MODULI AND MULTI-FIELD INFLATION

ZYGMUNT LALAK

Physics Department, University of Warsaw, ul. Hoża 69
00-681 Warsaw, Poland
Zygmunt.Lalak@fuw.edu.pl

Moduli with flat or run-away classical potentials are generic in theories based on super-symmetry and extra dimensions. They mix between themselves and with matter fields in kinetic terms and in the nonperturbative superpotentials. As the result, interesting structure appears in the scalar potential which helps to stabilise and trap moduli and leads to multi-field inflation. The new and attractive feature of multi-inflationary setup are isocurvature perturbations which can modify in an interesting way the final spectrum of primordial fluctuations resulting from inflation.

Keywords: supersymmetry; inflation; moduli.

1. Introduction

Last year's results reported by WMAP3 [1],[2] seem to confirm the inflationary paradigm with the spectral index $n_s < 1$ and agree with the cosmic concordance model strengthening the case for dark energy. Therefore it is an actual chalenge for the theory of fundamental interactions to accommodate any of the two ideas in realistic extensions of the Standard Model (BSMs).

Four dimensional supergravities stemming from string theories posses field dependent couplings controlled by vevs of moduli - fields which have flat or featureless classical scalar potential. These fields interact with gravity-strength couplings and appear to be natural candidates to play the role of the inflaton. In addition, they mix at the level of kinetic or potential energy with all matter-like fields present in the Lagrangian, which makes them a part of the inflationary dynamics even if the designed would-be inflaton is not a modulus but rather a matter field. Hence, the issue of generating supersymmetric inflation is intertwined with the issue of moduli stabilisation and it is a multi-field rather than a single field phenomenon, which may lead to important consequences. The presence of more than one active field makes the analysis of the dynamics more complicated. However, there are two crucial advantages of such an approach. Firstly, it becomes easier to model simultaneously succesful inflation, cancellation of the post-inflationary vacuum energy and trapping of the moduli. Secondly, the physics of inflationary fluctuations is markedly different in multi-field setups than in the standard single field inflationary models, which

makes the realistic spectrum of fluctuations more natural and easier to generate.

2. Modular inflation

One interesting illustration of the possible role of additional scalars in generating realistic inflation is the case of the dilaton in heterotic string effective Lagrangian [3]. It is well known that a pair of condensates (a racetrack) can easily generate a scalar potential with a nontrivial weakly coupled minimum for both scalar components of the dilaton superfield. An additional difficulty consists in the necessity to cancel the post-inflationary cosmological constant. To simplify the inflationary dynamics, let us assume that in addition to the dilaton there exists another modulus, the volume modulus T, with a no-scale Kähler potential, which automatically makes the tree-level scalar potential semi-positive definite. We assume in addition, that that no-scale modulus becomes stabilised in a different sector of the model, for instance with the help of non-trivial D-terms [3]. Let us note at this point that the two pehnomena – the cancellation of the cosmological constant and generation of the inflationary period are not independent, as discussed for instance in [4], but we ignore this complication in what follows.

The weakly-coupled racetrack cannot generate inflation. There are no regions where slow-roll parameters could be made sufficiently small simultaneously in all directions in (two-dimensional) field space. Moreover, the exponential scalar potential is so steep along the real component of the dilaton, that the field which starts its evolution from arbitrary initial conditions set at very high initial temperatures "overshoots" the shallow weakly-coupled minimum and runs away towards the "cold" asymptotic minimum at vanishing gauge coupling. Both problems can be solved if an additional superfield is present. In the case the mass, m, has a dependence on the vacuum expectation value, $< \chi >$, of a scalar field, χ, it is clear that the condensation scale and hence the dilaton potential which determines the condensation scale will also depend on $< \chi >$. This can arise through the moduli dependence of the couplings involved in the mass generation after a stage of symmetry breakdown. For example a gauge-nonsinglet field, Φ, may get a mass from a coupling to a field, Ψ, when it acquires a vacuum expectation value through a Yukawa coupling in the superpotential of the form $W = \lambda \Psi \Phi \Phi$. In general the coupling λ will depend on the complex structure moduli, $\lambda = \lambda(< \chi >)$ and so the mass of the Φ field will be moduli dependent, $m = \lambda(< \chi >) < \Psi >$. In what follows we will consider the implications of a very simple dependence of m on the $< \chi >$ which is sufficient to illustrate how χ can provide an inflaton if it is a (modulus) field which has no potential other than that coming from the m dependence of the condensation scale. In particular we take $m = \alpha + \beta < \chi >$ where α is a mass coming from another sector of the theory, possibly also through a stage of symmetry breaking. We shall assume for simplicity a canonical kinetic term, $K = \bar{\chi}\chi$, for the modulus χ, although in practice the form of its Kähler potential may be more complicated.

