Emergence of spacetime from string-theoretic Matrix Models

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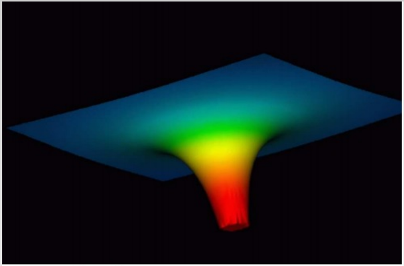

Credit: Pablo Laguna

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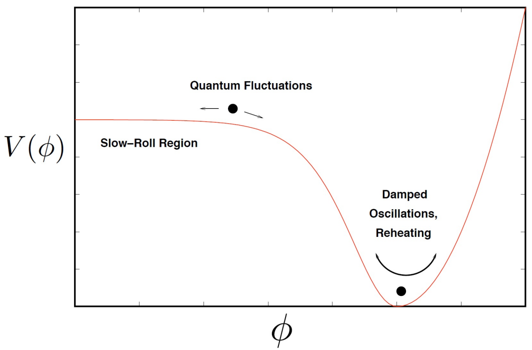

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Alternate description of the early-universe from *Matrix Theory*.

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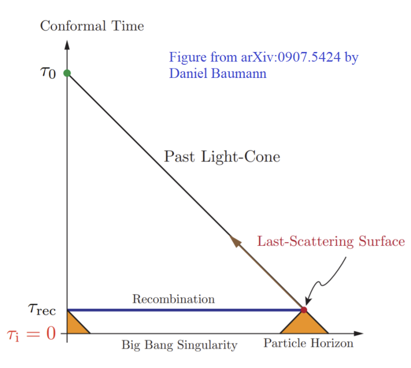

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- Solution to the standard horizon problem ⇐ Hubble radius smaller than causal horizon is all that is needed. [Brandenberger, 2011]
- Solution to the standard flatness problem ⇐ Spatially flat universe.
- Origin of structure ⇐ Scale invariant power spectrum of adiabatic perturbations. [Sunyaev & Zel'Dovich; Peebles & Yu, 1970]
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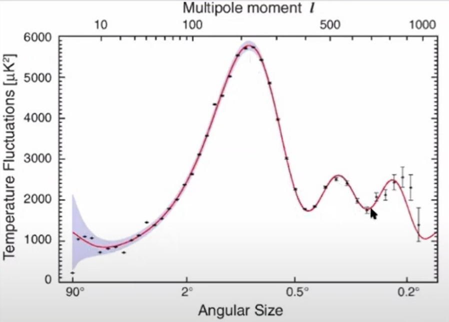

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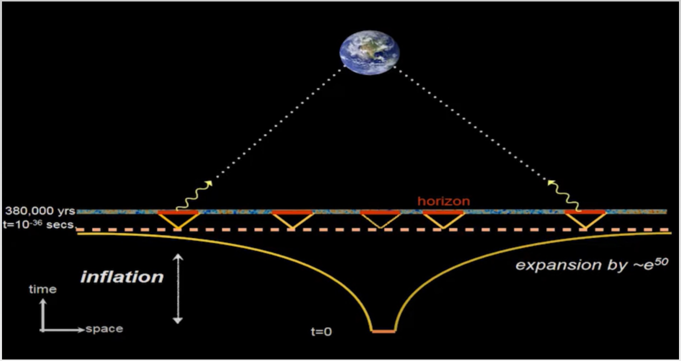

Photo credit: P. Adshead

Swampland implies short-lived meta-stable dS vacua can exist ⇒ Consistent with low-scale models of inflation.

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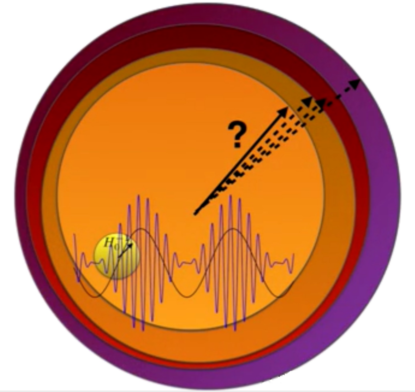

