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Predictions from non trivial Quark-Lepton complementarity

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The complementarity between the quark and lepton mixing matrices is shown to provide robust predictions. We obtain these predictions by first showing that the matrix V_M, product of the quark (CKM) and lepton (PMNS) mixing matrices, may have a zero (1,3) entry which is favored by experimental data.

We obtain that any theoretical model with a vanishing (1,3) entry of V_M that is in agreement with quark data, solar, and atmospheric mixing angle leads to $\theta_{13}^{PMNS} = (9^{+1}_{-2})^\circ$. This value is consistent with the present 90% CL experimental upper limit. We also investigate the prediction on the lepton phases. We show that the actual evidence, under the only assumption that the correlation matrix V_M product of CKM and $PMNS$ has a zero in the entry $(1,3)$, gives us a prediction for the three CP-violating invariants J, S_1, and S_2. A better determination of the lepton mixing angles will give stronger prediction for the CP-violating invariants in the lepton sector. These will be tested in the next generation experiments. Finally we compute the effect of non diagonal neutrino mass in $l_i \to l_j\gamma$ in SUSY theories with non trivial Quark-Lepton complementarity and a flavor symmetry. The Quark-Lepton complementarity and the flavor symmetry strongly constrain the theory and we obtain a clear prediction for the contribution to $\mu \to e\gamma$ and the τ decays $\tau \to e\gamma$ and $\tau \to \mu\gamma$. If the Dirac neutrino Yukawa couplings are degenerate but the low energy neutrino masses are not degenerate, then the lepton decays are related among them by the V_M entries. On the other hand, if the Dirac neutrino Yukawa couplings are hierarchical or the low energy neutrino masses are degenerate, then the prediction for the lepton decays comes from the CKM hierarchy.

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1. Introduction

The actual experimental situation is such that we are very close to obtain a theory of flavor that is able to explain in a easy way all the Standard Model masses and mixing[1−15]. The last but not least experimental ingredient have been the neutrino data. In fact, after the recent experimental evidences about neutrino physics we know very well almost all the parameters in the quark and lepton sectors. We measured all the quark and charged lepton masses, and the value of the difference between the square of the neutrino masses $\Delta m^2_{12} = m^2_1 - m^2_2$ and $|\Delta m^2_{23}| = |m^2_3 - m^2_2|$. We also know the value of the quark mixing angles and phases, and the two mixing angles θ_{12} and θ_{23} in the lepton sector. The challenger for the next future [16−18] will be to determine the sign of δm^2_{23} (i.e. the hierarchy in the neutrino sector), the absolute scale of the neutrino masses, and the value of the 3rd lepton mixing angle θ_{13} (in particular if is it zero or not). Finally, if θ_{13} is not too small, there is a hope to measure the CP violating phases.

From all these results we are able to extract strong constraints on the flavor structure of the SM. In particular the neutrino data were determinant to clarify the role of the discrete symmetry in flavor physics. The neutrino experiments confirm [19−27] that the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [28,29] lepton mixing matrix U_{PMNS} contains large mixing angles. For example the atmospheric mixing θ^{PMNS}_{23} is compatible with 45° and the solar mixing θ^{PMNS}_{12} is $\approx 34^\circ$ [30−42]. These results should be compared with the third lepton mixing angle θ^{PMNS}_{13} which is very small and even compatible with zero [43,44], and with the quark mixing angles in the CKM matrix [45,46]. The disparity that nature indicates between quark and lepton mixing angles has been viewed in terms of a 'Quark-Lepton complementarity' (QLC) [47−51] which can be expressed in the relations

$$\theta^{PMNS}_{12} + \theta^{CKM}_{12} \simeq 45^\circ\,; \qquad \theta^{PMNS}_{23} + \theta^{CKM}_{23} \simeq 45^\circ\,. \tag{1}$$

