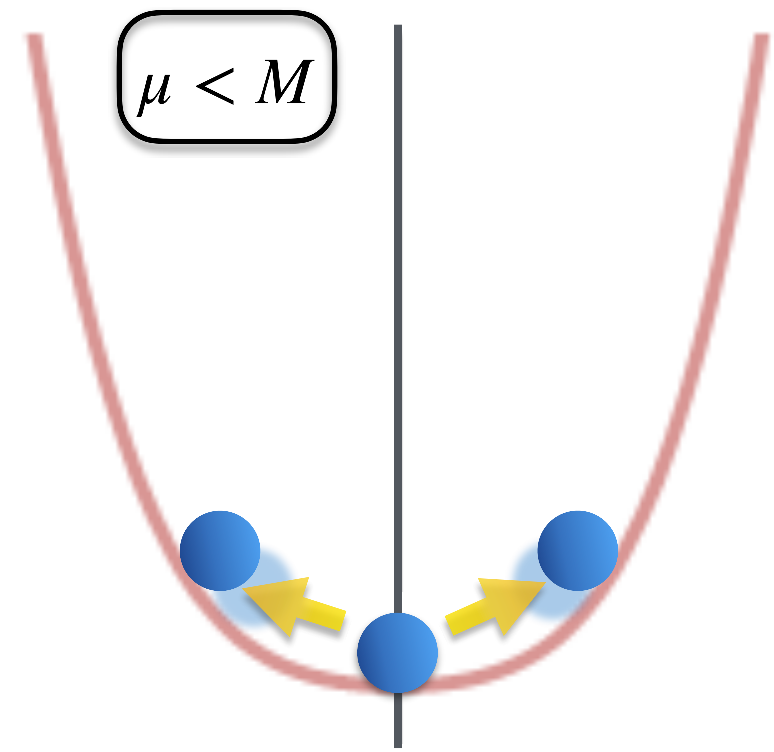

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At this point one cannot trace the minimum as $\dot{\chi}_{min} = \dfrac{\mu\dot{\mu}}{\sqrt{\lambda\left(\mu^2(t) - M^2\right)}}$ diverges**!**

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for the model of this talk $\kappa = 2$

Repetitio est mater studiorum

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Babichev, Gorbunov, Ramazanov(2020)

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Ramazanov, Babichev, Gorbunov, Vikman (2021)

DM χ coupled to ϕ - a multiplet of N *thermal* degrees of freedom

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Temperature at inverse phase transition

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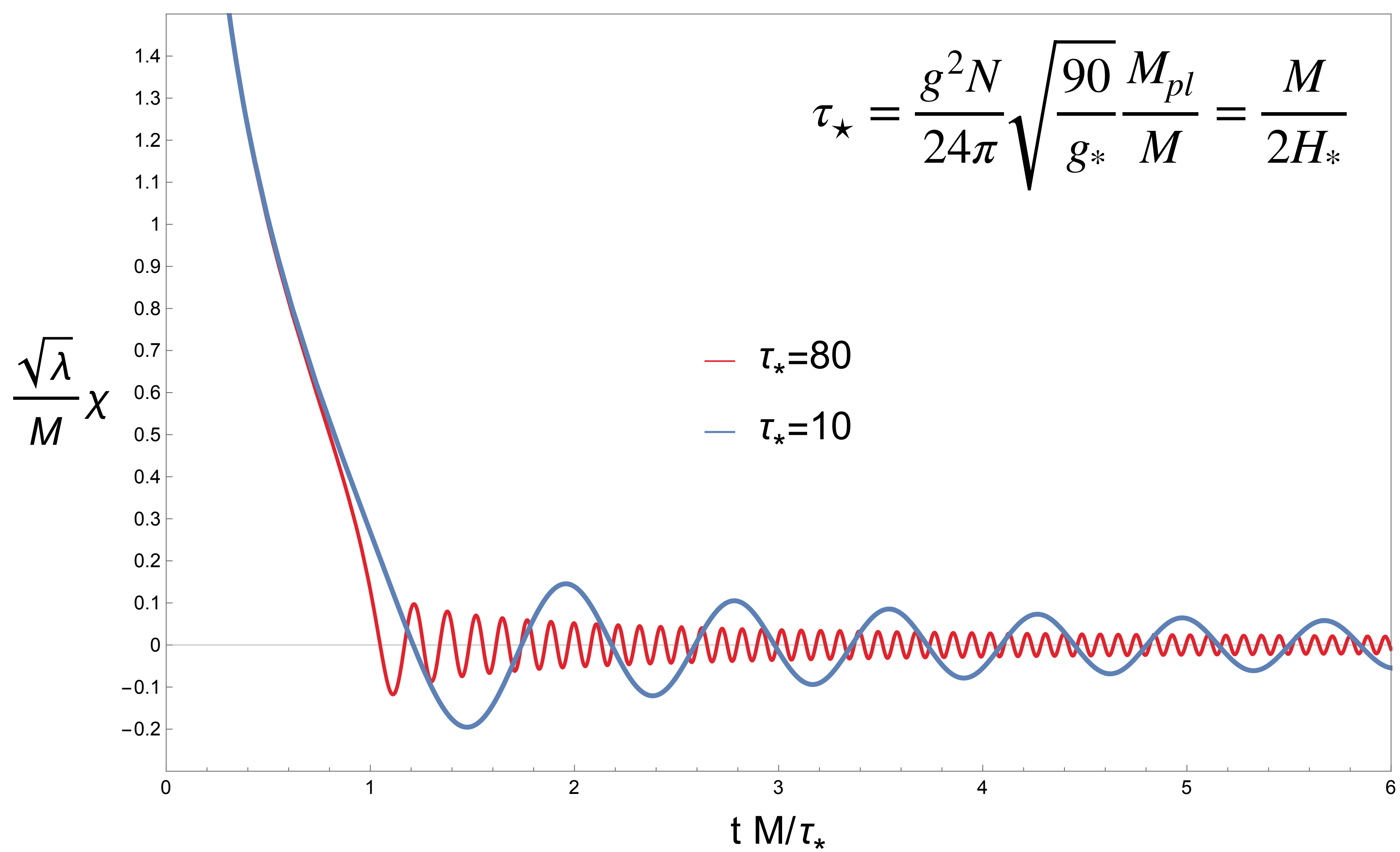

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which one uses in

$$\rho_\chi = \frac{\left(4M^{10}H_*^2\right)^{1/3}}{4\lambda}\left(\frac{a_*}{a_{eq}}\right)^3 = \varepsilon_{rad}\left(T_{eq}\right)$$

to obtain M

Mass of DM

$$M = \frac{\lambda^{3/5}}{g}\sqrt{\frac{12}{N}}\left(\frac{\pi^4 g_*^2(T_*)}{75}\left(\frac{M_{pl}}{T_{eq}}\right)^2\right)^{1/5} T_{eq}$$

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Recalling that $T_{eq} \simeq 0.8\,\text{eV}$

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weak coupling

Allowed Parameter Space

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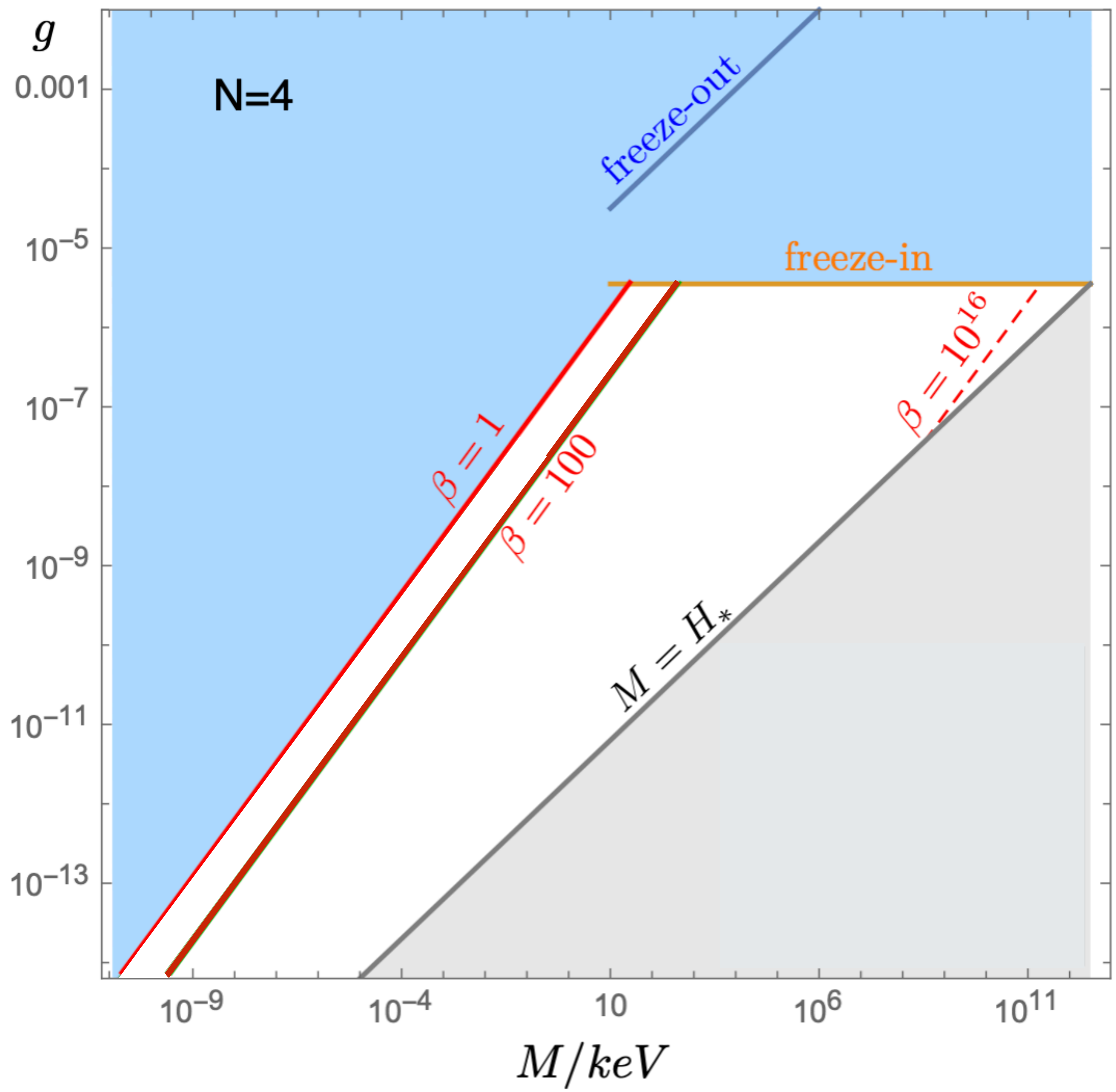