Photo credit: S. Shandera

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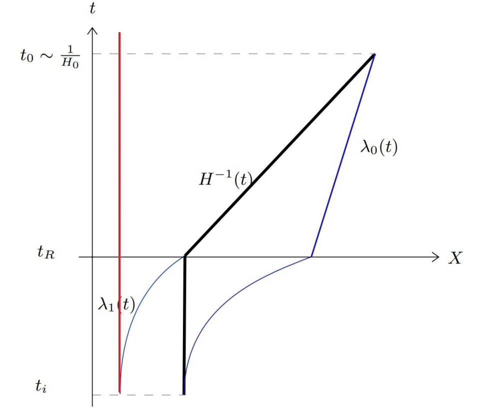

[Bedroya, Brandenberger, LoVerde, Vafa, 2019]

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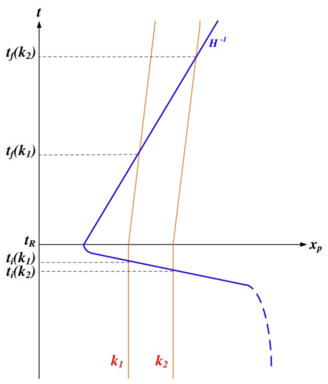
Symmetries of QG, *e.g.*, String Dualities, can play a pivotal role in this!
New symmetries (T-duality) ⇔ New states (Winding modes)

Alternatives

↪ Alternate descriptions of early-universe cosmology:

- Examples: *String Gas Cosmology* [Brandenberger & Vafa, 1989], *Ekpyrotic bounce* [Khoury, Ovrut, Steinhardt & Turok, 2001], *Early phase of topological gravity* [Agrawal, Gukov, Obied & Vafa, 2020], *CPT-symmetric universe* [Boyle & Turok, 2018-22], *Matter bounce* [Brandenberger, Wands, Wilson-Ewing, Cai & Wilson-Ewing, . . .], , . . .



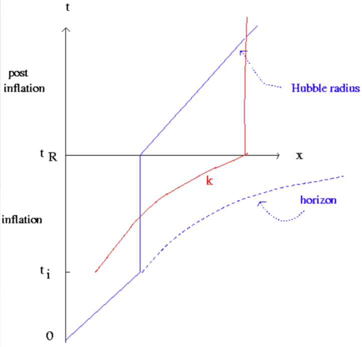


Figure Credit: R. Brandenberger

↪ The BFSS matrix model [Banks, Fischler, Shenker & Susskind, 1997]:

$$S_{\mathrm{BFSS}} = \frac{1}{g^2} \int \mathrm{d}t \mathrm{Tr} \left\{ \frac{1}{2} (D_t X_i)^2 - \frac{1}{4} [X_i, X_j]^2 + \frac{1}{2} \psi_\alpha D_t \psi_\alpha - \frac{1}{2} \psi_\alpha \gamma^i_{\alpha\beta} [X_i, \psi_\beta] \right\}$$

where $D_t \equiv \partial_t - i [A_t, \cdot]$ and $N \times N$ bosonic matrices $A(t), X_i(t)$ $(i = 1, \ldots, d)$ and $\psi_\alpha(t)$ $(\alpha = 1, \ldots, p)$.

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Matrix Models: Non-perturbative String Theory

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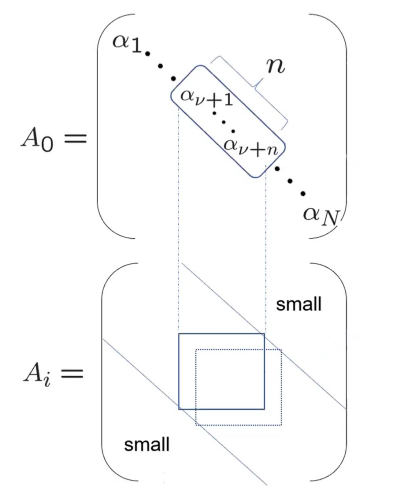
↪ A consistent, principled *top-down* approach to early-universe cosmology with the promise of rich phenomenology.

- **No** Fock-space EFT description: No **cosmological constant problem**!
- UV-complete: Eigenvalues never become trivial ⇒ **No singularities!**

Emergent $(3+1)$-d spacetime: Numerical evidence

[Aoki, Hirasawa, Ito, Kim, Nishimura, Tsuchiya, ...]