We are now in a position to write down the form of the superpotential corresponding to two gaugino condensates driven by two hidden sectors with gauge group $SU(N_1)$ and $SU(N_1)$ respectively. For simplicity we allow for a moduli dependence in the second condensate only. The race-track superpotential has the form

$$W_{npert} = \left(A N_1 M^3 e^{-S/N_1} - B N_2 M^3 e^{-S/N_2} \left(\frac{M^2}{(\alpha + \beta\chi)^2} \right)^\gamma \right) \chi^p . \qquad (1)$$

The coefficients A, B are related to the remaining string thresholds Δ_i by $A = e^{-\Delta_1/(2N_1)}/N_1$ etc. whose moduli dependence we do not consider here, γ is related to the difference of beta functions before and after a stage of symmetry breaking at the scale m.

We will now argue that the racetrack potential has all the ingredients to meet the challenges posed by successful inflation. There are two main aspects to this. Firstly the domain walls generated by the racetrack potential naturally satisfy the conditions needed for topological inflation. As a result there will be eternal inflation within the wall which sets the required initial conditions for a subsequent period of slow-roll inflation during which the observed density perturbations are generated. The second aspect is the existence of a saddle point(s) close to which the potential is sufficiently flat to allow for slow-roll inflation in the weak coupling domain. As noted above this is not the case for the pure dilaton potential but does occur when one includes a simple moduli dependence.

2.1. *Topological inflation*

As pointed out by Vilenkin [5] and Linde [6], "topological inflation" can occur within a domain wall separating two distinct vacua. The condition for this to happen is that the thickness of the wall should be larger than the local horizon at the location of the top of the domain wall (we call this the 'coherence condition'). In this case the initial conditions for slow-roll inflation are arranged by the dynamics of the domain wall which align the field configuration within the wall to minimise the overall energy. The formation of the domain wall is inevitable if one assumes chaotic initial conditions which populate both distinct vacuua and moreover walls extending over a horizon volume are topologically stable. Although the core of the domain wall is stable due to the wall dynamics and is eternally inflating, the region around it is not. As a result there are continually produced regions of space in which the field value is initially close to that at the centre of the wall but which evolve to one or other of the two minima of the potential. If the shape of the potential near the wall is almost flat these regions will generate a further period of slow-roll inflation, at which time density perturbations will be produced.

In the case of the racetrack potential the coherence condition necessary for topological inflation appears to be rather easily satisfied. This condition states that the physical width of the approximate domain wall interpolating between the minimum at infinity and the minimum corresponding to a finite coupling should be larger than the local horizon computed at the location of the top of the barrier that separates

them. The width of the domain wall, Δ, is such that the gradient energy stored in the wall equals its potential energy, $\left(\frac{2\delta}{\Delta}\right)^2 = V(s_{\max})$, where $\delta = \log(s_{\max}/s_{\min})$. [a]

The fact that the racetrack potential readily generates topological inflation offers solutions to *all* the problems related to modular inflation. With chaotic initial conditions for the dilaton at the Planck era the different vacuua will be populated because the height of the domain walls separating the minima is greater than the equilibrium temperature T_{eq} so thermal effects will not have time to drive the dilaton to large values. This avoids the initial thermal-roll problem. Then there will be regions of space in which the dilaton rolls from the domain wall value into the minimum at finite s, moving from larger to smaller values, and thus avoiding the thermal-roll problem. In fact once created the vacuum bag at finite s is stable, because it cannot move back into the core and over to the other vacuum — the border of the inflating wall escapes exponentially fast — so the respective region of space is trapped in the local vacuum. Finally, if after inflation the reheat temperature is lower than the critical temperature T_{crit} [b] the thermal effects will be too small to fill in the racetrack minimum at finite s and the region of space in this minimum will remain there, thus avoiding a late thermal-roll problem.

Although the existence of topological inflation seems necessary for a viable inflationary model, by itself it is not sufficient to generate acceptable density perturbations. What is needed is a subsequent period of slow-roll inflation with the appropriate characteristics to generate an universe of the right size ($> 50 - 60$ e-folds of inflation) and density perturbations of the magnitude observed.