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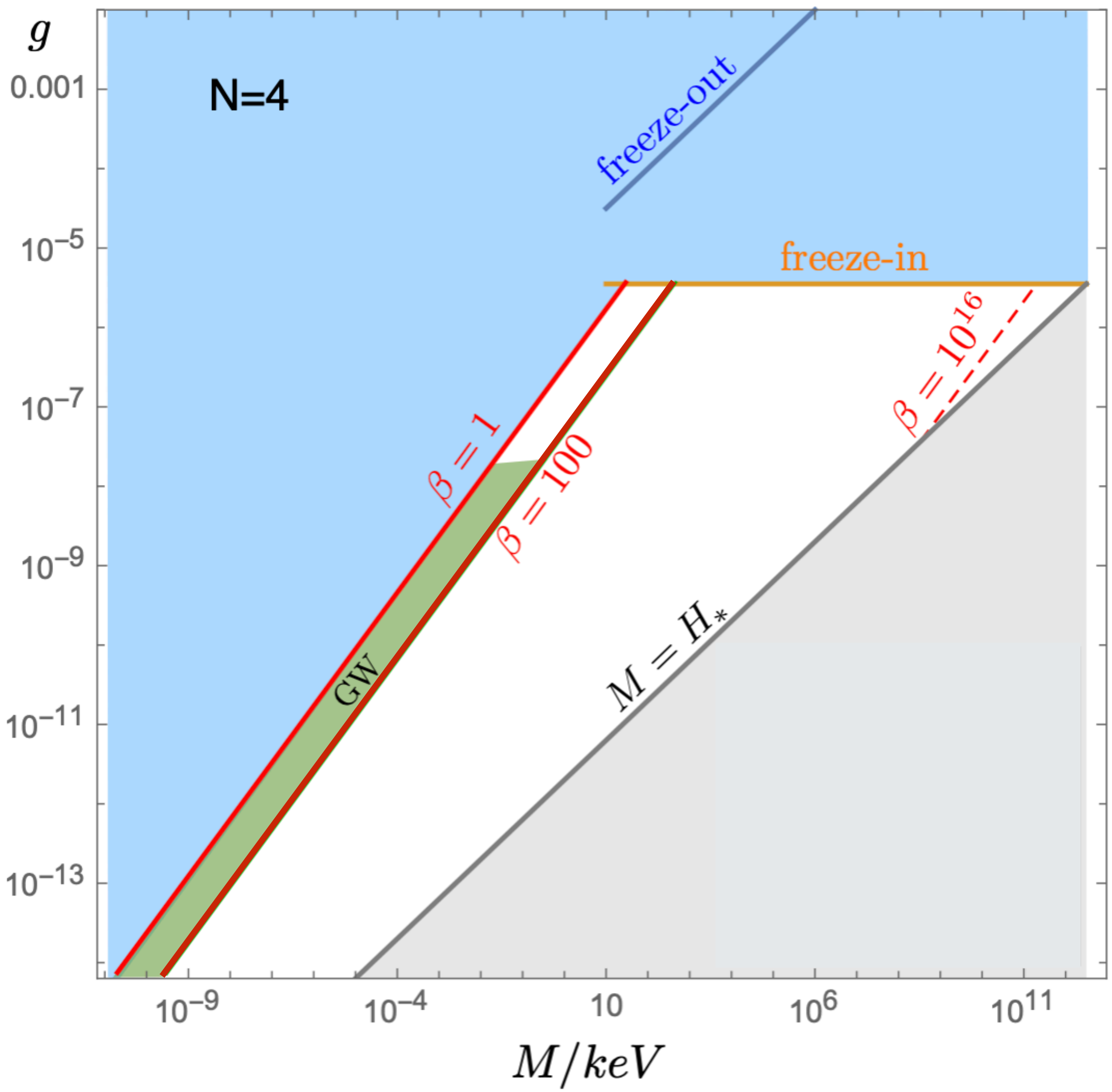

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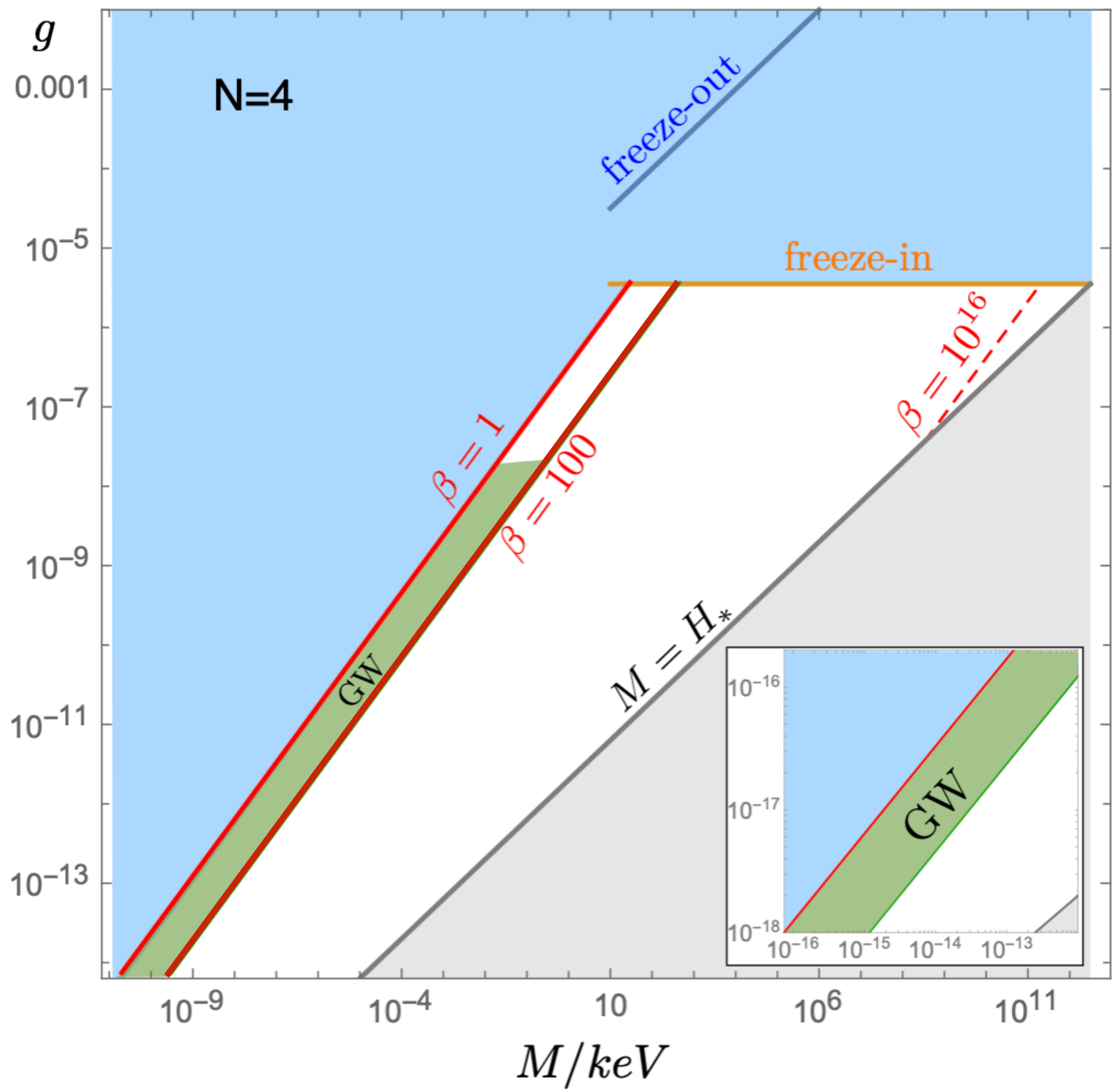
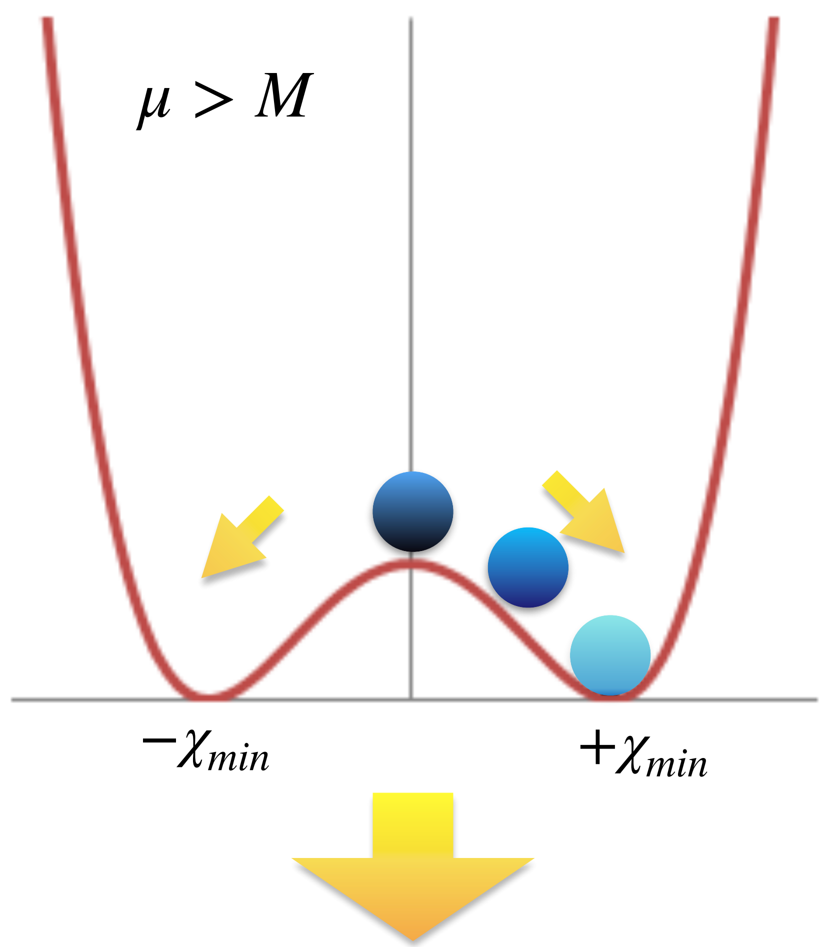