↪ Lorentzian IKKT model: $Z \sim \int \mathrm{d}A\,\mathrm{d}\Psi\, e^{iS_{\mathrm{IKKT}}}$



✓ Diagonalize $A_0 : \alpha_1 < \ldots < \alpha_N$.

✓ Define time via *coarse-graining*:

$$t(\nu) := \frac{1}{n}\sum_{i=1}^{n} \alpha_{\nu+i}, \quad \nu = 1, \ldots, N-n$$

✓ *Non-trivial* to obtain dynamical band-diagonal structure!

→ Time-dep $n \times n$ spatial matrices: $\left(\bar{A}_i\right)_{I,J}(t(\nu)) := \left(A_i\right)_{\nu+I,\nu+J}$

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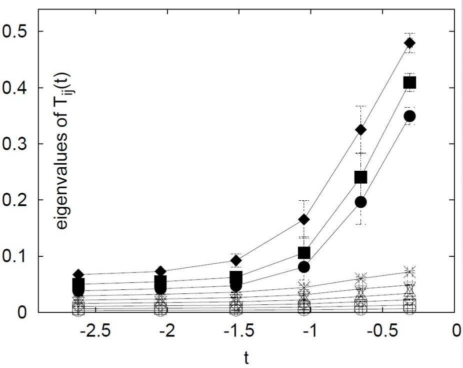
↪ *Emergence* of 3 large spatial dimensions:



✓ As order parameter, define moment of inertia tensor

$$T_{ij}(t) := \left\langle \frac{1}{n}\, \mathrm{Tr}\bar{A}_i(t)\bar{A}_j(t) \right\rangle$$

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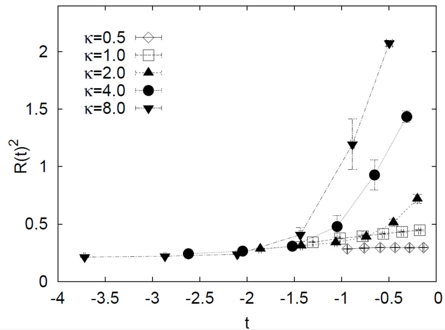

✓ Extent of a given spatial dimension parameter: $x_i^2(t) := \left\langle \frac{1}{n} \mathrm{Tr}\, \bar{A}_i(t)^2 \right\rangle$

→ Total extent of space parameter: $R^2(t) = \sum_{i=1}^{9} x_i^2(t)$

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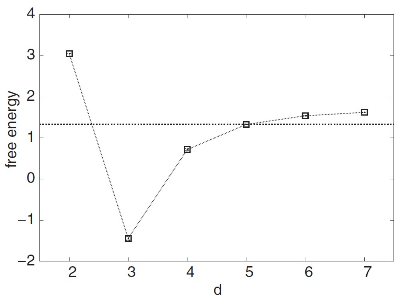

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Takeaway: A $(3+1)$-d universe emerges *dynamically* from non-perturbative (matrix model) description of superstring theory

Coarse-graining spacetime from abstract matrices

[S.B., Brandenberger & Laliberté, 2206.12468 (JHEP)]

↪ Emergent time identified from the diagonal elements of A_0 matrix, ordered as $A_0 = \mathrm{diag}(t_1, \ldots, t_N)$ with $t_i > t_j$ for $i > j$.

↪ In the diagonal A_0 basis, the eigenvalues of the A_i matrices **decay** when moving away from the diagonal ⇒ Band-diagonal structure.

✓ $t_{\max} \sim \sqrt{N}$, and discrete time eigenvalues scale as: $\Delta t \sim 1/\sqrt{N}$.

✓ In the $N \to \infty$ limit, *emergent* continuous and infinite time.

✓ Total **physical** extent of space: $\ell_{\mathrm{phys}} \sim \sqrt{N}$.

✓ Continuous and infinite extent of space when $N \to \infty$!

↪ Emergent metric: $g_{ij}(n, t) = \mathcal{A}(t)\, \delta_{ij}$, Assumption: $SO(3)$ symmetry.

No Flatness Problem! Independent of isotropy assumption.

