# Observers in black hole spacetimes: The inside story

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River approach to BH, flows and peculiar velocities of particles, especially under horizon

Application to particle collisions, new version of high energy collisions

Strategies of motion under horizon: either to live long or to see much

Blue/red shifts under horizon

Frames under horizon, limiting transitions from one frame to another

# Frames and observers: geometry probed by particles

Regular coordinates for Schwarzschild

Eddington-Finkelstein, Kruskal-Szekeres,Lemaitre, etc.

Unification: Fomin 1968 (local Lorentz transformation)
Martel and Poisson 2001
Bronnikov et al 2012
Lemos and Silva 2021
Toporensky and OZ 2022

Dynamics and geometry: physical realization of frame

Two faces: parameter e of transformation and energy/momentum of particle

Frames and limiting transition under horizon

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$dt = \frac{e_0}{f}\sqrt{F}\, dT - \frac{\sqrt{G}\, P_0}{f}\, d\rho$$

$$P_0 = \sqrt{e^2{}_0 - f}$$

$$dr = e_0 \sqrt{G}\, d\rho - \sqrt{F}\, P_0\, dT$$

$$e_0 = \frac{1}{\sqrt{1 - V^2}}$$

$$ds^2 = -F dT^2 + G d\rho^2 + r^2 d\Omega^2$$

If we put F=1 and will use previous radial coordinate, we have

$$d\tilde{t} = e_0 dt + \frac{P_0 dr}{f},$$

instead of T we use in this particular case $\tilde{t}$

$$ds^2 = -d\tilde{t}^2 + 2\frac{d\tilde{t}drV}{e_0} + \frac{dr^2}{e^2{}_0}$$

Generalization of GP metric

$e_0 \to 0$ Singular transformation. Both coordinates fail to be independent

Impossible to take limit in this metric directly

The proper distance grows indefinitely, metric becomes singular

## Another synchronous frame under horizon

T=-r, t=y, g=- f>0 Novikov's approach

$$ds^2 = -\frac{dT^2}{g} + g dy^2 + T^2 d\Omega^2.$$

$$ds^2 = -d\hat{t}^2 + g dy^2 + T^2(\hat{t}) d\Omega^2.$$

## What happens in the limit $e_0 \to 0$

Singular transformation. Term with $d\rho$ drops out

Works under horizon only, f<0 $P_0 = \sqrt{e^2{}_0 - f}$

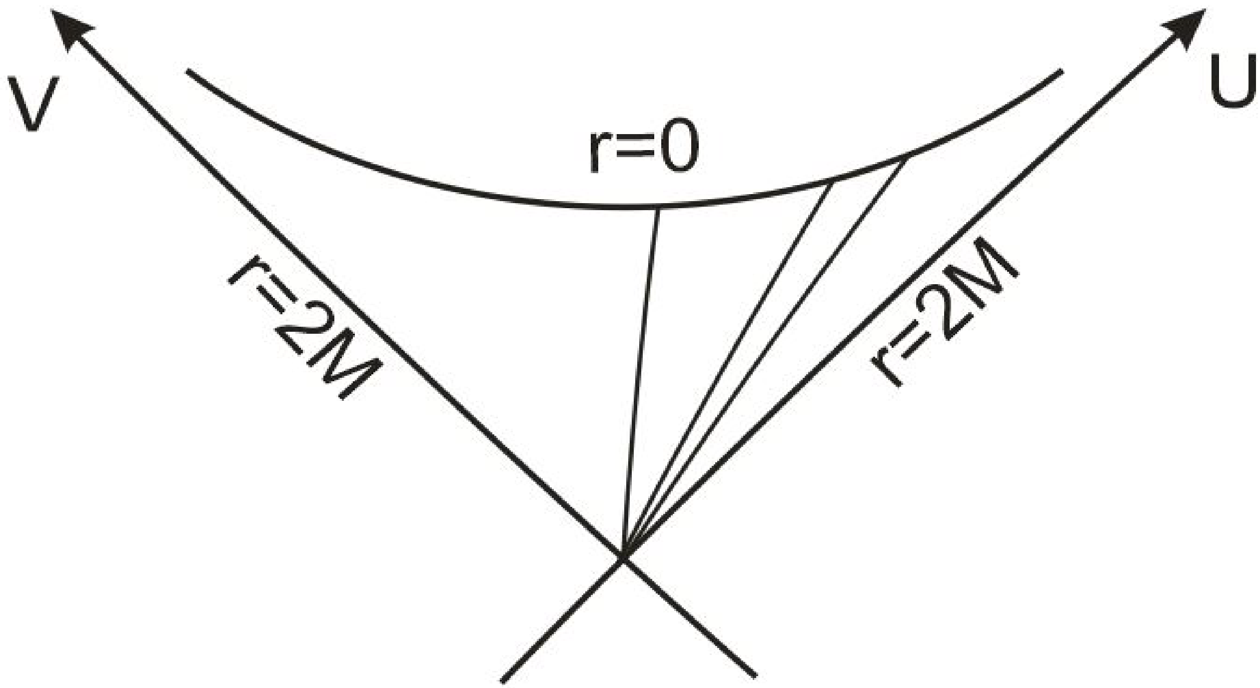
For a synchronous metric the limit is allowed, provided we make rescaling

$$\rho = e_0 \tilde{\rho}$$

$$ds^2 = -dT^2 + g(r(T))d\tilde{\rho}^2 + r^2 d\Omega^2 \qquad g = -f > 0$$

$$T = -\int^{r} \frac{d\tilde{r}}{g(\tilde{r})}$$

Novikov presentation,
Particular case of Kantowski-Sachs cosmology




Time-like geodesics for e_0=0

Another version: modification of GP system under horizon

From $\tilde{t}, r$ to $\tilde{t}, t$

$$ds^2 = -\frac{g}{P^2{}_0} d\tilde{t}^2 + \frac{g^2 dt^2}{P^2{}_0} + 2\frac{e_0 g dt d\tilde{t}}{P^2{}_0} + r^2(t,\tilde{t}) d\Omega^2$$

$$ds^2 = -d\tilde{t}^2 + \frac{g^2}{P^2{}_0}(dt + \frac{e_0 d\tilde{t}}{g})^2 + r^2(t,\tilde{t}) d\Omega^2$$

