Basics of νeutrino Oscillations

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Outline

Outline

Outline

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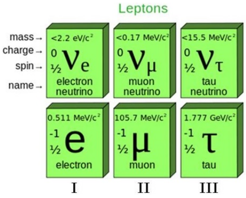
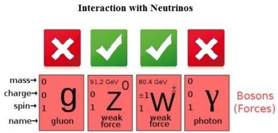
Identity

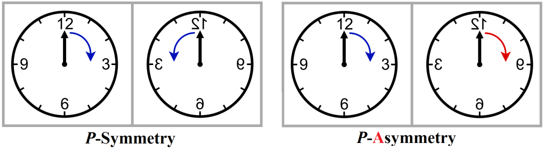
- νeutrinos are neutral spin-$\frac{1}{2}$ elementary fermions.
- νeutrinos are leptons: interact in the Standard Model (SM) via weak interactions only.
- νeutrinos exist in three flavors: electron-νeutrino, muon-νeutrino and tau-νeutrino: $\nu_\ell = (\nu_e, \nu_\mu, \nu_\tau)$.
- νeutrino flavors are created in weak interactions in association with the corresponding charged lepton $\ell = (e, \mu, \tau)$.
- νeutrinos are tiny massive $\mathcal{O}$(eV).



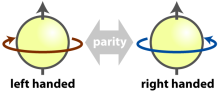


Parity

- Parity is the transformation under space reflection.
- Parity was assumed at the beginning to be a symmetry of nature (1927, Eugene Wigner).
- That's a 'mirrored' image of a natural system behaves in the same way as the 'mirror' image of that system does.

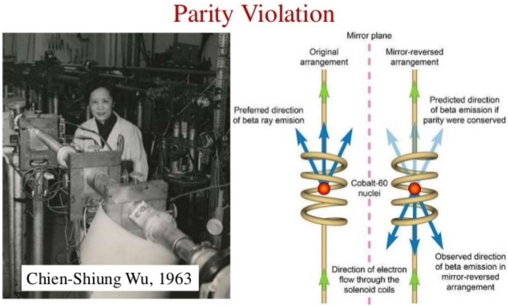


- Fermions are chiral. They are absolutely classified into left- and right-handed chiralities. They are helicial too, according to their relative spin projection in the direction of motion.



Parity Violation

- Parity is a symmetry for both strong and electromagnetic interactions.
- In 1963, *Wu* found that Parity is maximally violated in the weak interactions.
- Only left-handed fermions feel the weak interactions.

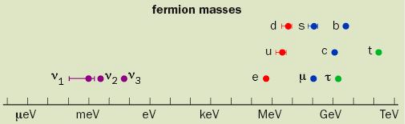


- (Known) νeutrinos are only left-handed.

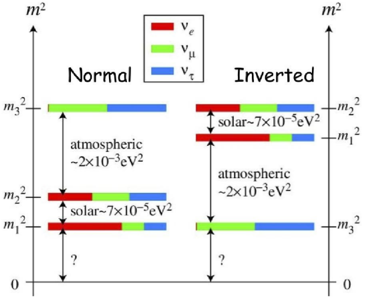
Outline

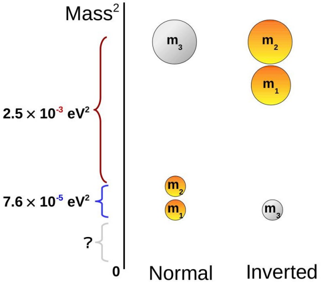
νeutrino mass hierarchies

- νeutrinos masses are of order of little eV's.



- Limits from solar and atmoshperic νeutrino experiments propose the normal and inverted hierarchies for the νeutrino masses.





νeutrino mixing

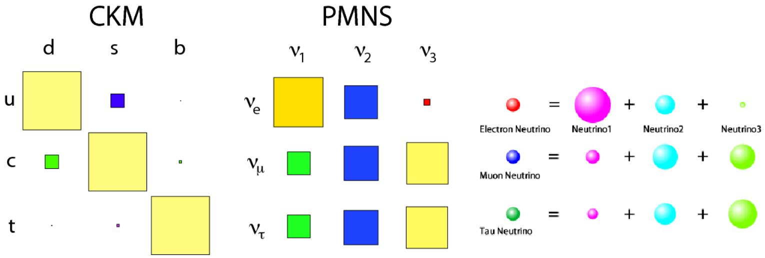
- νeutrinos nonzero masses yields to νeutrino mixing phenomena [1].
- That is, νeutrino flavors $\nu_e,\ \nu_\mu,\ \nu_\tau$ do not have definite masses.
- Instead, νeutrino flavors are linear combinations of three other mass states $\nu_1,\ \nu_2,\ \nu_3$ with definite masses $m_1,\ m_2,\ m_3$.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & -0.15 \pm 0.03 \\ -0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & -0.45 \pm 0.06 & 0.77 \pm 0.06 \end{bmatrix}$$

CKM & PMNS Mixing matrices

- The CKM quark mixing matrix is almost diagonal.
- The PMNS mixing matrix of νeutrinos is equlibrated; all elements have approximately the same order.