2.2. *Inflation in the weak coupling regime*

In this subsection we present an example of a viable inflationary model in which the inflaton is the pseudo-Goldstone boson associated with the phase θ of the field χ.

Numerical analysis of the complete Lagrangian in the (S, χ) hyperplane shows that typically there are inflationary solutions. A very nice example corresponds to the choice of parameters $A = 1.5$, $B = 8.2$, $N_1 = 10$, $N_2 = 9$, $p = 0.5$, $\alpha = 1$, $\beta = 2.3$, and $\gamma = 10^{-4}$. There is a weakly interacting minimum at $s = 152.6$, $\phi = 0$, $x = 0.42$, and $\theta = 3.16$ (we remind the reader that $S = s + i\phi$ and $\chi = xe^{i\theta}$). The structure of the potential in the neighbourhood of $s = 152.6$ is shown in Figure 1 from which it may be seen that there is a maximum at $s \approx 160$. There is a domain wall between the weakly interacting minimum and the non-interacting minimum at $s = \infty$. As may be seen from Figures 2 and 3 it has a saddle point at $s = 162.2$, $\phi = 0$, $x = 0.074$, $\theta = 3.152$. Inflation occurs within the domain wall and there is further slow-roll inflation outside the wall as the wall inflates to a size not supported by the dynamics generating the wall.

---

[a]One should use here the canonically normalised variable $z = \log(s)$, s being the real part of the dilaton.
[b]Above T_{crit} the minimum may disappear due to thermal corrections in the hot gauge sector.
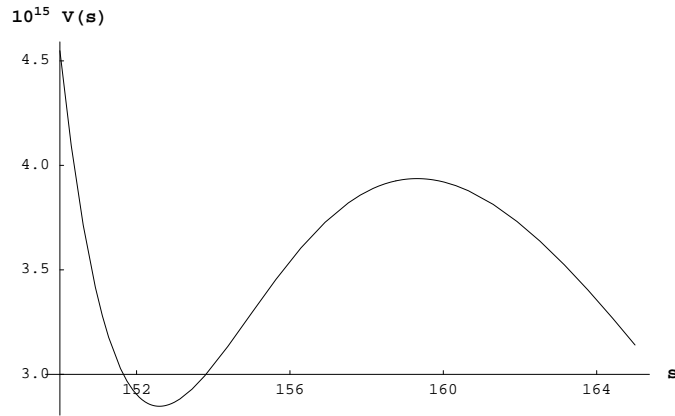
Fig. 1.   The s dependence of the potential in the neighbourhood of the weakly coupled minimum.
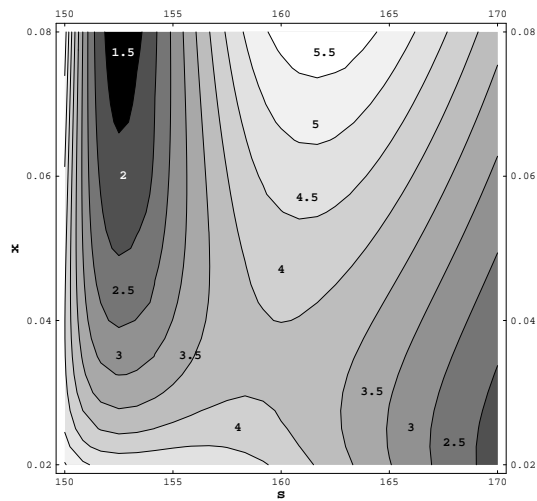

Fig. 2.   The contour plot in the s, x plane in the neighbourhood of the saddle point. The numbers on the contours multiplied by 10^{-15} give the corresponding vaules of the scalar potential in the units of M_P^4.

The initial conditions or this slow-roll deserves some comment. The s field starts very close to the saddle point at $s = 162.2$. The same is true for the fields ϕ and x which, at the saddle point, have masses larger than the Hubble expansion parameter at this point. However the field θ is a pseudo-Goldstone field and acquires a mass-squared proportional to γ. As may be seen from Figure 3, since γ is small, the potential is very flat in the θ direction and the mass of θ is much smaller than the Hubble expansion parameter. As a result the vev of θ during the eternal domain
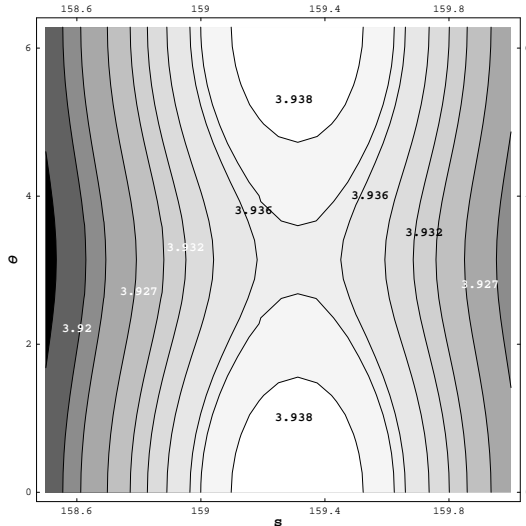
6 *Zygmunt Lalak*

