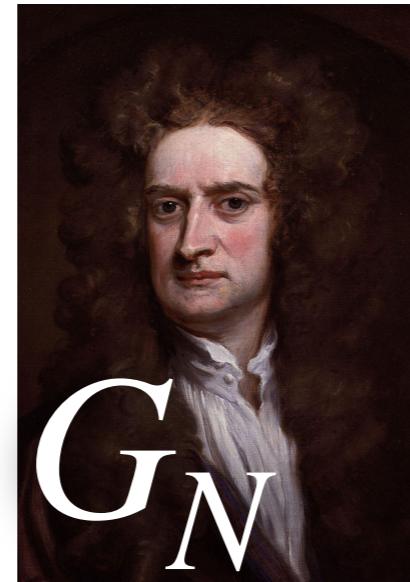
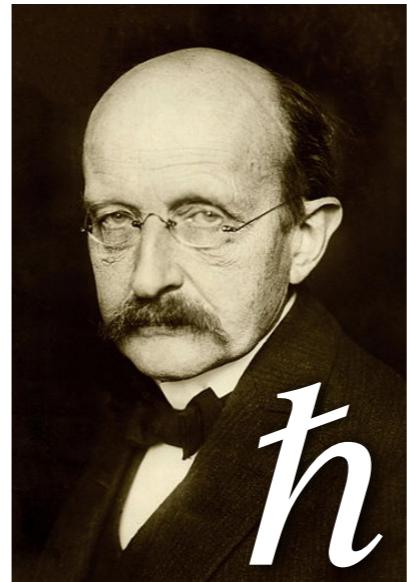


# Frozen Axions & Global Dynamics



for  
&



*Alexander Vikman*

07.02.2023



Institute of Physics  
of the Czech  
Academy of Sciences

CEICO

# Losing the trace to find dynamical Newton or Planck constants

**Pavel Jiroušek,<sup>a,b</sup> Keigo Shimada,<sup>c</sup> Alexander Vikman<sup>a</sup>  
and Masahide Yamaguchi<sup>c</sup>**

<sup>a</sup>CEICO — Central European Institute for Cosmology and Fundamental Physics,  
FZU — Institute of Physics of the Czech Academy of Sciences,  
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<sup>b</sup>Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University,  
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JCAP04(2021)028

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## Global Dynamics for Newton and Planck

arXiv 2107.09601

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- $\Lambda$ ,  $G_N$  and even  $\hbar$  as global degrees of freedom can be *frozen axions* for a confined Yang-Mills / QCD.
- The origin of the values of these global degrees of freedom should be in quantum cosmology - they are remnants of the BIG BANG - ideal “Landscape” for poor people

# What a Strange Theory!

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# $\Lambda$ as a global dynamical degree of freedom

*first way to lose the trace*

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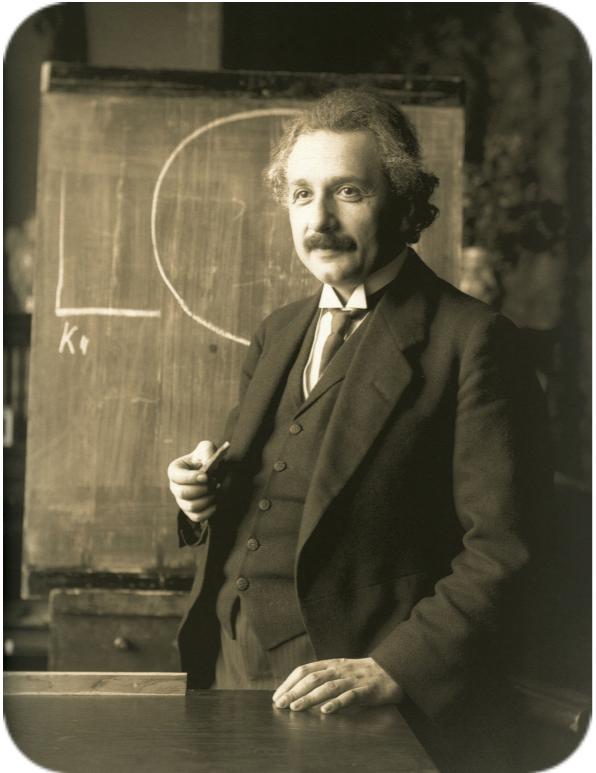
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IN: Königlich Preußische Akademie der Wissenschaften (Berlin). Sitzungsberichte (1919): 349–356.