What was before?

A direct phase transition?

Early universe spontaneously Broken Phase



Domain Walls!

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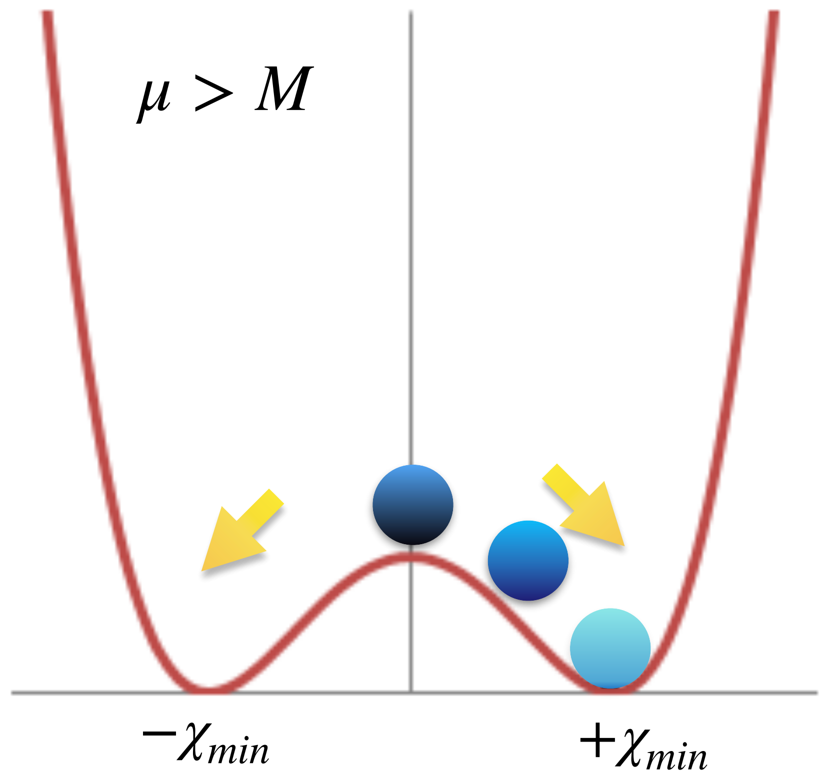

To avoid too much friction

$$\mu \simeq H_i$$

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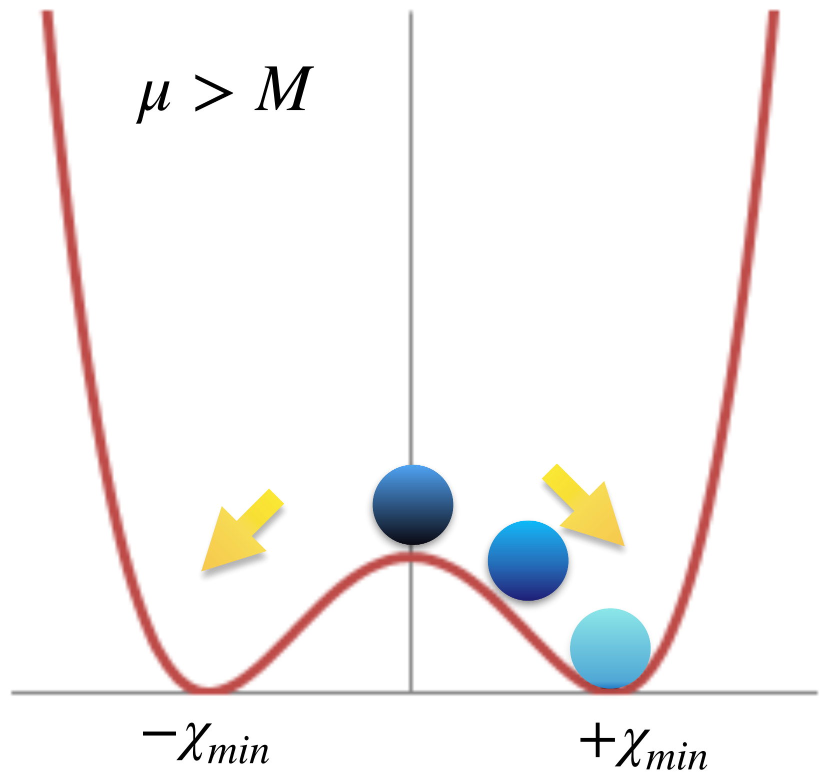

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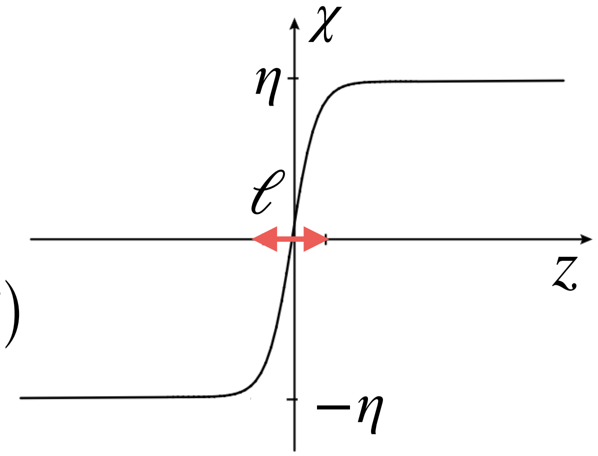

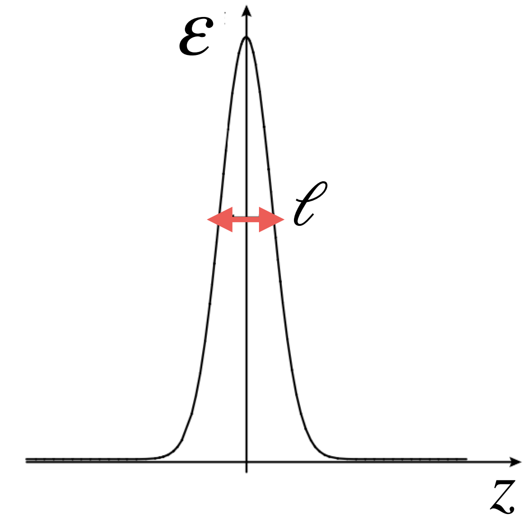

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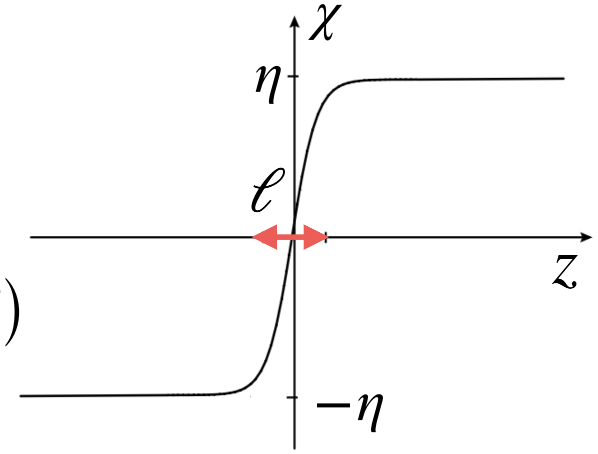