Under horizon t is spacelike, so we have 1 spacelike and 1 timelike coordinates
In GP system two timelike under horizon.
Nondiagonal term defines flow velocity $-\frac{e_0}{g}$

can be interpreted as a velocity with respect to frame where fiducial observer has $e_0 = 0$
Metric dual to GP, arranged for region under horizon, has smooth limit to
$e_0 = 0$

General radially moving references frames in the black hole background

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 (d\theta^2 + d\varphi^2 \sin^2\theta).$$

$$d\tilde{t} = e_0 dt + \frac{dr}{f} P_0$$

$$P_0 \equiv \sqrt{e_0^2 - f}\,.$$

# Velocities and their behavior

Frame attached to free falling observer

$$e_0$$

Energy of observer

$$e$$

energy of particle

$$V^{(1)} = \frac{P_0 e - P e_0}{e_0 e - P P_0}$$

$$P_0 \equiv \sqrt{e_0^2 - f}$$

$$V^{(3)} = \frac{\mathbf{L}\sqrt{-f}}{rP}$$

## Limiting transitions. Horizon limit

$e > 0$

$$V^{(1)} \to \frac{1 - e^2 + \frac{L^2}{r_+^2}}{1 + e^2 + \frac{L^2}{r_+^2}}$$

$e_0 = 1$

$$V^{(3)} \to \frac{2Le}{r_+ (1 + e^2 + \frac{L^2}{r_+^2})}$$

Static observer: $V_{st}^{(1)} \to 1$ $V_{st}^{(3)} \to 0$ Universal properties

Particle hits horizon radially

$$V^{(3)} = V_{st}^{\ (3)} \frac{\sqrt{f}}{1 + vV^{(1)}_{\ \ st}} = V_{st}^{\ (3)} \frac{\sqrt{1 - v^2}}{1 + vV^{(1)}_{\ \ st}}$$

$$V^{(1)}_{\ \ st} \to -1$$

Big Lorentz boost compensates small angular velocity in static frame.
As a result, in Lemaitre frame component $V^{(3)}$ is finite and nonzero

In a similar way, radial velocity can take any value.

All this is a bright manifestation of the known relativistic effect according to which a vector, not collinear to the direction of motion, rotates under a Lorentz transformation.

Vicinity of singularity

$$r \to 0, f \to \infty$$

This means that any initial differences in radial motion for different particles disappear near the singularity, and the radial motion of any particle tends to the motion of the frame.

Pure radial case

$$V^{(1)} \approx \frac{1-\varepsilon}{\sqrt{|f|}} \to 0$$

Non-radial case

$$V^{(1)} \approx \frac{1}{\sqrt{|f|}} \to 0$$

Nonradial motion with nonzero L

The situation with angular velocity is opposite. If

$$L \neq 0 \qquad V^{(3)} \to \pm 1$$

Pure radial motion appears to be unstable—an arbitrary small deviation grows infinitely and results in an ultrarelativistic motion in an angular direction. If initially the directions of the vectors L are distributed randomly, the corresponding particles have mutual ultrarelativistic relative velocities near a singularity.

Horizon limit in general

If $e$ and $e_0$ have the same sign

$$V^{(1)} \to V_H{}^{(1)} = \frac{e_0^2(1+\frac{L^2}{r_g^2}) - e^2}{e_0^2(1+\frac{L^2}{r_g^2}) + e^2}$$

$$V^{(3)} \to V_H{}^{(3)} = \frac{2ee_0 L}{r_g[e^2 + e_0^2(1+\frac{L^2}{r_g^2})]}.$$

For different signs

$$|V^{(1)}| \to 1$$

$$V^{(3)} \to 0$$

## Near singularity

$$L \neq 0 \qquad V^{(3)} \to sign\mathbf{L} = \pm\mathbf{1}$$

$$V^{(1)} \approx \frac{e_0}{P_0} \approx \frac{e_0}{\sqrt{-f}} \to 0$$

pure radial motion appears to be unstable—an arbitrary small deviation grows infinitely and results in an ultrarelativistic motion in an angular direction. If initially the directions of the vectors *L* are distributed randomly, the corresponding particles have mutual ultrarelativistic relative velocities near a singularity.