Outline

Beats Lagrangian

- Lagrangian for forced oscillations (ω: free oscillation frequency) [2]

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 + xF(t), \tag{1}$$

- *Euler-Lagrange* equation

$$\ddot{x} + \omega^2 x = F(t)/m. \tag{2}$$

- Free oscillations (A: amplitude and α: initial phase)

$$x(t) = A\cos(\omega t + \alpha), \tag{3}$$

- Oscillatory force

$$F(t) = f\cos(\gamma t + \beta). \tag{4}$$

Beats amplitude

- Near the resonance ($\gamma = \omega$), $\gamma = \omega + \epsilon$, $\frac{\epsilon}{\omega} \ll 1$, the motion is small oscillations with varaible amplitude

$$x(t) = C(t) \exp(i\omega t), \tag{5}$$

where

$$C^2 = a^2 + b^2 + 2ab \cos(\epsilon t + \beta - \alpha), \tag{6}$$

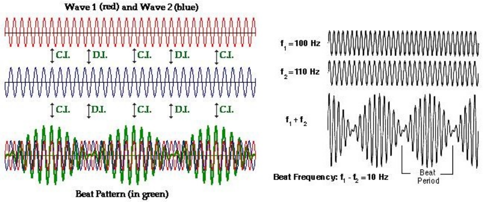
and a, b are constants.

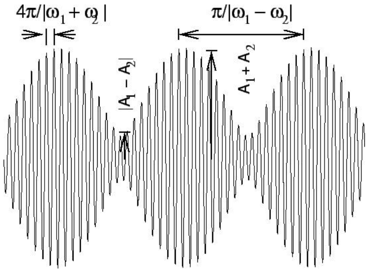
- The amplitude varies slowly with frequency ϵ between the limits

$$|a - b| \leq C \leq a + b. \tag{7}$$

- This phenomena is called the *"beats"*. ϵ is the beat frequency.

- Click BEATS Enjoy!





νeutrino two flavor mixing

- Mixing only two flavors ν_α, ν_β of two mass states ν_i, ν_j

$$\nu_\alpha = \nu_i \cos\theta_{ij} + \nu_j \sin\theta_{ij}, \tag{8}$$
$$\nu_\beta = -\nu_i \sin\theta_{ij} + \nu_j \cos\theta_{ij}, \tag{9}$$

- When a $|\nu_\alpha(0), \vec{p}\rangle$ νeutrino is produced with momentum $\vec{p}$ at time $t = 0$, the ν_i and ν_j components will have slightly different energies E_i and E_j due to their slightly different masses.
- In QM, their associated waves have slightly different frequencies, and their interference gives rise to the '*beats*' phenomenon.
- As a result, the original beam of ν_α develops a ν_β component whose intensity oscillates as it travels through space, meanwhile, the intensity of the ν_α νeutrino beam itself is correspondingly reduced.
- This is the '*νeutrino oscillations*' phenomenon and its occurence follows from simple QM.

Propagation

- At time $t = 0$, the initial state (8) is produced with momentum $\vec{p}$

$$|\nu_\alpha(0), \vec{p}\rangle = |\nu_i, \vec{p}\rangle \cos\theta_{ij} + |\nu_j, \vec{p}\rangle \sin\theta_{ij}. \tag{10}$$

- After time t, Schrödinger equation for the mass states ν_i, ν_j dictates

$$|\nu_\alpha(t), \vec{p}\rangle = a_i(t)|\nu_i, \vec{p}\rangle \cos\theta_{ij} + a_j(t)|\nu_j, \vec{p}\rangle \sin\theta_{ij}, \tag{11}$$

where

$$a_{i,j}(t) = \exp\left(-iE_{i,j}t\right) \tag{12}$$

are the usual oscillating time factors associated with any quantum mechanical stationary state.

Propagation

- Inverting eqs. (8,9)

$$\nu_i = \nu_\alpha \cos\theta_{ij} - \nu_\beta \sin\theta_{ij}, \tag{13}$$

$$\nu_j = \nu_\alpha \sin\theta_{ij} + \nu_\beta \cos\theta_{ij}. \tag{14}$$

- Substituting into eq. (11) gives the oscillation formula

$$|\nu_\alpha(t), \vec{p}\rangle = A(t)|\nu_\alpha(0), \vec{p}\rangle + B(t)|\nu_\beta(0), \vec{p}\rangle, \tag{15}$$

where

$$A(t) = a_i(t)\cos^2\theta_{ij} + a_j(t)\sin^2\theta_{ij}, \tag{16}$$

$$B(t) = \sin\theta_{ij}\cos\theta_{ij}[a_j(t) - a_i(t)]. \tag{17}$$

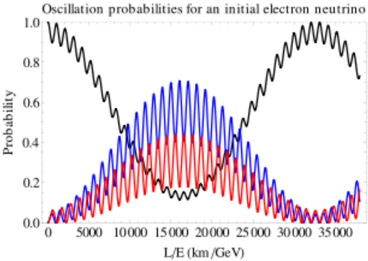
Oscillation Probability

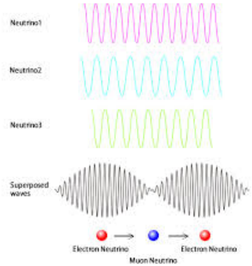
- The probability of finding a ν_β state is therefore