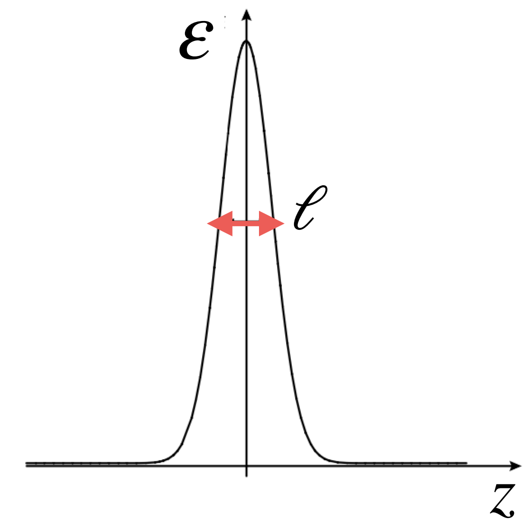

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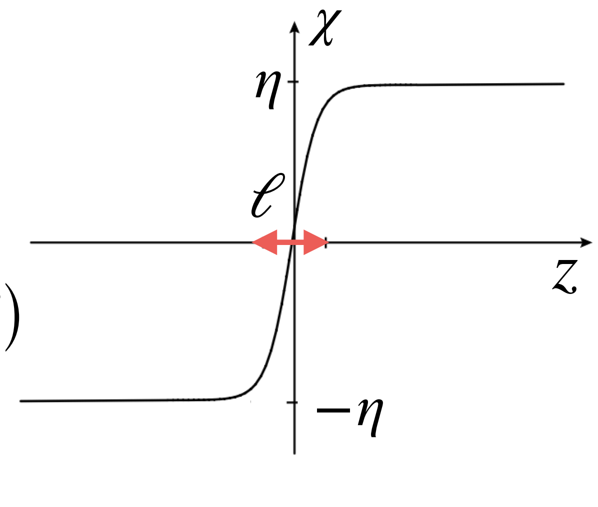

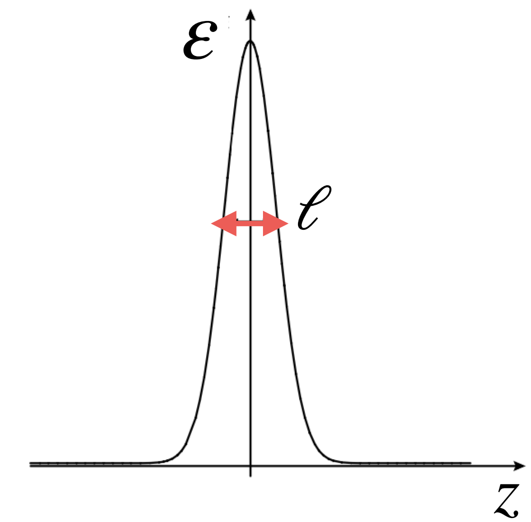

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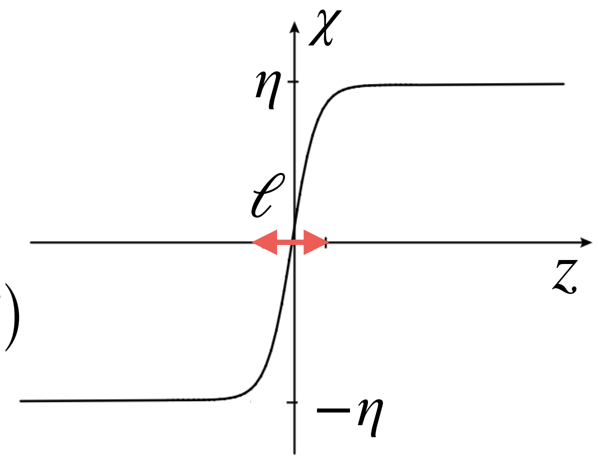

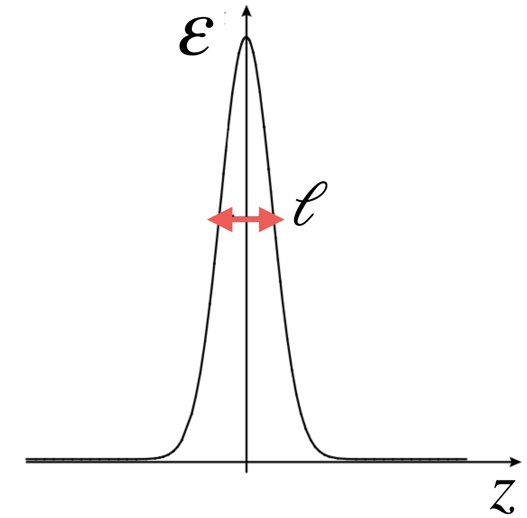

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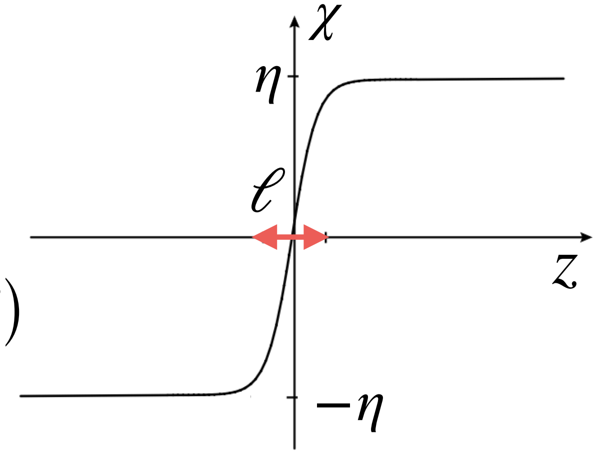

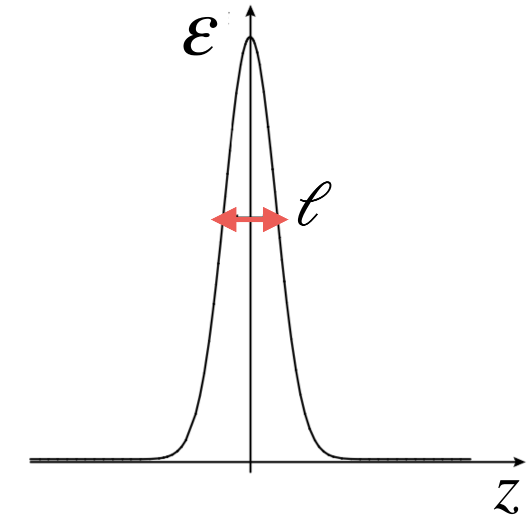

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GW

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Einstein's formula

$$P \sim \dddot{Q}_{ij}^2 / M_{Pl}^2$$

works well for domain wall network!!!

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$$f = H_i$$

On the estimation of gravitational wave spectrum from cosmic domain walls

#7

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On the estimation of gravitational wave spectrum from cosmic domain walls #7

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our case

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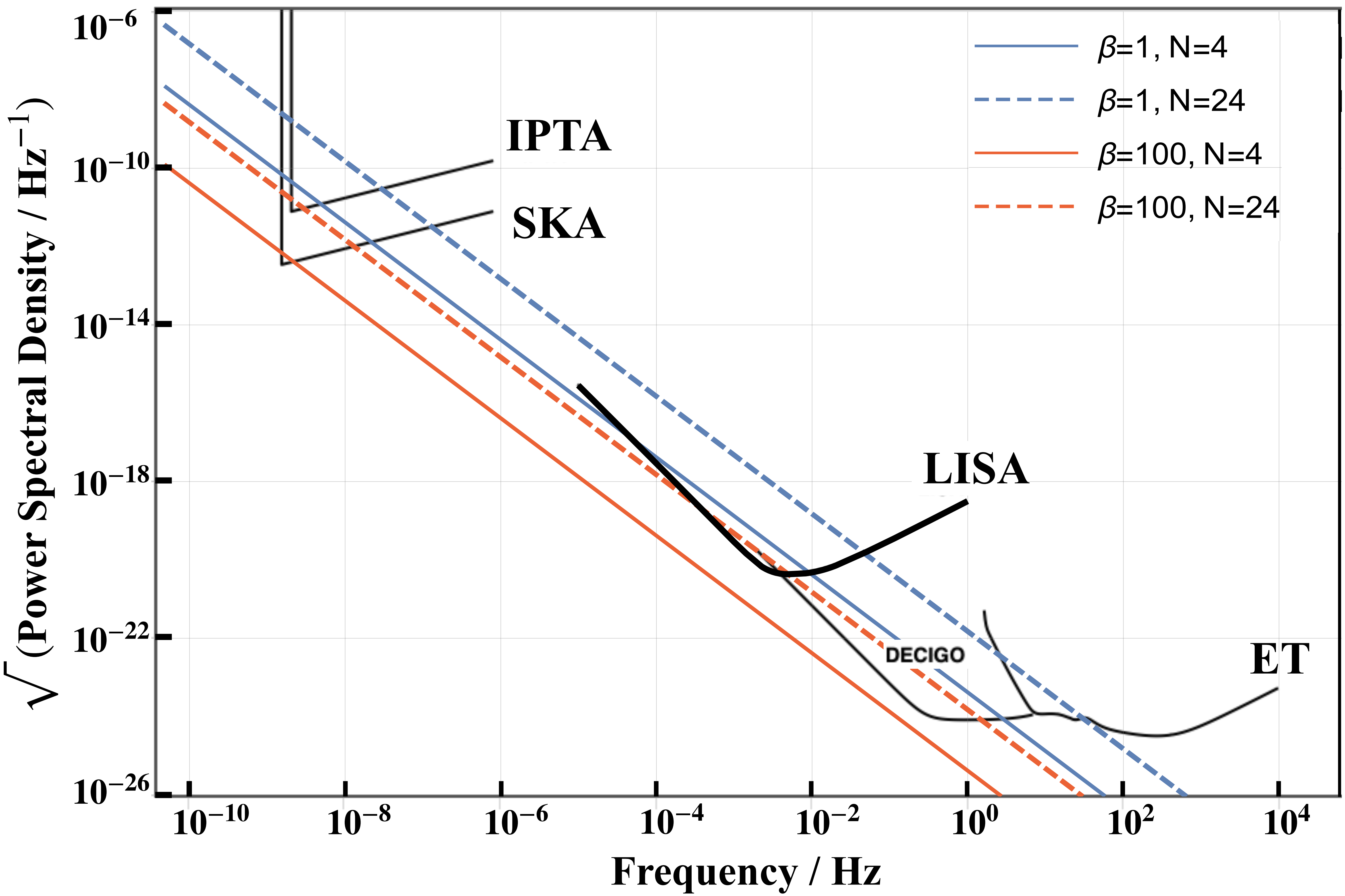