Possible consequences of QLC have been investigated in the literature and in particular a simple correspondence between the U_{PMNS} and U_{CKM} matrices has been proposed [52−55] and analyzed in terms of a correlation matrix [56−64]. The relations in eq. (1) are related to the parametrization used for the CKM and PMNS mixing matrix. From a more general point of view, we can define a correlation matrix V_M as the product of the PMNS and CKM mixing matrices, $V_M = U_{CKM}\, U_{PMNS}$. A lot of efforts have been done to obtain the *most favorite* pattern for the matrix V_M [47,53−59]. The naive QLC relations in eq. (1) seems to implies V_M to be Bi-Maximal, i.e. in the standard parametrization it contains two maximal mixing angle, and a third angle to be zero. Despite the naive relations between the PMNS and CKM angles, a detailed analysis shows that the correlation matrix $V_M = U_{CKM}U_{PMNS}$ is phenomenologically compatible with a TriBi-Maximal pattern, and only marginally with a Bi-Maximal pattern. From actual experimental evidences a non trivial Quark-Lepton complementarity arises [58], i.e. we learn that V_M Bi-Maximal, although it is not ruled out by the experiments, is excluded at 90% CL in non SUSY models, or in SUSY models with $\tan\beta < 40$ where the RGE correction are negligible [65−69], and a non trivial Quark-Lepton complementarity arises [58]. Future experiments on neutrino physics, and in particolar in the determination of θ_{23} and the CP violating parameter J, will be able to better clarify if a trivial Quark-Lepton complementarity, i.e. V_M Bi-Maximal, is ruled out in favor of a non trivial Quark-Lepton complementarity, i.e. V_M TriBi-Maximal or even more structured [5]. Unitarity then implies $U_{PMNS} = U^{\dagger}_{CKM} V_M$ and one may ask where do the large lepton mixings come from? Is this information implicit in the form of the V_M matrix? This question has been widely investigated in the literature, but its answer is still open. However the fact V_M has a clear non trivial structure and the strong indication of gauge coupling unification allow us to obtain in a straightforward way constraints on the high energy spectrum too. Within this framework we get some informations about flavor physics from the correlation matrix V_M itself. It is very impressive that for some discrete flavor symmetries such as A_4 dynamically broken into Z_3, as in Refs. 6 and 10, or S_3 softly broken into S_2, as in Ref. 4, the TriBi-Maximal structure appears in a natural way. In fact in some Grand Unification Theories (GUTs) the direct QLC correlation between the CKM and the $PMNS$ mixing matrix can be obtained. In this class of models, the V_M matrix is determined by the heavy Majorana neutrino mass matrix [47,54]. Moreover as long as quarks and leptons are inserted in the same representation of the underlying gauge group, we need to include in our definition of V_M arbitrary but non trivial phases between the quark and lepton matrices. Hence we will generalize the relation $V_M = U_{CKM} \cdot U_{PMNS}$ to

$$V_M = U_{CKM} \cdot \Omega \cdot U_{PMNS} \tag{2}$$

where the quantity Ω is a diagonal matrix $\Omega = \mathrm{diag}(e^{\mathrm{i}\omega_i})$ and the three phases ω_i are free parameters (in the sense that they are not restricted by present experimental evidence).

In this paper we will show how the investigation of the correlation matrix V_M based on eq. (2) implies that there is a zero texture of V_M, namely $V_{M_{13}} = 0$. The conclusion for matrix V_M is that the correlation between the matrices U_{CKM} and U_{PMNS} is rather nontrivial. Then, by using this fact we will report the predictions that can be obtained from experimental data and QLC for θ_{13}^{PMNS}, CP violating parameters in the lepton sector and the lepton number violating decays. The plan of the work is the following. In section **2** we study the numerical ranges of V_M entries with the aid of a Monte Carlo simulation and we will show that the vanishing of the $(1,3)$ entry is favored by the data analysis. After that we present the matter from a different point of view: we start from a zero $(1,3)$ V_M entry (e.g. a Bi-Maximal or TriBi-Maximal matrix) and we derive the consequent predictions. In section **3** we get a small value for θ_{13}^{PMNS} with a sharp prediction

$$\theta_{13}^{PMNS} = (9 \pm \tfrac{1}{2})^\circ \,, \tag{3}$$

for the U_{PMNS} lepton mixing angle through

$$U_{PMNS} = (U_{CKM} \cdot \Omega)^{-1} \cdot V_M \tag{4}$$

In sec. **4**, with the aid of the Monte Carlo simulation, we study the numerical correlations of the lepton CP violating phases J, S_1, and S_2 with respect to the mixing angle θ_{12}^{PMNS}. In Sec. **5** we compute the value of the contribution to the $l_i \to l_j\gamma$ processes from a non diagonal Dirac neutrino Yukawa coupling. By using the non trivial Quark-Lepton complementarity and the see-saw mechanism we will compute the explicit spectrum of the heavy neutrino. This will allow us to investigate the relevance of the form of V_M in the $l_i \to l_j\gamma$.