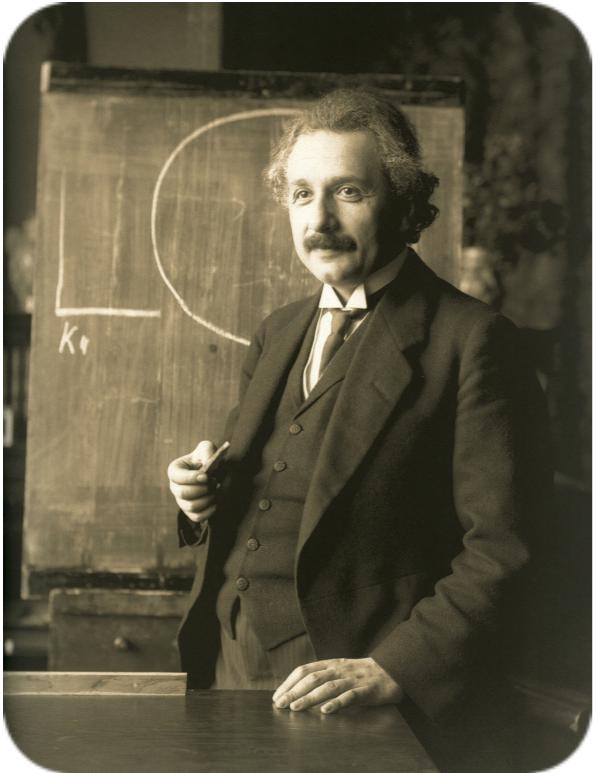
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Von A. EINSTEIN.

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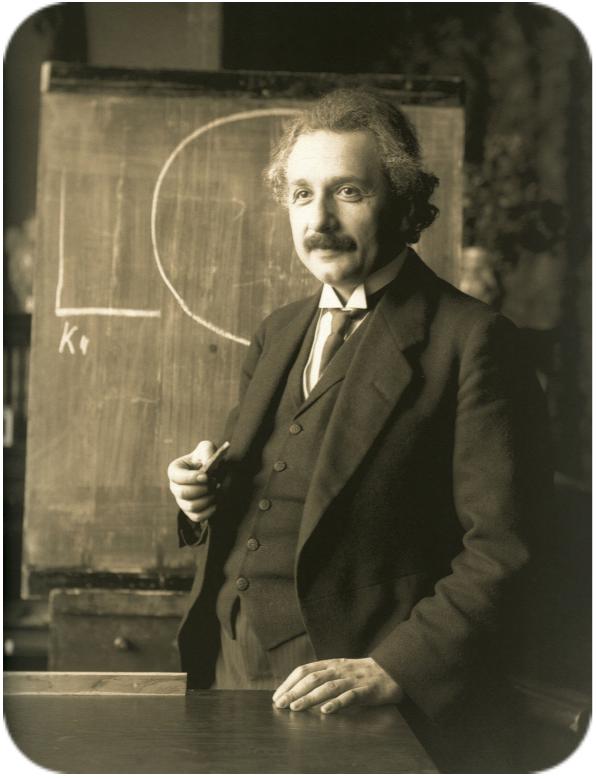
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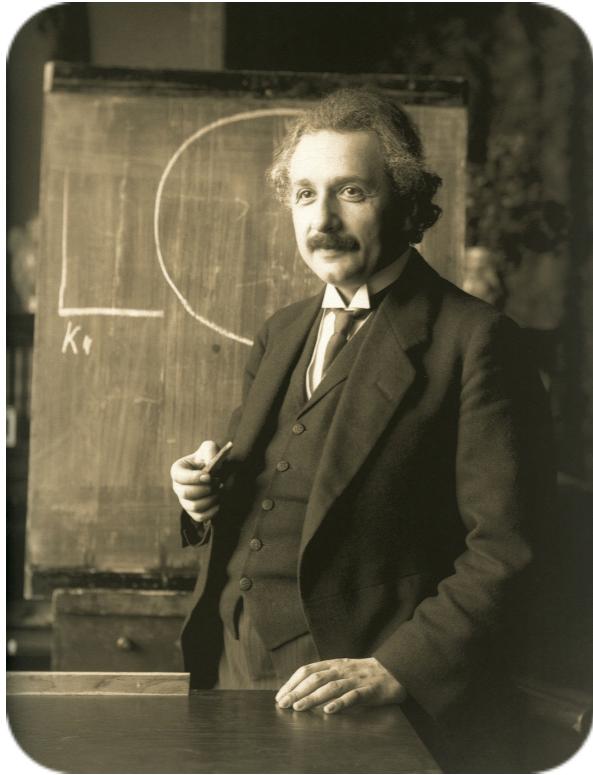
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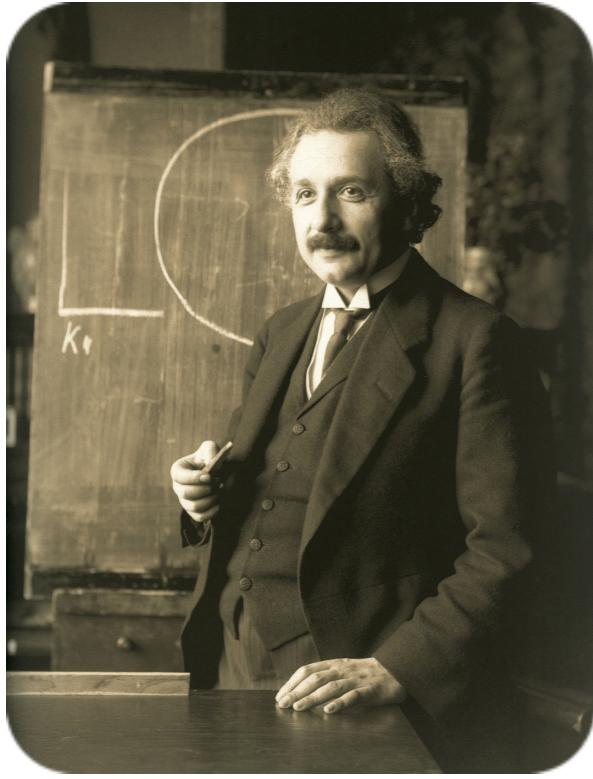