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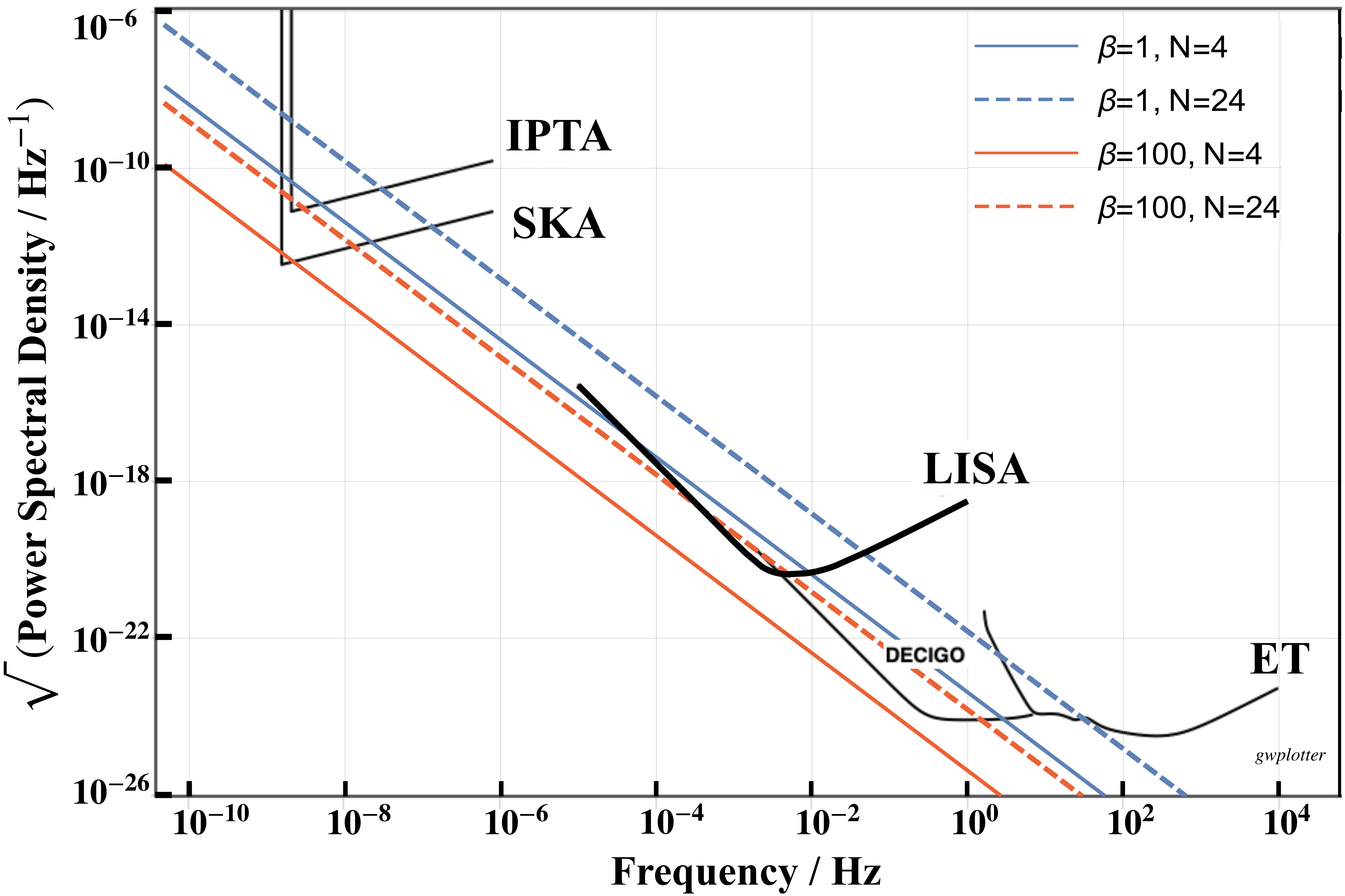

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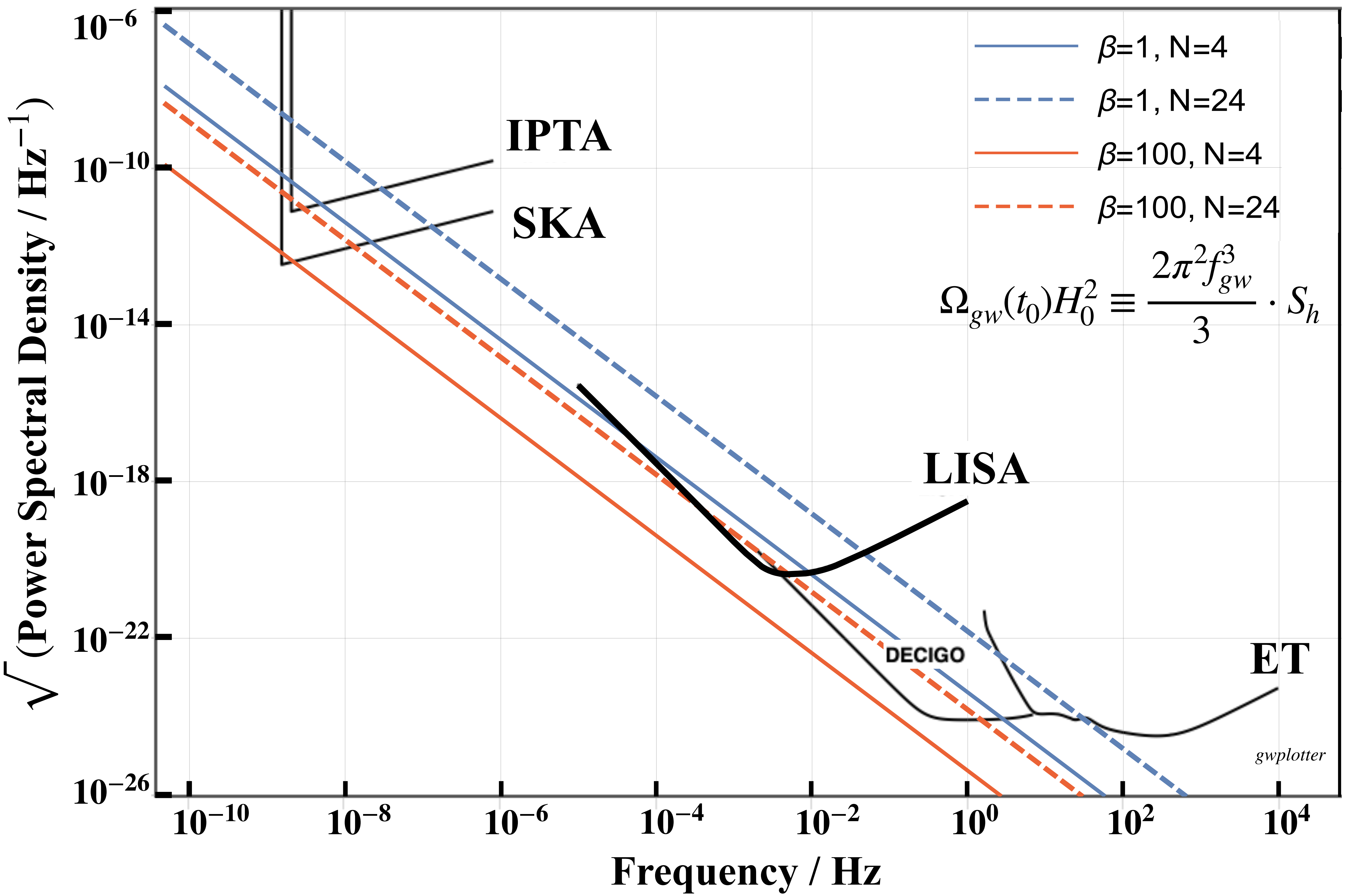