If $e_0 = 0$

$$V^{(1)} = -\frac{e}{P} \approx -e\frac{r}{\sqrt{-f}\,|\mathbf{L}|} \to 0$$

$$L = 0$$

$$V^{(3)} = 0$$

$$V^{(1)} \approx \frac{e_0 - e}{\sqrt{-f}} \to 0$$

## Classification of frames

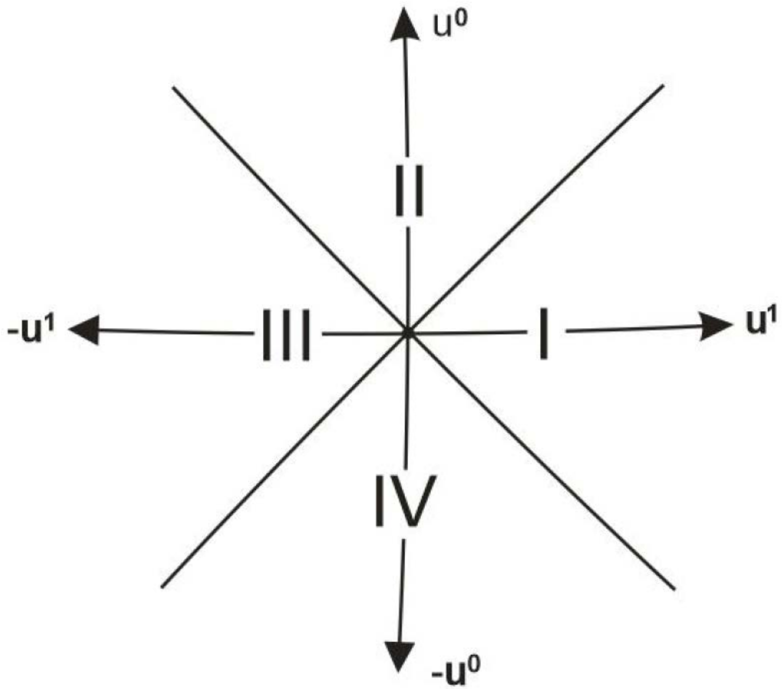



FIG. 1: The Kruskal diagram for the Schwarzschild metric

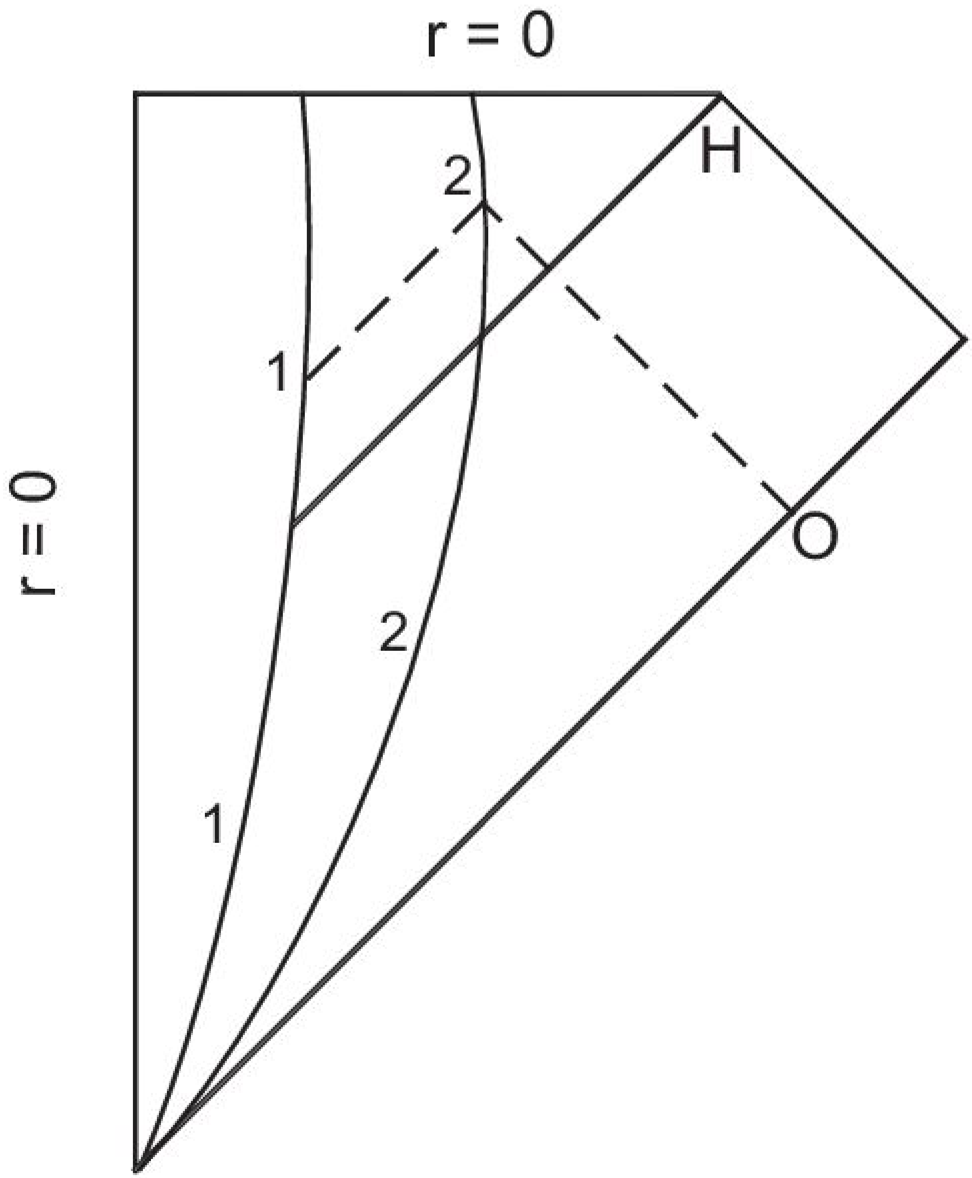
| Frame | Regions covered |
|---|---|
| $(C, +)$ | I, II |
| $(C, -)$ | III, II |
| $(E, +)$ | IV, I |
| $(E, -)$ | IV, III |

E – expanding, C – contracting, sign of e_0 indicated

# MOTION WITH ANGULAR MOMENTUM AND HORIZON ASYMPTOTICS

| Frame / Particle | $e > 0,\ P > 0$ | $e < 0,\ P > 0$ | $e > 0,\ P < 0$ | $e < 0,\ P < 0$ |
|---|---|---|---|---|
| $e_0 > 0,\ P_0 > 0$ | $V_H^{(1)};\ V_H^{(3)}$ | $+1;\ 0$ | $+1;\ 0$ | – |
| $e_0 < 0,\ P_0 > 0$ | $-1;\ 0$ | $-V_H^{(1)};\ V_H^{(3)}$ | – | $-1;\ 0$ |
| $e_0 > 0,\ P_0 < 0$ | $-1;\ 0$ | – | $-V_H^{(1)};\ V_H^{(3)}$ | $-1;\ 0$ |
| $e_0 < 0,\ P_0 < 0$ | – | $+1;\ 0$ | $+1;\ 0$ | $V_H^{(1)};\ V_H^{(3)}$ |

Straightforward approach
What observer will see?



## Incomplete treatment and misconceptions

In their popular book, Gurevich, L. E., Gliner, E. B.: General relativity after Einstein. Moscow, (1972) (In Russian) authors wrote that when the world line of an observer approaches the singularity, he sees a surrounding world to fade

Hamilton: exponential redshift *inside* the event horizon
Surface of star does not approach would-be (past) horizon, as was supposed
In Hamilton, A.J.S., Polhemus, G.: Stereoscopic visualization in curved spacetime: seeing deep inside a black hole. New J. Phys. **12**, 123027 (2010

## Radial motion

## Main scenarios

(i) Photon enters region under horizon from outside $q = |\omega_0| > 0$

(ii) Two-step character

At first, a photon is emitted by observer 1 already under the horizon. Further, it is received by observer 2. In doing so, the angular momentum *l* of a photon does not change during the travel between two events. We assume that observer 2 falls from infinity and has $p_2 > 0$. The role of observer 1 can be played, say, by a star collapsing surface (then, $p_1 > 0$) or some particle from dust cloud that is able to radiate. We also assume that a photon is sent to meet observer 2, so absorption has a kind of head-on collision: $p_2 > 0$, $q < 0$.

p radial momentum of massive particle per unit mass, L angular momentum
q radial momentum of photon, l angular momentum