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$\Lambda$  is merely an integration constant!

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known under the name “unimodular” gravity

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Let's make  $f(x)$  internal / dynamical function, which would still be irrelevant, but save general covariance!

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- Scale invariance, as there is no fixed scale in the action

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four volume of space time     $\mathcal{T}(t_2) - \mathcal{T}(t_1) = \int_{t_1}^{t_2} dt \int d^3\mathbf{x} \sqrt{-g}$

Four-volume of spacetime is  
canonically conjugated  
to the cosmological constant



Heisenberg uncertainty relation

$$\delta\Lambda \times \delta \int_{\Omega} d^4x \sqrt{-g} \geq 4\pi \ell_{Pl}^2$$

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Global Solution to Global Problem!

actually  $\delta\epsilon_\lambda \simeq \epsilon_r$  close to final singularity,  
dS avoids “end of time”!

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*Stevens Institute of Technology*

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DAVID FINKELSTEIN\*

*Belfer Graduate School of Science*

*Yeshiva University*

*New York, New York 10033*

(Received 13 January 1971)

# Axionic Cosmological Constant

## Cosmological Constant and Fundamental Length

*In usual formulations of general relativity, the cosmological constant  $\Lambda$  appears as an inelegant ambiguity in the fundamental action principle. With a slight reformulation,  $\Lambda$  appears as an unavoidable Lagrange multiplier, belonging to a constraint. The constraint expresses the existence of a fundamental element of space-time hypervolume at every point. The fundamental scale of length in atomic physics provides such a hypervolume element. In this sense, the presence in relativity of an undetermined cosmological length is a direct consequence of the existence of a fundamental atomic length.*

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Chern-Simons Current  $C^\mu$   
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# Axionic Cosmological Constant, Comment II

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Hammer, Jiroušek, Vikman arXiv:2001.03169

PHYSICS REPORTS (Review Section of Physics Letters) 104, Nos. 2–4 (1984) 143–157. North-Holland, Amsterdam

## Foundations and Working Pictures in Microphysical Cosmology

Frank WILCZEK

I would like to briefly mention one idea in this regard, that I am now exploring. It is to do something for the  $\Lambda$ -parameter very similar to what the axion does for the  $\theta$ -parameter in QCD, another otherwise mysteriously tiny quantity. The basic idea is to promote these parameters to dynamical variables, and then see if perhaps small values will be chosen dynamically. In the case of the

# Chern-Simons Current

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introduce the shifts  $C^\mu \rightarrow C^\mu + \epsilon^\mu \quad \nabla_\mu \epsilon^\mu = 0$

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$$g^2(q) = \frac{48\pi^2}{(11n - 2f) \log(q^2/\Lambda_{QCD}^2)}$$

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# $G_N$ as a global dynamical degree of freedom

*Second way to lose the trace*

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Side product:  $\hbar$  as global  
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$\bar{G}_N$  is merely an integration constant!