Fig. 3. The θ dependence of the potential in the neighbourhood of the saddle point. Note that the slope *along* the direction of θ is much smaller than the slope in the direction of s. The numbers on the contours multiplied by 10^{-15} give the corresponding vaules of the scalar potential in the units of M_P^4.

wall inflation undergoes a random walk about the saddle point and so its initial value can be far from saddle point.

With these initial conditions it is now straightforward to determine the nature of the inflationary period after the fields emerge from the region of the domain wall. This corresponds to the roll of the fields, s, ϕ and x from the saddle point to the weakly interacting minimum, but allows for θ to be far from the saddle point. The s, ϕ and x fields rapidly roll to their minima. However the gradient in the direction of the θ field is anomalously small due to the pseudo-Goldstone nature of the field. Quantitatively, in the neighbourhood of the weakly interacting minimum, we find a negative eigenvalue of the squared mass matrix corresponding to the phase θ, and its absolute value is about 10^4 times smaller than the positive eigenvalues. This is much smaller than the Hubble expansion parameter at the start of the roll and so the θ field indeed generates slow-roll inflation. The remaining degrees of freedom can be integrated out along the inflationary trajectory. Inflation stops after about 7800 e-folds at $\theta_e = 3.54$ and the pivot point corresponds to $\theta_\star = 4.71$. The value of η at this point is $\eta_\star = -0.0089$ so the the spectral index is $n_\star = 0.98$, consistent with the WMAP3 value at 2σ. The agreement with the normalisation of the spectrum is also readily achieved (we remind the reader that the expectation value of t can be considered as a free parameter for the purpose of tuning the overall height of the inflationary potential, as it is fixed in a separate sector of the model). Note that the 'running' of the spectral index is very small, $d \ln n_s / d \ln k < \times 10^{-5}$, hence probably undetectable.

To summarise, the moduli dependent racetrack potential has a saddle point which lead to a phenomenologically acceptable period of slow-roll inflation with the inflaton being a component of the moduli. No fine-tuning of parameters is required and the initial conditions are set naturally by the first stage of topological inflation. After inflation there will be a period of reheat and the nature of this depends on the non-inflaton sector of the theory which we have not specified here. From the point of view of the racetrack potential the main constraint on this sector is that the reheat temperature should be less than T_{crit} to avoid the thermal roll problem. However T_{crit} is quite high, much higher than the maximum reheat temperature allowed by considerations of gravitino production, so we expect this constraint will be comfortably satisfied in any acceptable reheating model.

3. Inflation from matter fields mixed with moduli

The inflationary scheme described above encounters some problems when post-inflationary phenomenology is considered. First of all, the scale of the gravitino mass tends to be somewhat large with respect to the Fermi scale when the scale of inflation is made close to the upper limit imposed by COBE normalisation. Secondly, one might wish to rely on more traditional dynamics than topological inflation and topological trapping. In addition, in supersymmetric extensions of the Standard Model there are typically many quasi-flat directions involving combinations of scalars which belong to the matter rather than moduli sector. Hence, it is natural to consider matter-scalar driven inflation. However, as already explained in the introduction, and discussed at length in [4], matter scalars always mix with moduli. It turns out that very often the curvature along the direction of the relevant modulus is so large, that it spoils successful inflation along the direction of the matter scalar - the fields acquire large velocity transverse to the would-be inflationary trajectory and inflation ends prematurely. In the paper [4] various ways out of this rather generic problem have been discussed. It turns out that sometimes certain amount of tuning and creative model building are necessary to generate sufficiently long epoch of slow-roll inflation. The discussion of this issue is rather model dependent, and for more details and further references we recommend papers [4] and [7].