**Table 1** Behavior of the frequency near the horizon

| | $\omega$ |
|---|---|
| $pq > 0$ | Finite |
| $pq < 0$ | Infinite |
| $q = 0, p \neq 0$ | Infinite |
| $p = 0, q \neq 0$ | Infinite |
| $p = q = 0$ | Finite |

## Vicinity of horizon

Special case of emission

Finite shift
$$\frac{\omega_2}{\omega_1} = \frac{V_1}{V_2}$$

V Kruskal coordinate along the future horizon

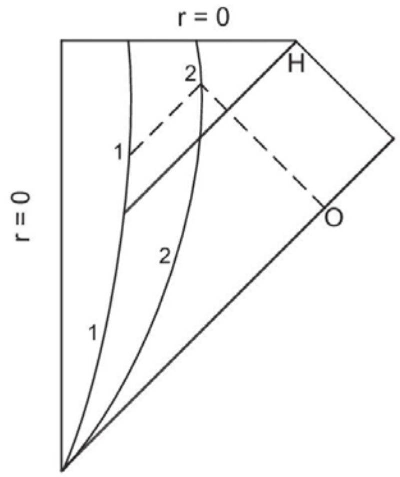


**Table 2** Behavior of the frequency near the singularity

| | $\omega$ |
|---|---|
| $\mathcal{L}l > 0$ | Finite nonzero |
| $\mathcal{L}l < 0$ | Infinite blueshift |
| $\mathcal{L} = 0, l \neq 0$ | Infinite blueshift |
| $\mathcal{L} \neq 0, l = 0$ | Infinite blueshift |
| $\mathcal{L} = 0 = l$ | Infinite redshift |

Particular case with radical changes
From (almost) infinite redshift near horizon
to infinite blueshift at singularity

Let a surface of a collapsing star radiate pure radial photons, l = 0.
Assuming that an observer has L = 0 and taking into account Table 2, we come to the conclusion that in the course
of his travel, the behavior of frequency ω2 changes radically. There is a strong redshift near the horizon and in an intermediate region but, as the singularity is approached, this changes to unbounded blueshift!

**Table 3** Behavior of $\frac{\omega_2}{\omega_1}$ for the cases when a photon is emitted near the horizon and is absorbed near the singularity

| | $p_1 > 0$ | $p_1 = 0$ |
|---|---|---|
| $l = 0, \mathcal{L}_2 \neq 0$ | $\frac{1}{\sqrt{\lvert T_2 \rvert}}$ | $\frac{1}{\sqrt{\lvert T_2 \rvert}}$ |
| $l = 0, \mathcal{L}_2 = 0$ | $\sqrt{\lvert T_2 \rvert}$ | $\sqrt{\lvert T_2 \rvert}$ |
| $l \neq 0, \mathcal{L}_2 l > 0$ | | finite |
| $l \neq 0, \mathcal{L}_2 l < 0$ | | $\frac{1}{T_2^2}$ |
| $l \neq 0, \mathcal{L}_2 = 0$ | | $\frac{1}{\lvert T_2 \rvert}$ |

$T_2$ is time near singularity for observer

**Table 4** Properties of $\frac{\omega_2}{\omega_1}$ for different types of scenario with $l = 0, \mathcal{L}_2 \neq 0$

| Scenario | Vicinity of horizon | Intermediate | Singularity |
|---|---|---|---|
| i, ii (typical) | Finite | Finite | Unbounded blueshift |
| ii (special) | unbounded redshift | unbounded redshift | Unbounded blueshift |

There is no qualitative difference between scenarios i and ii (typical). But there is an essential difference between subcases ii (typical) and ii (special).

## Relation with BSW effect (indefinite growth of energy In CM frame)

Schwarzschild – no BSW effect, provided both

$$e > 0 \qquad e_0 > 0$$

R - region White holes (Grib and Pavlov): possible

If particle comes from mirror universe,

$$e_0 < 0$$

BSW possible but in weak version

Lemaitre time to reach horizon is infinite for one particle and finite for another. Bifurcation point, its vicinity.

Frame and particle dynamics

$$ds^2 = -\frac{f}{e_0^2} d\tilde{t}^2 + \frac{2d\tilde{t}dr}{e_0^2} P_0 + \frac{dr^2}{e_0^2} + r^2 d\omega^2.$$