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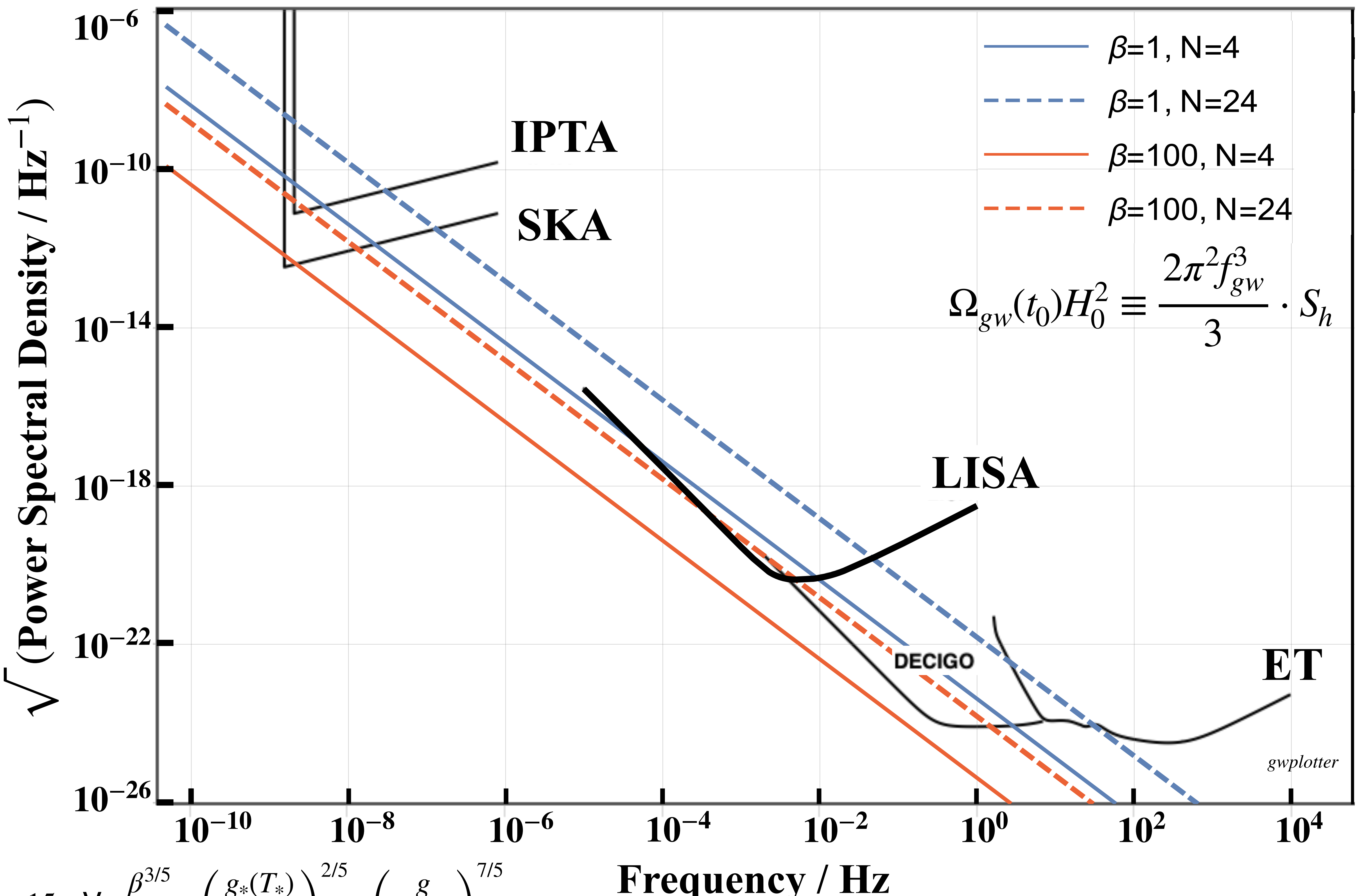

$$M \simeq 15\ \text{eV} \cdot \frac{\beta^{3/5}}{\sqrt{N}} \cdot \left(\frac{g_*(T_*)}{100}\right)^{2/5} \cdot \left(\frac{g}{10^{-8}}\right)^{7/5}$$

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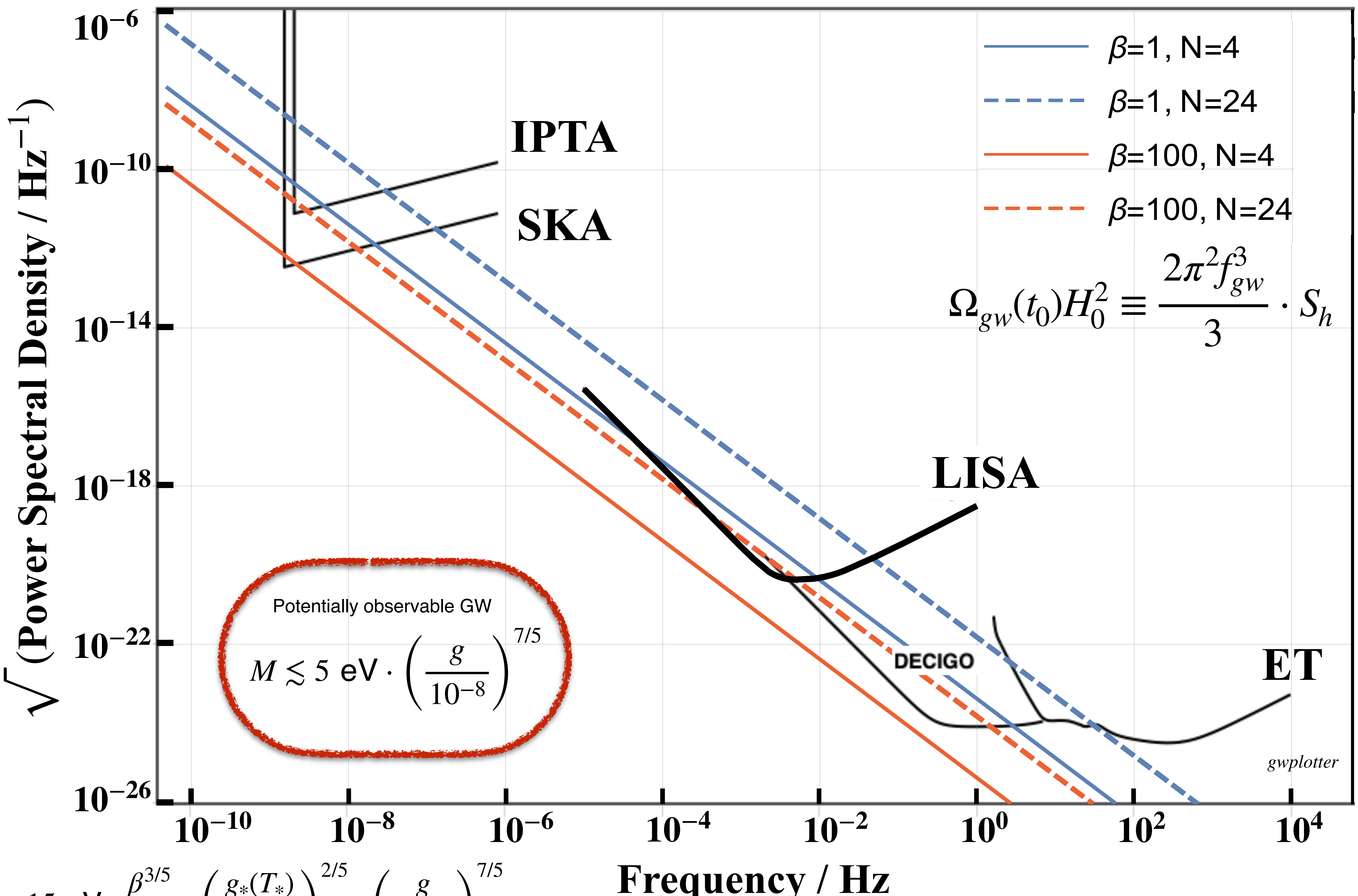