4. New features of multi-field inflation

It is interesting to put aside fine details of the construction of multi-field inflationary models and ask for generic phenomena which occur in such a setup. The most interesting feature concerns quantum fluctuations during the inflationary period, which serve as primordial fluctuations triggereing formation of large scale structure. In the single-inflaton models fluctuations essentially do not evolve after horizon crossing, staying frozen until the second, post-inflationary horizon crossing, when the physical wavelength of a fluctuation with a given wave number becomes smaller than
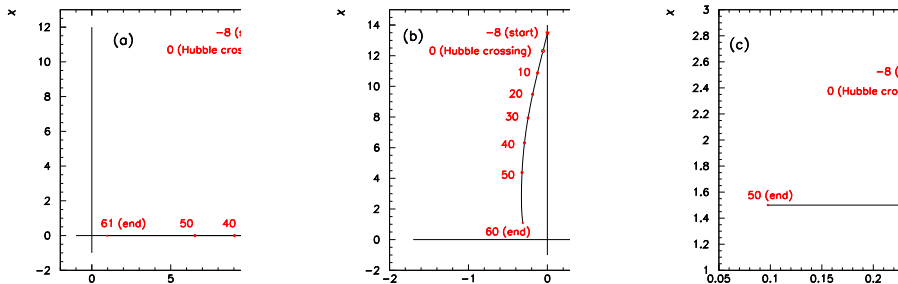
8   *Zygmunt Lalak*

Fig. 4.   Examples of classical inflationary trajectories for double inflation with canonical kinetic terms (left), double inflation with non-canonical kinetic terms (center) and roulette inflation (right). Subsequent tens of efolds are indicated along the curves.

the FRW horizon. However, when additional active inflatons are present, fluctuations in various fields get coupled and can evolve significantly after the first horizon crossing.

In fact, this offers novel possibilities in shaping the spectrum of inflationary fluctuations. The point is that in the direction transverse to the classical inflationary trajectory the potential doesn't have to be as flat as along the classical trajectory. Hence, transverse fluctuations (isocurvature ones) do not need to have the spectrum as flat as that of the curvature fluctuations which borne as fluctuations in the 'momentary' inflaton. If the coupling between both types of fluctuations is large enough, the isocurvature fluctuations can feed the curvature perturbations, and their not-so-flat spectrum becomes imprinted on the final spectrum of primordial fluctuations resulting from inflation. As the result, it becomes reasonably easy to obtain the scalar spectral index significantly smaller than 1, as suggested by WMAP3. Examples of such solutions are presented in [7]. Here we shall only illustrate the above statements. In the Figure 4 examples of bent and curved classical inflationary trajectories are given. All of them give rise to significant isocurvature perturbations. To be more specific let us consider the so-called roulette inflation model which has been investigated recently in [8] (see also [9]). In the notation of [7] this model can be effectively described by

$$b(\phi) = b_0 - \frac{1}{3} \ln \left(\frac{\phi}{M_P} \right) \tag{2}$$

and

$$V(\phi, \chi) = V_0 + V_1 \sqrt{\psi(\phi)} e^{-2\beta_1 \psi(\phi)} + V_2 \psi(\phi) e^{-\beta_1 \psi(\phi)} \cos(\beta_2 \chi), \tag{3}$$

where $\psi(\phi) = (\phi/M_P)^{4/3}$ and b_0, V_i, β_i are functions of the parameters of the underlying string model. A generic feature of the potential (3) is that it has an infinite number of minima arranged periodically in χ and a plateau for large values
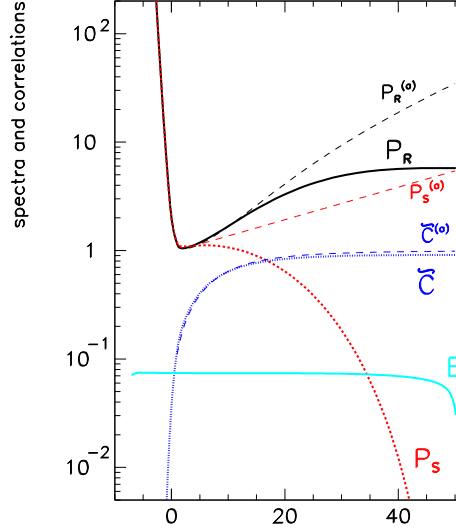
Fig. 5.   *Predictions for the spectra and correlations of the perturbations in roulette inflation. Thick lines show the numerical results for $\mathcal{P}_\mathcal{R}$ – power spectrum of curvature fluctuations, $\mathcal{P}_\mathcal{S}$ – power spectrum of isocurvature perturbations, normalized to the single-field result, respectively. The coupling B between the curvature and isocurvature perturbations is also shown.*