$$d\rho = \frac{dr}{P_0} + d\tilde{t}$$

Generalization of Lemaitre

$$ds^2 = -d\tilde{t}^2 + \frac{P_0^2}{e_0^2} d\rho^2 + r^2 d\omega^2.$$

$$\frac{d\rho}{d\tau} = \frac{e_0 (eP_0 - e_0 P)}{f}$$

If $e = e_0$ $\rho = const$

## Redshifts/blueshifts inside a black hole

Photon angular momentum l=0

$$\frac{\omega}{\omega_0} = \frac{1+V^{(1)}}{(1+v)\sqrt{1-v^2{}_p}}$$

If a motion of observer is also radial, $V^{(1)} = V = v_p,$

$$\frac{\omega}{\omega_0} = \frac{\sqrt{1+V}}{(1+v)\sqrt{1-V}}$$

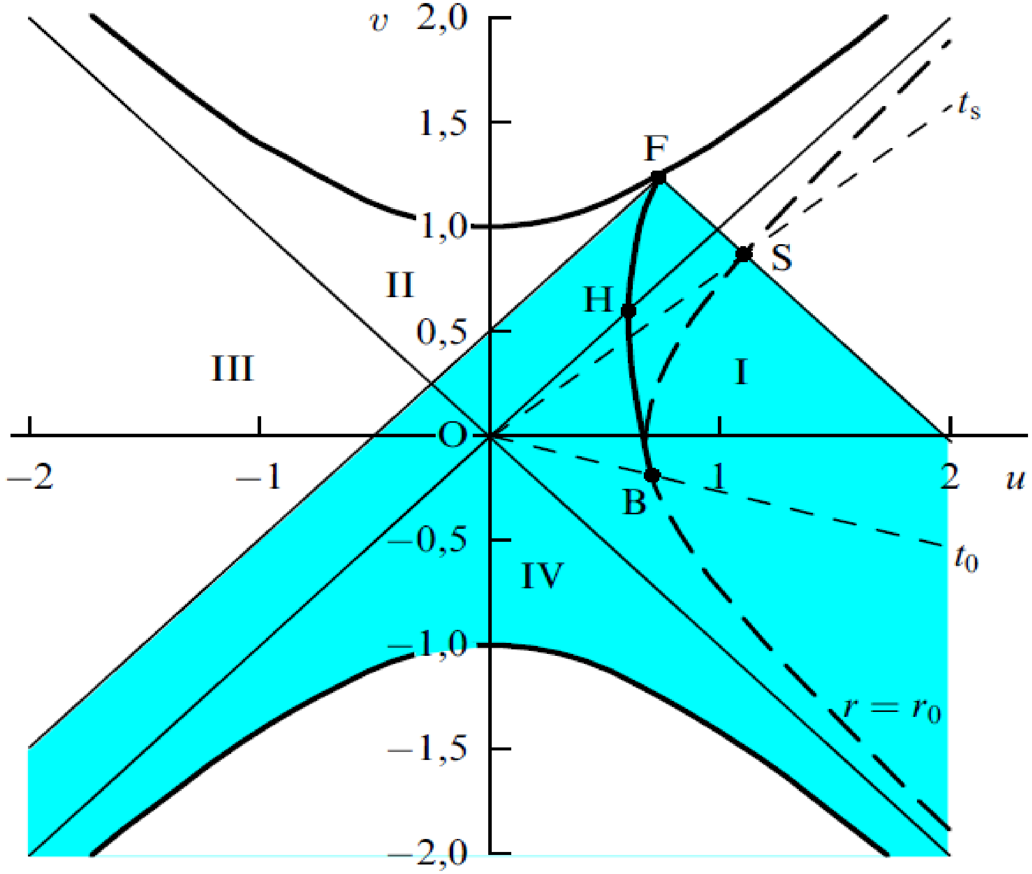
# What Can a Falling Observer See?

Proper time finite, time t of remote observer infinite. Finiteness of speed of light – only finite part of Universe is accessible to remote observer (Krasnikov 2008, Grib and Pavlov 2009)

Assumptions
1) Observer falls from right region R1
2) It moves along geodesics

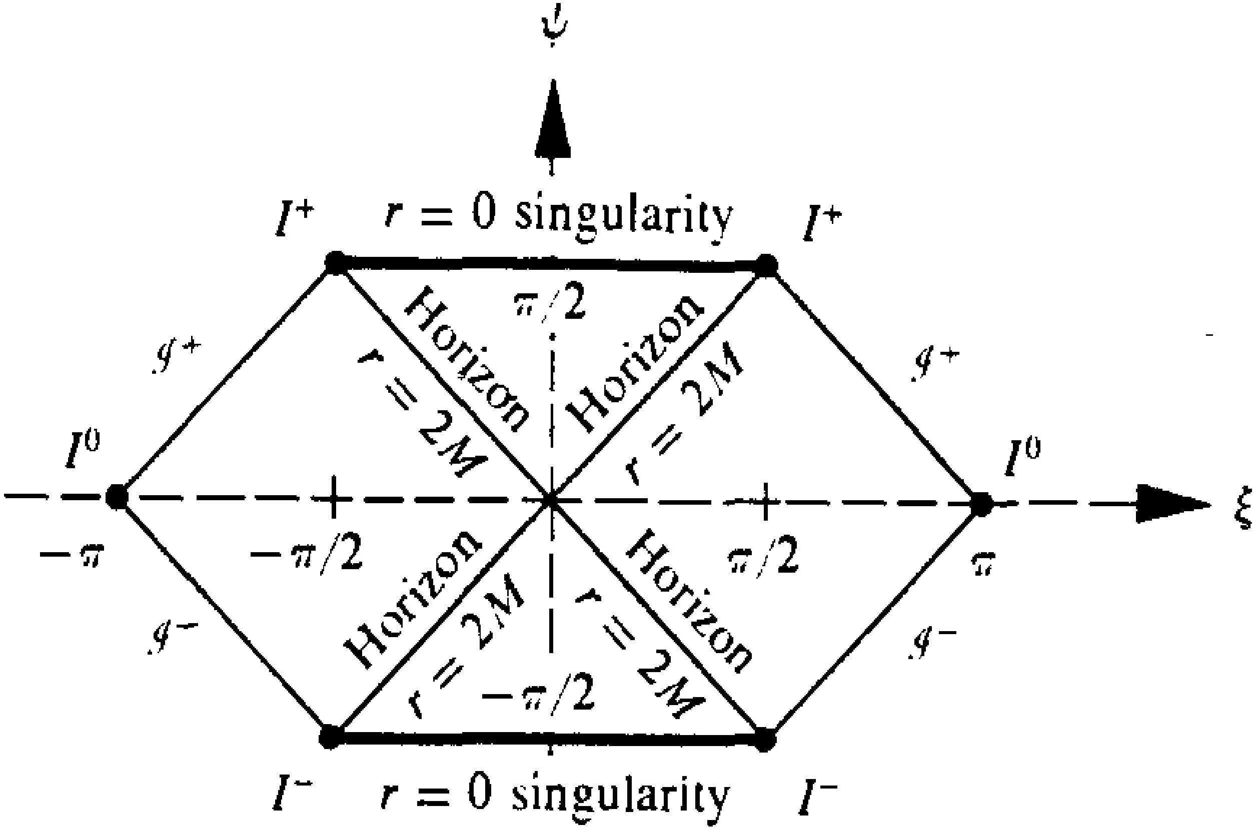
But now an astrounaut should switch from E>0 to E=0. Engine!

Grib and Pavlov 2009



Another task: How to maximize lifetime after crossing horizon?

G. F. Lewis and J. Kwan, *Publ. Astron. Soc. Aust.* **24** (2007) 46. Numerics



Equations of motion for free particle give us

$$\tau = \int_{r_1}^{r_+} \frac{dr}{Z}, Z = \sqrt{\varepsilon^2 + g(1 + \frac{L^2}{m^2 T^2})}$$

Hitting the singularity, $r_1 = 0$

We want to maximize the proper time: L=E=0

$$\tau_{\max} = \frac{\pi}{2} r_+$$

What changes, if particle moves not freely? $\varepsilon = \varepsilon(T)$

But structure of equations remains, the formula for proper time remains. The limit indicated above remains.