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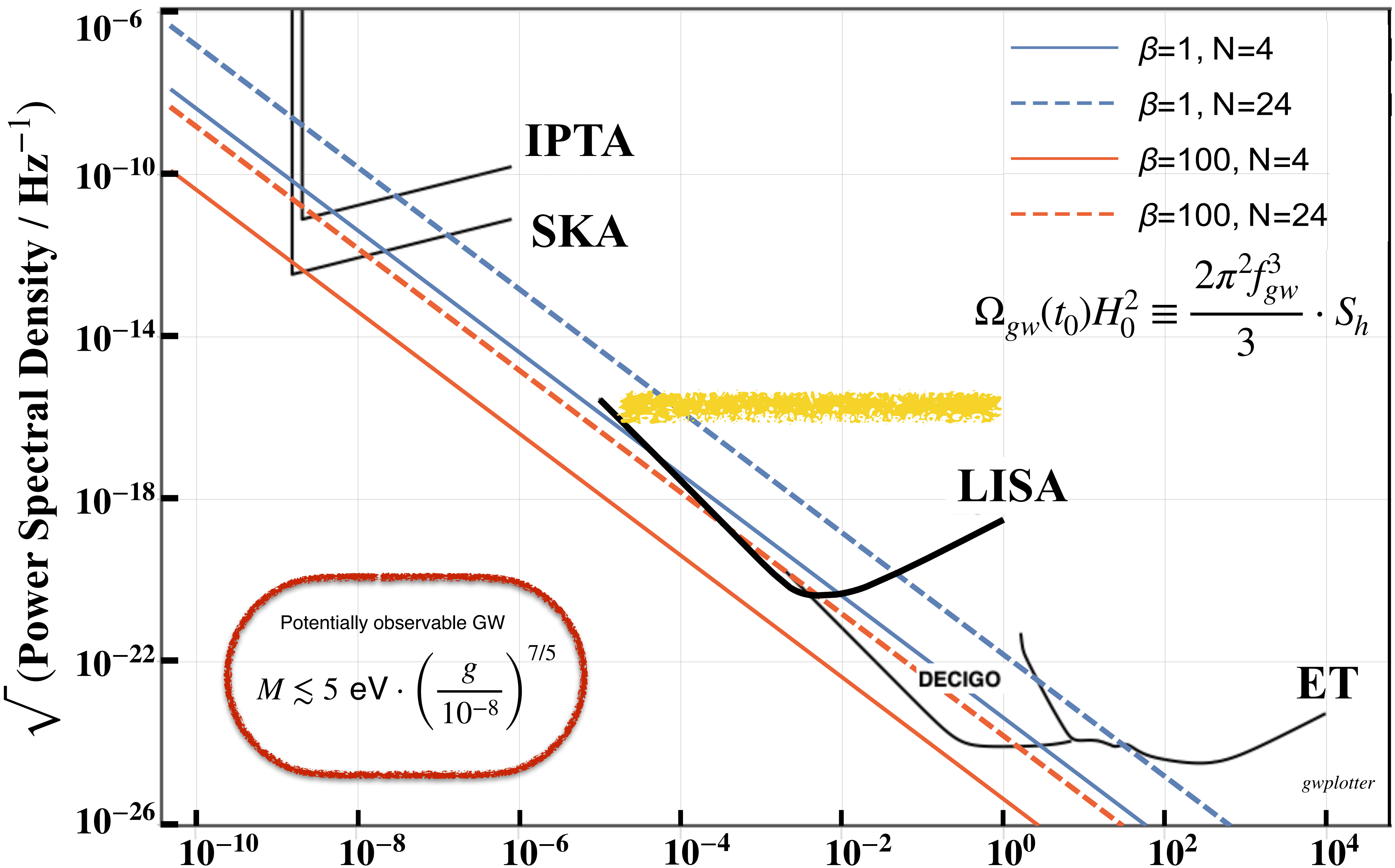

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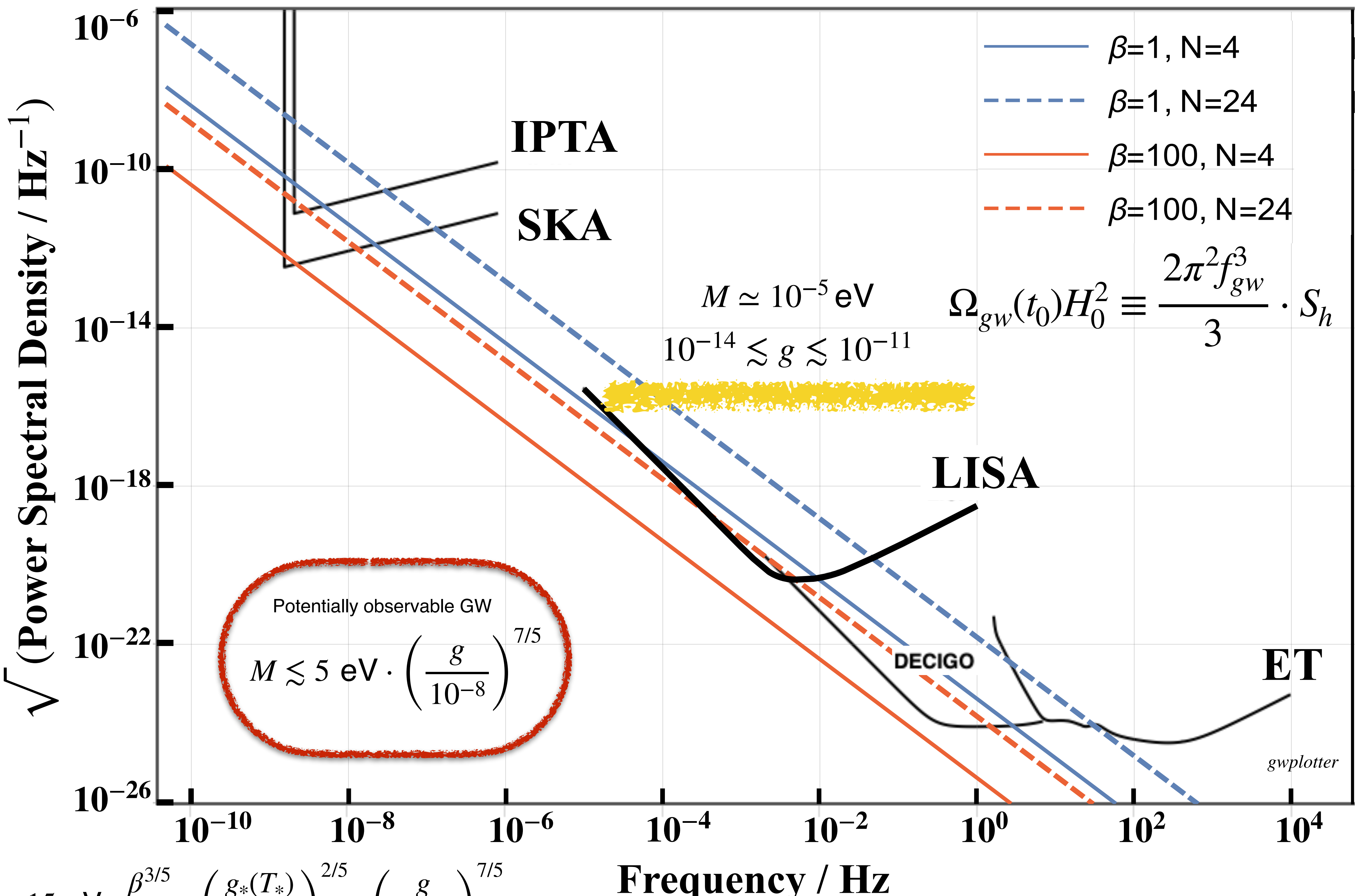

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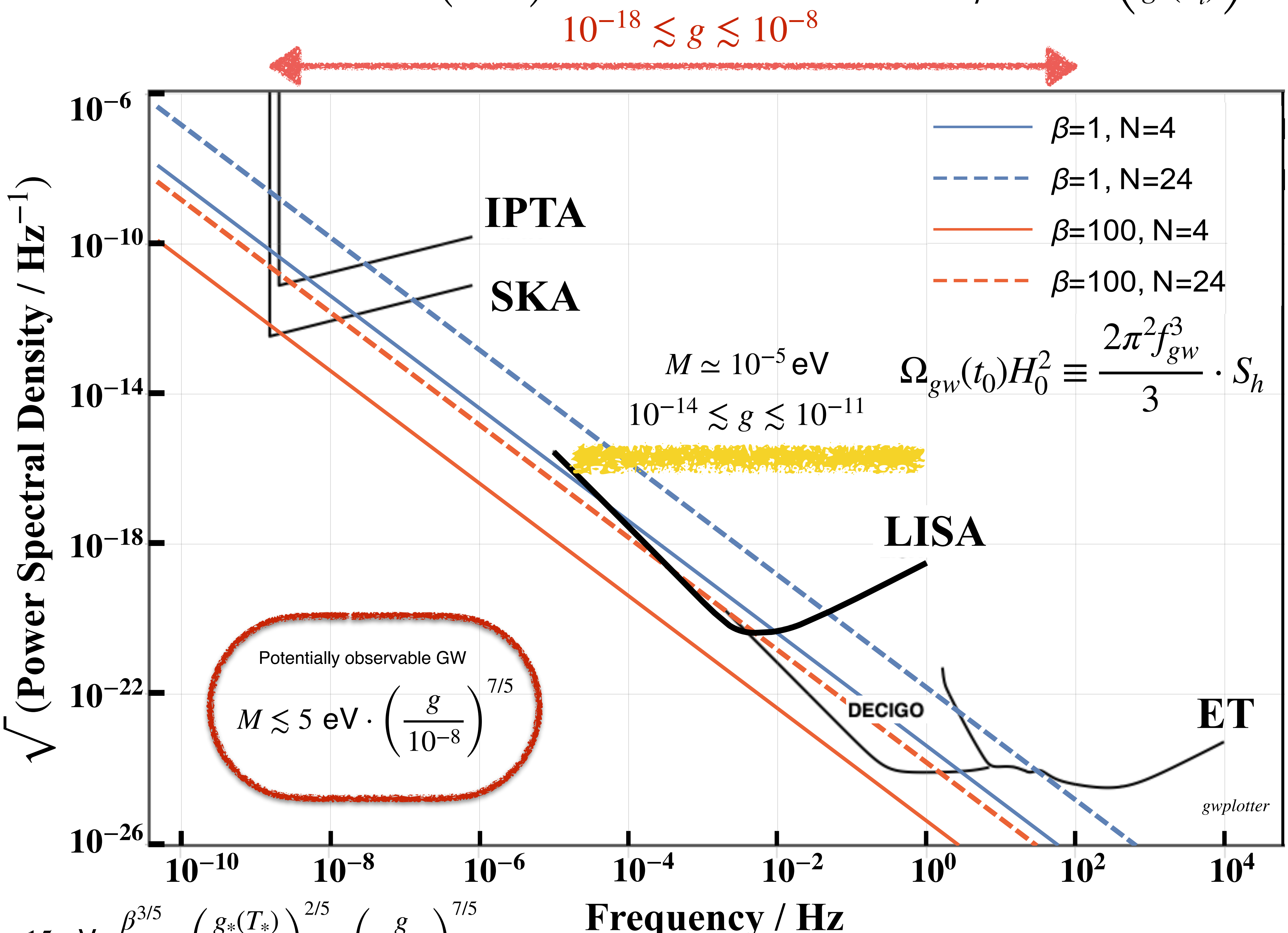