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Coarse-graining spacetime from abstract matrices

[S.B., Brandenberger & Laliberté, 2206.12468 (JHEP)]

$\hookrightarrow$ Emergent time identified from the diagonal elements of A_0 matrix, ordered as $A_0 = \text{diag}(t_1, \ldots, t_N)$ with $t_i > t_j$ for $i > j$.

$\hookrightarrow$ In the diagonal A_0 basis, the eigenvalues of the A_i matrices **decay** when moving away from the diagonal $\Rightarrow$ Band-diagonal structure.

- ✓ $t_{\text{max}} \sim \sqrt{N}$, and discrete time eigenvalues scale as: $\Delta t \sim 1/\sqrt{N}$.
- ✓ In the $N \to \infty$ limit, *emergent* continuous and infinite time.
- ✓ Total **physical** extent of space: $\ell_{\text{phys}} \sim \sqrt{N}$.
- ✓ Continuous and infinite extent of space when $N \to \infty$!

$\hookrightarrow$ Emergent metric: $g_{ij}(n,t) = \mathcal{A}(t)\,\delta_{ij}$, Assumption: $SO(3)$ symmetry.

No Flatness Problem! Independent of isotropy assumption.

Cosmology from a thermal state in BFSS

↪ Consider a thermal state in the BFSS model.

↪ Start with the Euclidean BFSS model and consider its compactification on a thermal circle: BFSS $\xrightarrow{T\to\infty}$ IKKT (natural to assume thermal state)

⇝ $S_{\rm BFSS}(\beta) = \frac{1}{2g^2}\int_0^\beta \mathrm{d}t\, \mathrm{Tr}\left\{(D_t X_i)^2 - \frac{1}{2}[X_i, X_j] + \text{fermions}\right\}, \quad \beta = 1/T.$

⇝ Fourier expand fields as $X_i(t) = \sum_n X_i^n e^{in\omega t}$ in Matsubara frequencies $\omega = 2\pi T$.

↪ $S_{\rm BFSS} = S_0 + S_{\rm kin} + S_{\rm int}$, where $S_0 =: S_{\rm IKKT}^{\rm bosonic}$ (zero-mode action).

✓ One can just as well start with a thermal state in the IKKT model (by compactifying the A_0 matrix through the method of images) and use that to derive cosmology ($T \to 1/T$). We find the same background and a scale invariant spectrum, although with slight $\mathcal{O}(1)$ factors different in the amplitude. [Laliberté & S.B., 2304.10509 (JHEP)]

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✓ Justified to use Einstein's equations since we are considering IR modes (as in *String Gas*) with the BFSS producing the source term.

Thermal state formalism: [Nayeri, Brandenberger & Vafa, 2005]

↪ Thermal fluctuations considered in *String gas cosmology* and *Warm Inflation*. Unlike in standard inflation or *ekpyrosis*, perturbations not sourced by quantum vacuum fluctuations.

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Cosmological observables from the BFSS theory

We find scale-invariant spectrum for *IR modes* of observational interest

[S.B., Brandenberger & Laliberté, 2107.11512]

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✓ Single $U(N)$ matrix QM is known to be equivalent to $2d$ String Theory.

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- The collective field formalism:

$$\phi_k = \mathrm{Tr}\left(e^{ikM}\right) = \sum_{i=1}^{N} e^{ik\lambda_i}$$

$$\phi(x) = \int \frac{\mathrm{d}k}{2\pi} e^{-ikx}\,\mathrm{Tr}\left(e^{ikM}\right) = \mathrm{Tr}\left(\delta(x-M)\right) = \sum_{i=1}^{N} \delta\left(x-\lambda_i\right)$$

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✓ Change of variables $M \to \phi$ leads to $(1+1) - d$ field theory:

$$H_\phi = \int \mathrm{d}x \left[\frac{1}{2} \partial_x \pi(x) \; \phi(x) \; \partial_x \pi(x) + \frac{\pi^2}{6} \phi^3(x) - (\mu_F - V(x)) \, \phi(x) \right]$$

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Collective field formalism for multi-matrix models

$\hookrightarrow$ A direct transformation to collective field for more than one matrices can only be implemented numerically $\rightarrow$ Impossible to solve the 'Schwinger-Dyson' equations analytically.