Naive first principle “explanation” based on the minimal action principle (the curve maximizing a proper time between two fixed points is a geodesic) is not applicable here since the black hole singularity is not a spacetime point, it is hypersurface. It seems that these ideas became some kind of folklore. The fact that the proper time of the fall has its maximum at the geodesic (so, if a falling object is equipped by an engine, it is better to switch it off at some point!) is often considered as a rather counter-intuitive, since naïve approach prompts to decelerate, whereas the true best strategy requires to “give up” at some point.

More general question. How to maximize proper time from initial point O to spacelike hypersurface $\Sigma$ ?
Let us introduce a synchronous frame such that Σ is described by equation $\tilde{t} = \tilde{t}_0$ =const

$$ds^2 = -d\tilde{t}^2 + \gamma_{ik} dx^i dx^k$$

We can introduce a three-dimensional velocity of a particle according to

$$v^i = \frac{dx^i}{d\tilde{t}}$$

Along trajectory $$ds^2 = d\tilde{t}^2 (1 - v^2)$$

$$\tau = \int d\tilde{t} \sqrt{1 - v^2}$$

We take v=0. Trajectory is geodesic, x=const.

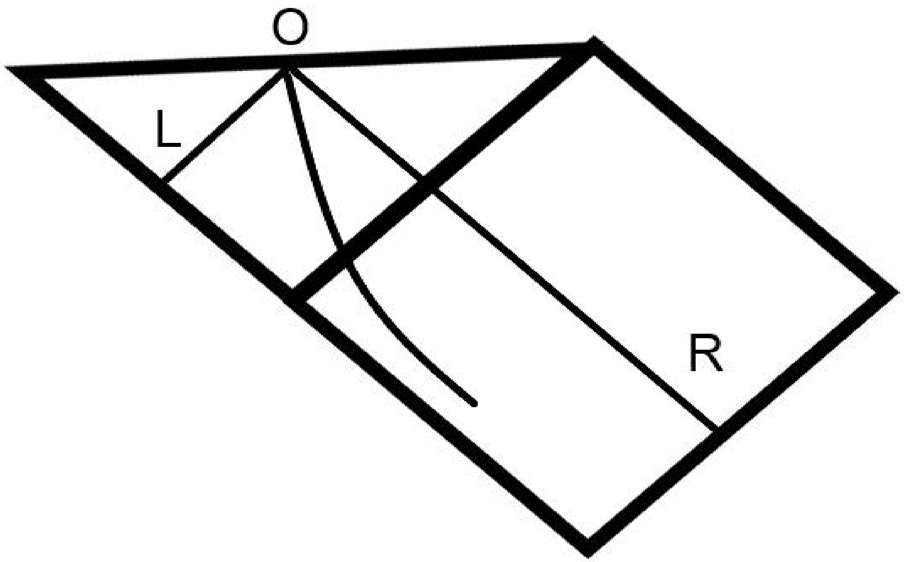
Strategies of motion under the black hole horizon

$$ds^2 = -fdt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

Under horizon

$$r = -T, t = y, f = -g$$

$$ds^2 = -\frac{dT^2}{g} + gdy^2 + T^2 d\Omega^2$$




Trajectory of an observer inside a black hole and his individual horizon

$$\tilde{t}_L = \tilde{t}_s - r + 2\sqrt{r_+ r} + 2r_+ \ln \frac{\sqrt{r_+} - \sqrt{r}}{\sqrt{r_+}}$$

Light geodesics

$$\tilde{t}_R = \tilde{t}_s - r - 2\sqrt{r_+ r} + 2r_+ \ln \frac{\sqrt{r_+} - \sqrt{r}}{\sqrt{r_+}}$$

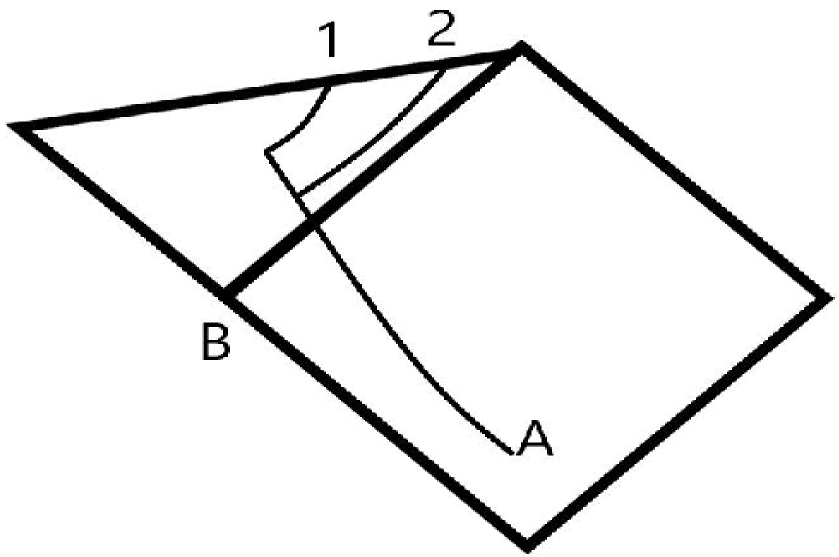
Lemaitre time from point 1 to singularity for timelike observer with E=0

$$\tilde{t} = r_{+} \ln \frac{\sqrt{r_{+}} + \sqrt{r_{1}}}{\sqrt{r_{+}} - \sqrt{r_{1}}} - 2\sqrt{r_{+} r}$$