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta(0), \vec{p} | \nu_\alpha(t), \vec{p} \rangle|^2$$
$$= |B(t)|^2 = \sin^2(2\theta_{ij}) \sin^2[\frac{1}{2}(E_j - E_i)t]. \quad (18)$$

- The probability of finding a ν_α state is therefore

$$P(\nu_\alpha \to \nu_\alpha) = |\langle \nu_\alpha(0), \vec{p} | \nu_\alpha(t), \vec{p} \rangle = |A(t)|^2 = 1 - |B(t)|^2. \quad (19)$$





Remarks

- No oscillation with vanishing mixing $\theta_{ij} = 0$.
- For small mixing θ_{ij} and large energy difference, oscillation is negligible.
- These formulas assume that the νeutrinos are propagating in a vacuum. This is usually a very good approximation, because of the enormous mean free paths for νeutrinos to interact with matter.
- *Mikheyev-Smirnov-Wolfenstein* (MSW) effect shows that νeutrino oscillations can be enhanced when νeutrinos traverse very long distances in matter due to weak interactions with matter's electrons analogous to the electromagnetic process leading to the refractive index of light in a medium.
- The MSW effect was dramatically confirmed in the 'Sudbury νeutrino Observatory (SNO)', and resolved the solar νeutrino problem.

Detection

- The time t traveled by a νeutrino is determined by the distance L of the νeutrino detector its source.
- νeutrino are always ultra-relativistic and their momenta are much greater than their possible masses and they approximately travel at the speed of light.
- So $t \approx L$ and

$$E_j - E_i = (m_j^2 + p^2)^{1/2} - (m_i^2 + p^2)^{1/2} \approx \frac{m_j^2 - m_i^2}{2p}. \tag{20}$$

- Thus the oscillation probability (18) can be written as

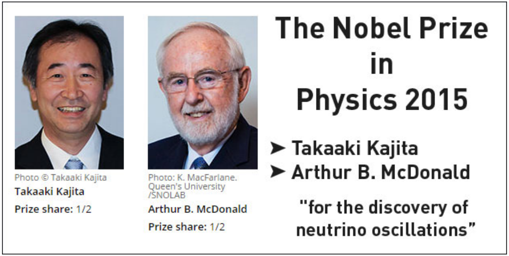
$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta_{ij}) \sin^2[L/L_0], \tag{21}$$

where the oscillation length

$$L_0 = \frac{4E}{m_j^2 - m_i^2}. \tag{22}$$

Celebration

- The oscillation lengths are typically of order 100 km or more.
- Oscillations can be safely neglected under normal laboratory conditions.
- The Nobel Prize in Physics 2015 for the discovery of νeutrino oscillations. Click ▸ NPNO, ▸ NPNO.pdf.



Outline

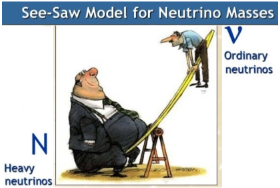
Open Problems

- νeutrino mass heirarchy. Why νeutrino masses are tiny?
- νeutrino absolute masses. What they are?
- νeutrino: Majorana? Are νeutrinos their own antiparticles?

Outline

Type I seesaw

- It is thought that νeutrinos not only mix among each others, but also with their right-handed much heavy cousins which balance their tiny masses on seesaw.



- SM+type I seesaw Lagrangian

$$\mathcal{L}_\nu = Y^\nu_{ij}\bar{\nu}_{L_i}(v+h)\nu_{R_j} + M_{R_{ij}}\bar{\nu}^c_{R_i}\nu_{R_j} + h.c. \tag{23}$$

- Light and heavy νeutrino masses for $M_R \gg$ Higgs vev (v)

$$m_{\nu_\ell} = \frac{m_D^2}{M_R}, \quad m_{\nu_h} \approx M_R, \quad m_D = Y^\nu v \tag{24}$$

Assignment to think

- What happens to the νeutrino oscillation pattern at very large time scale compared to the oscillation scale itself?
- Quarks mix too via the CKM matrix. Are there quark oscillations?
- Assume there are quark oscillations. On which scale they would be?
- Read about *Wu* experiment.

Thank you!
Questions?

References

[1] B. Martin and G. Shaw, Particle physics.
John Wiley & Sons, 2013.

[2] L. D. Landau and E. M. Lifshits, Quantum Mechanics, vol. v.3 of Course of Theoretical Physics.
Butterworth-Heinemann, Oxford, 1991.