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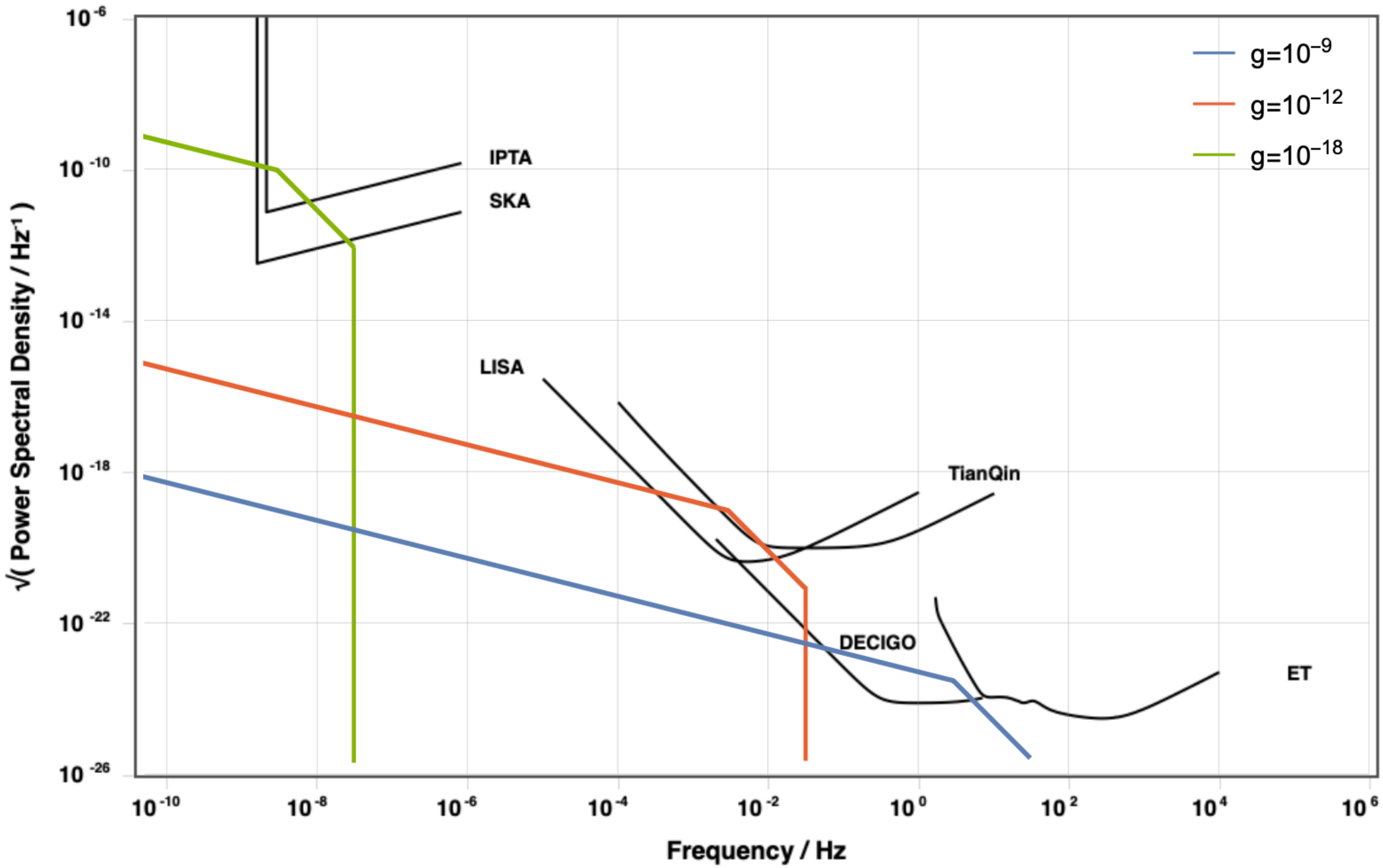
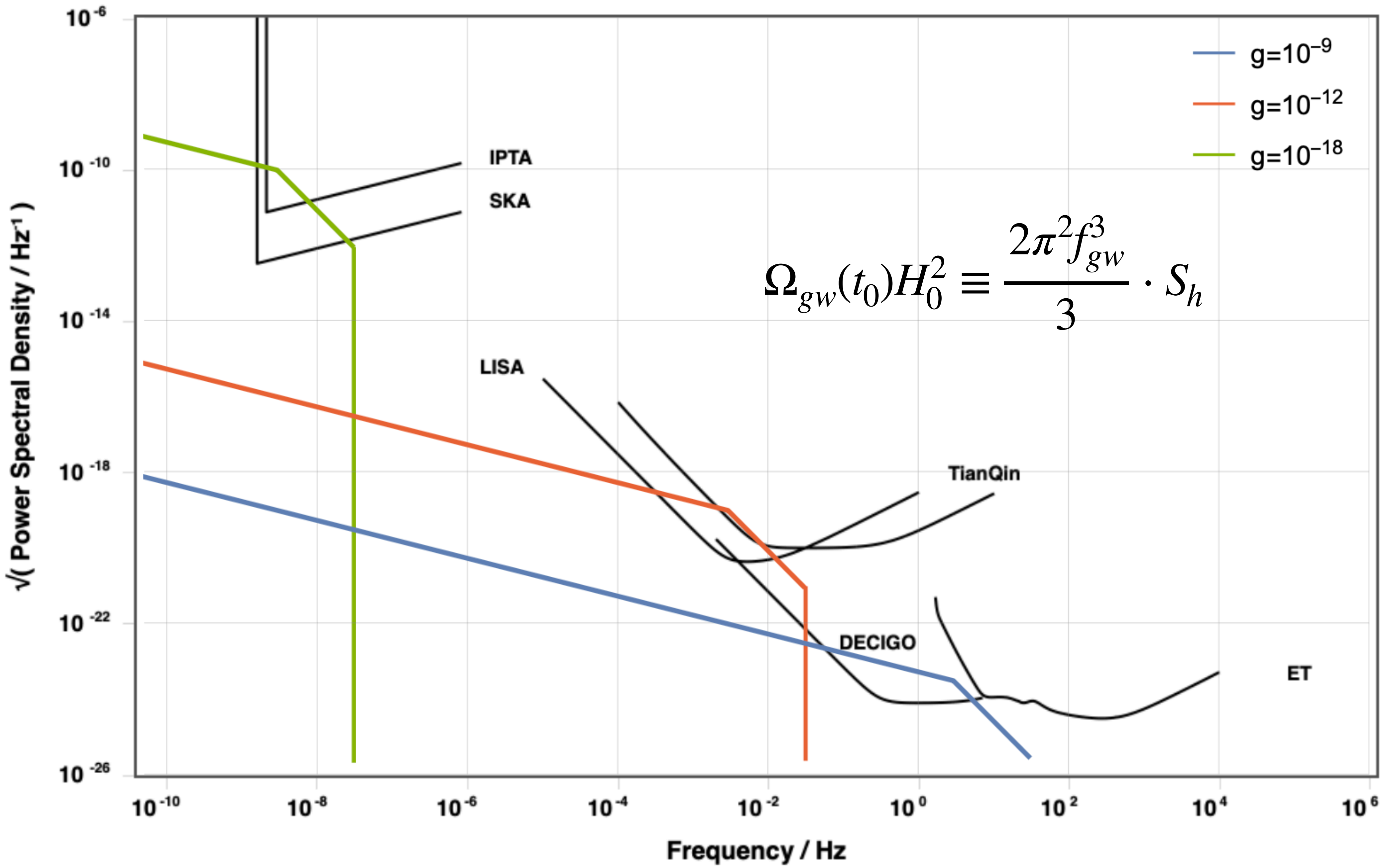


$\Omega_{gw}(IR) \sim f^2$ $\Omega_{gw}(UV) \sim f^{-1}$ Cutoff $\ell = (\lambda/2)^{-1/2}\eta^{-1}$



$$\Omega_{gw}(t_0)H_0^2 \equiv \frac{2\pi^2 f_{gw}^3}{3} \cdot S_h$$

$\Omega_{gw}(IR) \sim f^2$ $\Omega_{gw}(UV) \sim f^{-1}$ Cutoff $\ell = (\lambda/2)^{-1/2}\eta^{-1}$

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