of ϕ, admitting a large variety of inflationary trajectories, which may end at different minima even if they originate from neighboring points in the field space – hence the model has been dubbed *roulette inflation*. In this work, we adopt the parameter set no. 1 (in Planck units: $b_0 = -11$; $V_0 = 9.0 \times 10^{-14}$; $V_1 = 3.2 \times 10^{-4}$; $V_2 = 1.1 \times 10^{-5}$; and $\beta_1 = 9.4 \times 10^5$; $\beta_2 = 2\pi/3$) from [8] and choose the particular inflationary trajectory shown in Figure 4. For this trajectory, the factor $b_\phi M_P$ is rather large, of the order 10^3, but the effect of the non-canonical kinetic terms is strongly suppressed by a very small value of ϵ on the plateau of the potential. The smallness of ϵ also suppresses the energy scale of inflation and one needs a smaller number of efolds than in the models described above. For definiteness, we assumed that there are ~ 50 efolds between the moment that the scale of interest crosses the Hubble radius and the end of inflation.

The largest portion of the inflationary trajectory in this example lies on the plateau of the potential (3), the slow-roll parameter ϵ is very small, which makes the direct impact of the non-canonicality negligible. The trajectory is, however, strongly curved in the field space and the interaction between the isocurvature and curvature modes is still important. As shown in [7] one can accurately predict the

spectra and correlations in the vicinity of the Hubble crossing, with deviations on super-Hubble scales resulting from the sourcing of the curvature perturbations by the isocurvature ones. Eventually, most of the curvature perturbations arise through this effect.

5. Summary and outlook

Multi-field inflation, with more than a single field active during the inflationary period, is a rather generic phenomenon in unification theories based on supersymmetry and extra dimensions. This fact has several implications for inflationary model building and also for the post-inflationary dynamics.

First of all, in the case of modular inflation one can use some of the active fields to trap run-away moduli via the topological inflation while the remaining ones could be used to create slow-roll inflation. A minimal example of such a scheme presented here following [3] has several attractive features:

- There is an initial period of topological inflation which sets the initial conditions for slow roll inflation and avoids the rapid roll and thermal roll problems usually associated with the racetrack potential. As a result there is no difficulty in having our universe settle in the weakly coupled minimum of the dilaton potential and not in the runaway non-interacting minimum.
- The racetrack potential with simple moduli dependence has saddle points which lead to slow roll inflation capable of generating the observed density fluctuations with a spectral index smaller than but close to 1. Due to the initial period of topological inflation the initial conditions for the slow roll inflation are automatically set without fine tuning.

Another far-reaching feature of multi-field inflation is the presence of isocurvature perturbations. In some cases they may have a dramatic impact on the final spectrum of primordial fluctuations which trigger structure formation. Quite often they easily lower the scalar spectral index below 1, thus naturally reproducing the tendency implied by WMAP3 data.

Acknowledgments

References

1. D. N. Spergel *et al.*, "Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology," arXiv:astro-ph/0603449.

2. W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, "Inflation model constraints from the Wilkinson microwave anisotropy probe three-year data," arXiv:astro-ph/0605338.
3. Z. Lalak, G. G. Ross and S. Sarkar, "Racetrack inflation and assisted moduli stabilisation," Nucl. Phys. B **766** (2007) 1 [arXiv:hep-th/0503178].
4. J. Ellis, Z. Lalak, S. Pokorski and K. Turzynski, "The price of WMAP inflation in supergravity," JCAP **0610**, 005 (2006) [arXiv:hep-th/0606133].
5. A. Vilenkin, "Topological inflation," Phys. Rev. Lett. **72**, 3137 (1994) [arXiv:hep-th/9402085].
6. A. D. Linde and D. A. Linde, "Topological defects as seeds for eternal inflation," Phys. Rev. D **50**, 2456 (1994) [arXiv:hep-th/9402115].
7. Z. Lalak, D. Langlois, S. Pokorski and K. Turzynski, "Curvature and isocurvature perturbations in two-field inflation," arXiv:0704.0212 [hep-th], *to appear in JCAP*.
8. J. R. Bond, L. Kofman, S. Prokushkin and P. M. Vaudrevange, arXiv:hep-th/0612197.
9. J. P. Conlon and F. Quevedo, JHEP **0601**, 146 (2006) [arXiv:hep-th/0509012].