When $r_1 \to r_+$ $\tilde{t} \to \infty$

Finite for E>0

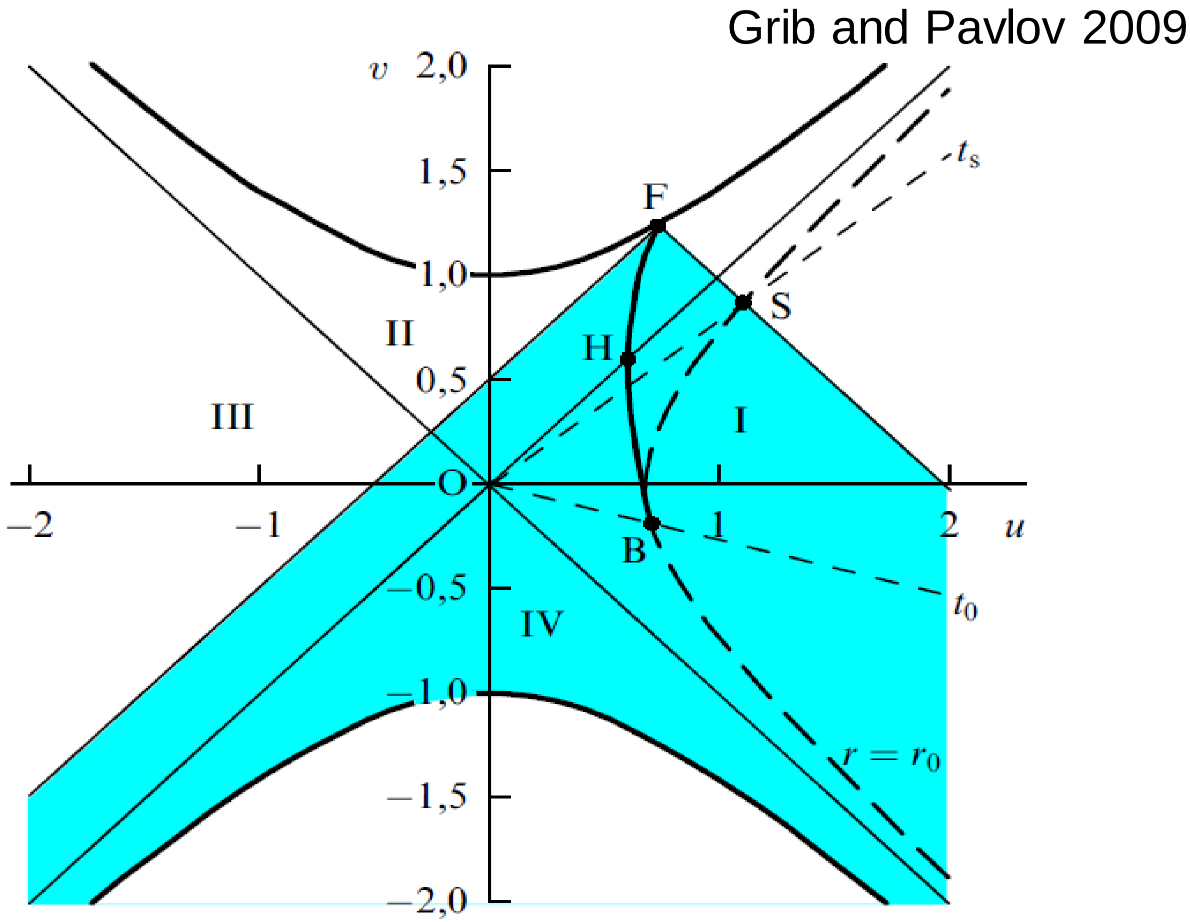
Any massive particle moving from R1 zone (our world)
into T zone along a geodesic has ε > 0. Now, let an observer come from R1, being
equipped with some engine. Let him turn it on in the point r under the horizon in
the T region in such a way that he sits on the trajectory ε = 0. He turns the engine
off afterwards, thus making a transition from the geodesic trajectory with ε > 0 to
the one with ε = 0. Then, the Lemaıtre time from r to singularity is given by the above formula




Schematic picture of trajectories of an astronaut changing his path to the $E = 0$ trajectory

Ability of engine. Ability of austronaut to survive. Bends more to horizon, sees more and more from outer Universe. Engine is more powerful – austronaut sees bigger part of Universe.

Grib and Pavlov 2009



## Two Strategies of an Astronaut

If one uses an engine near the horizon to make E=0, two goals at once: maximizing proper time till the singularity and maximizing the possible future of the universe seen during this fatal fall.
What happens if an engine is turned on deeply inside T region?
These two goals may require different strategies.

For example, suppose that the observer inside the
horizon found himself at a trajectory with $\varepsilon = 0$, but some fuel remains.
Is it
reasonable to use the fuel more? If we want to make the proper time
before hitting
singularity as large as possible, the answer is obviously “no” — the
trajectory with
$\varepsilon = 0$ is optimal. But what about the Lemaıtre time till the singularity?

More general formula for observer with any E between point 1 and singularity

$$\tilde{t} = \int_0^{r_1} \frac{dr}{v - v_p}$$

Under horizon

$$v > v_p$$

Therefore, the bigger the $v_p$, the bigger is the Lemaıtre time. So, the astronaut should use the remaining fuel — the fight against gravity makes sense! Ironically, not for the fighter — his proper time till singularity decreases while Lemaıtre time increases.

If an astronaut understands
that he/she is actually on the trajectory with $\varepsilon < 0$ and wants to achieve the maximum possible proper time, it is necessary to decrease $v_p$ in order to reach
$\varepsilon = 0$. On the contrary, such an astronaut should increase $v_p$ as much as possible
to maximize the Lemaıtre time (allowing to see more future of the outer word).
In other words, a researcher inside the horizon should pay by the time of his own life for satisfying his curiosity!

Existential question: to live long but boring life or a short long but to learn something?

Better to combine both but under Schwarzschild horizon this is impossible

## Contradictory strategies

| | Goal 1 | Goal 2 | | |
|---|---|---|---|---|
| E>0 | Decrease E to E=0 | Decrease E to E=0 | | |
| | | | | |
| E=0 | Do nothing | Increase v_p | | |
| E<0 | Decrease v_p | Increase v_p | | |

Goal 1: to make survival proper time bigger

Goal 2: to see the maximum from outer Universe

$$\varepsilon = \frac{1 - v v_p}{\sqrt{1 - v_p^2}}$$

Under horizon $$\frac{\partial \varepsilon}{\partial v_p} < 0$$

From Andrew J. S. Hamilton and Jason P. Lisle,
**The river model of black holes**
**2008**