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$$S_{\text{eff}} = \int \mathrm{d}\tau \left[\frac{1}{2l_s} \left(\sum_i^N \dot{\lambda}_i^2 + \sum_i^N \dot{\rho}_i^2 \right) \right] - \frac{1}{2} \sum_{i<j, i=1}^N \mathrm{Tr} \log \left(1 - \frac{1}{2l_s^4} \int \mathrm{d}s \, |s - \tau| \, (\lambda_i(s) - \lambda_j(s))^2 \right)$$

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To get a time-local effective action, we need to add a mass term $\mathrm{Tr}\left(m^2 Y^2\right)$ to the action. Then, to leading order:

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Possible to get a $(2+1)$-d collective field action! Starting point for connection to String Theory $\rightarrow$ *Geometry from Entanglement*

[with Brandenberger, Dasgupta, Lei & Pasiecznik]

Collective field: Integrating out off-diagonal elements

$$
\begin{aligned}
H \;=\;& \frac{l_s}{2}\int \mathrm{d}x\,\mathrm{d}y\;[\partial_x\pi(x,y)\phi(x,y)\partial_x\pi(x,y)+\partial_y\pi(x,y)\phi(x,y)\partial_y\pi(x,y)]\\
&+\frac{l_s\pi^2}{6}\int \mathrm{d}x\,\mathrm{d}y\,\phi^3(x,y)+\frac{l_s}{8}\int \mathrm{d}x\,\mathrm{d}y\,\frac{(\partial_y\phi(x,y))^2}{\phi(x,y)}\\
&-\frac{1}{16 m l_s^4}\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x'\,\mathrm{d}y'\,\phi(x,y)(x-x')^2\phi(x',y')+\frac{l_s m^2}{2}\int \mathrm{d}x\,\mathrm{d}y\,y^2\phi(x,y)\\
&-\lambda\left[\int \mathrm{d}x\,\mathrm{d}y\,\phi(x,y)-N\right]
\end{aligned}
$$

- Has the right expression when we turn off the second matrix (compactifying y-direction to zero)!
- Yet to understand physical interpretation of taking the massless limit of this model to preserve connection to String Theory!

Conclusions

✓ It has been notoriously difficult to find accelerating solutions in string theory → Inflation as a coherent state over warped Minskowski? dS *does* have such an interpretation!

[S.B., Dasgupta & Tatar, JHEP, 2020; S.B., Dasgupta, Guo & Kulinich, JHEP, 2024]

✓ M-theory consistency rules out large classes of bounce models.

[Bernardo, S.B., Dasgupta Mir & Tatar, Phys. Rev. Lett., 2021]

⋆ A new path towards a **UV-complete paradigm** for the early universe:

✓ Numerical evidence for the emergence of 3 large spatial dimensions from full String Theory.

✓ Analytically extract a coarse-grained time, space and metric.

✓ Thermal fluctuations ⇒ Scale invariant primordial perturbations.

✓ Horizon problem, Flatness problem and formation of structure from first-principles in a fundamental quantum gravity theory.

✓ No vacuum energy problem → Transition from non-geometric emergent phase to radiation dominated era. No cosmological constant.

✓ Possible to work with the full IKKT model **alone** and consider a thermal state in it. [Laliberté & S.B., JHEP, 2023]

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Looking ahead

⋆ A new path towards a UV-complete paradigm for the early-universe.
Ambitious, promising but a lot remains to be done:

- Details of non-geometric phase → Connection to non-commutative geometry emergent in the UV? [H. Steinacker; A. Chaney & A. Stern; ...]
- Physical explanation of the SSB phase? Connection with String Gas? Interestingly, non-geometric phase has $p = 0$ (quasi-static phase).

↪ Strings as solitonic states in Matrix models → Annihilation of string loops into radiation. Collective field formalism!

- Derive background cosmology from BFSS model (combine MC simulations and analytical insights). *Derive* Friedmann equations!
- A gauge-invariant notion of entanglement for matrices using the Collective Field → Area-law for toy models. [Frenkel & Hartnoll; Das, Kaushal, Mandal, Liu, Trivedi; Hampapura, Harper & Lawrence, ...]
- Observable consequences: NG, PBHs from the Poisson part of the UV spectrum, Primordial B fields, ...

⋮