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# Heisenberg uncertainty relation

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$$\frac{\delta G_N}{G_N} \times \frac{\delta \int_{\mathcal{V}} d^4x \sqrt{-g} \, R}{\ell_{Pl}^2} \geq 8\pi$$

## Heisenberg uncertainty relation

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$$\frac{\delta G_N}{G_N}\times\frac{\delta\int_{\mathcal{V}}d^4x\sqrt{-g}\,R}{\ell_{Pl}^2}\geq8\pi$$

$$\frac{\delta \ell_{Pl}}{\ell_{Pl}}\times\frac{\delta\int_{\mathcal{V}}d^4x\sqrt{-g}\,R}{\ell_{Pl}^2}\geq4\pi$$

# Minimal Quantum Fluctuations of $G_N$

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# Frozen Axion for $G_N$

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$$S\left[g,\mathcal{A},\nu\right]=\int\!d^4x\sqrt{-g}\left[-\frac{1}{2}\nu^2\,R+\frac{1}{2}\,(\partial\nu)^2+\frac{\nu}{f_\alpha}\widetilde{\mathcal{F}}_{\gamma\sigma}\widetilde{\mathcal{F}}^{\gamma\sigma}-V_\alpha(\nu)\right.$$

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Again formal limit / “confinement”  $g \rightarrow \infty$

New vacuum energy density  $V_\alpha(\nu)$

# Changing Matter via Henneaux–Teitelboim

$$S\left[g,\beta,L,\Phi_m\right]=\int d^4x \sqrt{-g}\,\beta\left(\mathcal{L}_m-\nabla_\lambda L^\lambda\right)$$

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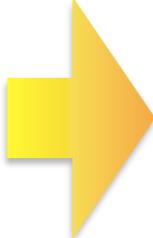
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Effective Planck Quantum  $\bar{\hbar} = \frac{\hbar}{\beta}$  Effective Newton Constant  $\bar{G}_N = \beta G_N$

Planck length  $\ell_{Pl} = \sqrt{\hbar G_N}$  remains invariant!

# Action Conjugated!

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$$\int_{\mathcal{V}} d^4x \sqrt{-g} \mathcal{L}_m > \delta \int_{\mathcal{V}} d^4x \sqrt{-g} \mathcal{L}_m$$



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# Unimodular, Unicurvature and Unimatter

for the globally dynamical  $\Lambda$

Unimodular  $S[g, \Lambda] = \int d^4x \Lambda \left( 1 - \sqrt{-g} \right)$

Henneaux-Teitelboim construction with fixed  $W^\mu = \delta_t^\mu \frac{t}{\sqrt{-g}}$

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each of these three constraints could be a gauge condition... but they yield dynamics!

Dynamics is the same as without fixing  $W^\mu$ ,  $C^\mu$  or  $L^\mu$  !

*Thanks a lot for attention!*

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Jiroušek, Vikman (2018)

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$$V^\mu = \Omega^{-4}(x) V'^\mu$$

# Action

$$S_g\left[h,V\right]=-\frac{1}{2}\int\!d^4x\sqrt{-h}\left[\left(\nabla_{\alpha}^{h)}V^{\alpha}\right)^{1/2}R\left(h\right)+\frac{3}{8}\cdot\frac{\left(\nabla_{\mu}^{h)}\nabla_{\alpha}^{h)}V^{\alpha}\right)^2}{\left(\nabla_{\sigma}^{h)}V^{\sigma}\right)^{3/2}}\right]\;.$$

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$$S_g [h, V] = -\frac{1}{2} \int d^4x \sqrt{-h} \left[ \left( \nabla_{\alpha}^{h)} V^{\alpha} \right)^{1/2} R(h) + \frac{3}{8} \cdot \frac{\left( \nabla_{\mu}^{h)} \nabla_{\alpha}^{h)} V^{\alpha} \right)^2}{\left( \nabla_{\sigma}^{h)} V^{\sigma} \right)^{3/2}} \right].$$

# Equations of motion

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$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$