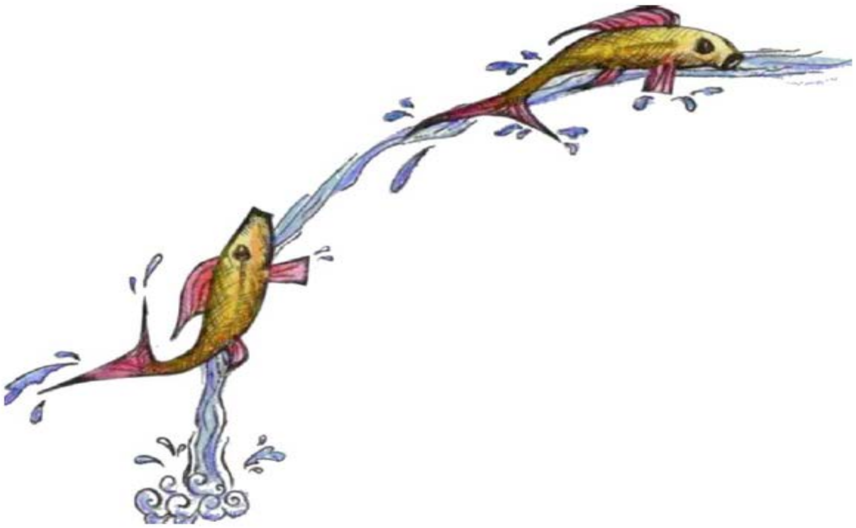


Fig. 1. (Color online) The fish upstream can make way against the current, but the fish downstream is swept to the bottom of the waterfall. Figure 1 of Ref. 5 presents a similar depiction.

River model: flat space flows through a flat background
Particles: analogues with special relativity
Direct physical meaning outside horizon
Inside: coordinate velocities > c, formally superluminal Motion
Region under horizon. Non-stationary, analogues with cosmology
Hubble flow and peculiar velocities as deviation from it
Now, we use the same concepts.
In terms of physical quantities, local description with tetrads
Application to high energy collisions. BSW effect: collision outside extremal horizon.
Now: inside Schwarzschild background
This version of high energy collisions is possible due to particles with zero “radial” momentum.

## River model and black holes

$$ds^2 = -fdt^2 + \frac{dr^2}{f} + r^2 d\omega^2$$

$$f = 1 - \frac{2M}{r}$$

Schwarzschild metric

Space flowing into black hole. Special role of Gullstrand-Painleve (GP) metric

A.J.S. Hamilton and J.P. Lisle, The River model of black holes, Am. J. Phys. 76 (2008) 519 [gr-qc/0411060]

Works perfect outside horizon. Inside? Some relevant quantities like coordinate velocities > c

# Under horizon

Metric is nonstationary. Analogy with cosmology.
Hubble flow + peculiar velocities
Tetrads attached to observer. Approach typical of special relativity
One more motivation: consideration of high energy collisions inside black hole
Generalized GP system

$$\tilde{t} = t + \int dr' \frac{f}{v} \qquad v = \sqrt{1-f}$$

$$ds^2 = -d\tilde{t}^2 + (dr + v d\tilde{t})^2 + r^2 d\omega^2 \qquad \frac{dr}{d\tilde{t}} = -v$$

Nonsingular on horizon, v is velocity of “flow” because of “river of space”

$$\tilde{t} = const$$

Flat cross-sections
Proper distance between points 1 and 2 $\left| r_2 - r_1 \right|$

Under horizon $f = -g < 0$

Choice of tetrads
Observer commoving with flow

$$h_{(0)}{}^{\mu} = \frac{\partial}{\partial \tilde{t}} - v\frac{\partial}{\partial r}$$

$$\frac{dr}{d\tilde{t}} = -v + v_p$$

Three-velocity

$$V_{(i)} = -\frac{h_{(i)\mu}u^{\mu}}{h_{(0)\mu}u^{\mu}} \qquad V_{(1)} = v_p$$

Free moving observer with $\varepsilon = \frac{E}{m} = 1$

Tetrads attached to it, vector (0) coincides with its four-velocity

Then, $\frac{dr}{d\tilde{t}} = -v$ Flow of space

Generic particle

$$\frac{dr}{d\tilde{t}} = -v + v_p$$

Here, $v_p$ is peculiar velocity deviation from flow

Properties of peculiar velocity

Outside horizon

$$v_p = \frac{v - V}{1 - vV}$$

V measured by static observer

$$\varepsilon = \frac{1 - v v_p}{\sqrt{1 - v^2{}_p}}$$

$$v = \sqrt{1 - f}$$

Inside horizon

$$v_p = \frac{\tilde{v} - V}{1 - \tilde{v}V}$$

V measured by observer with $\varepsilon = 0$

$$\tilde{v} = \frac{1}{v} = \frac{1}{\sqrt{1 + g}}$$

$$P = \frac{v - v_p}{\sqrt{1 - v^2{}_p}}$$

valid both outside the horizon and inside

Kinematics of collisions in terms of peculiar velocities

$$E^2_{c.m.} = -P_\mu P^\mu = m^2_1 + m^2_2 + 2m_1 m_2 \gamma$$

$$\gamma = \frac{\varepsilon_1 \varepsilon_2 - \sigma P_1 P_2}{f}$$

Under horizon

$$\gamma = \frac{P_1 P_2 - \sigma \varepsilon_1 \varepsilon_2}{g}$$

$$\gamma = \frac{1 - v_{p1} v_{p2}}{\sqrt{1 - v^2_{p1}} \sqrt{1 - v^2_{p2}}}$$

Like in Special Relativity

Banados, Silk, West (BSW) effect 2009

Collision between two particles near extremal Kerr BH

Particle 1 fine-tuned (critical)

Particle 2 not fine-tuned
(usual)

Kinematic explanation in terms of velocities from viewpoint of ZAMO

Particle 1 (critical) has small velocity near horizon $V_1 < 1$

Particle 2 (usual) has big velocity near horizon (almost c) $V_2 \to 1$

Rapid particle hits slow target

High energy collisions of massive particles under horizon

BSW effect in terms of peculiar velocities: situation is opposite

For critical particle $v_p \to 1$

For usual particle $v_p < 1$

Two mutually complementary descriptions

Particle 1 is critical $\varepsilon_1 = 0$ Zero-momentum

$$\gamma \approx \frac{\varepsilon_2}{\sqrt{g}} \gg 1$$

near horizon (small g)

# Summary

1) The results for absorption of light near the singularity are qualitatively different in the pure radial and nonradial cases.
2) The change of frequency during a fall of an
observer can unfold in different ways for typical and special scenarios.
3) Blueshift and particle collisions: counterpart of BSW effect inside
4) Role of p=0 (zero-momentum observers)
5) River model, separation to flow and peculiar velocities – qualitative explanation of behavior near singularity.
6) Role of zero-momentum observers:
Counterpart of BSW effect.
Transformation of frames, fiducial observers inside (counterpart of E=m observers outside).

Thank you!