2. Which V_M does the phenomenology imply?

In this section we investigate the value of the V_M matrix entries concentrating in particular in the (1,3) entry and the important mixing angle $\theta_{13}^{V_M}$ to which it is directly related. We then explicitly study the allowed values of the V_M angles, finally concluding that $\sin^2\theta_{13}^{V_M} = 0$ is the value most favored by the data. We will be using the Wolfenstein parameterization [70] of the U_{CKM} matrix in its unitary form [71] where one has the relation

$$\sin\theta_{12}^{CKM} = \lambda \qquad \sin\theta_{23}^{CKM} = A\lambda^2 \qquad \sin\theta_{13}^{CKM} e^{-\mathrm{i}\delta^{CKM}} = A\lambda^3(\rho - \mathrm{i}\eta) \tag{5}$$

to all orders in λ. The lepton mixing matrix U_{PMNS} is parameterized as

$$U_{PMNS} = U_{23}\cdot\Phi\cdot U_{13}\cdot\Phi^\dagger\cdot U_{12}\cdot\Phi_m. \tag{6}$$

Here Φ and Φ_m are diagonal matrices containing the Dirac and Majorana CP violating phases, respectively $\Phi = \mathrm{diag}(1,1,e^{\mathrm{i}\phi})$ and $\Phi_m = \mathrm{diag}(e^{\mathrm{i}\phi_1}, e^{\mathrm{i}\phi_2}, 1)$, so that

$$U_{PMNS} = \begin{pmatrix} e^{\mathrm{i}\,\phi_1} c_{12}\, c_{13} & e^{\mathrm{i}\,\phi_2} c_{13}\, s_{12} & s_{13} e^{-\mathrm{i}\,\phi} \\ e^{\mathrm{i}\,\phi_1}\left(-c_{23}\, s_{12} - e^{\mathrm{i}\,\phi}\, c_{12}\, s_{13}\, s_{23}\right) & e^{\mathrm{i}\,\phi_2}\left(c_{12}\, c_{23} - e^{\mathrm{i}\,\phi}\, s_{12}\, s_{13}\, s_{23}\right) & c_{13}\, s_{23} \\ e^{\mathrm{i}\,\phi_1}\left(-e^{\mathrm{i}\,\phi}\, c_{12}\, c_{23}\, s_{13} + s_{12}\, s_{23}\right) & e^{\mathrm{i}\,\phi_2}\left(-e^{\mathrm{i}\,\phi}\, c_{23}\, s_{12}\, s_{13} - c_{12}\, s_{23}\right) & c_{13}\, c_{23} \end{pmatrix}$$

The investigation we perform for the V_M matrix starts from the fundamental equation $V_M = U_{CKM}\cdot\Omega\cdot U_{PMNS}$ and uses the experimental ranges and constraints on lepton mixing angles. We resort to a Monte Carlo simulation with two-sided Gaussian distributions around the mean values of the observables. We use the updated values for the CKM and $PMNS$ mixing matrix, given at 95%CL by [72]

$$\begin{aligned} \lambda = 0.2265^{+0.0040}_{-0.0041}\ , &\quad A = 0.801^{+0.066}_{-0.041}\ , \\ \overline{\eta} = 0.189^{+0.182}_{-0.114}\ , &\quad \overline{\rho} = 0.358^{+0.086}_{-0.085}\ , \end{aligned} \tag{7}$$

$$\text{with} \qquad \rho + i\eta = \frac{\sqrt{1-A^2\lambda^4}(\overline{\rho}+i\overline{\eta})}{\sqrt{1-\lambda^2}\,[1-A^2\lambda^4(\overline{\rho}+i\overline{\eta})]}\ ; \tag{8}$$

and [a] [30–42]

$$\begin{aligned} \sin^2\theta_{23}^{PMNS} = 0.44\times\left(1^{+0.41}_{-0.22}\right)\ , &\quad \sin^2\theta_{12}^{PMNS} = 0.314\times\left(1^{+0.18}_{-0.15}\right)\ , \\ \sin^2\theta_{13}^{PMNS} = \left(0.9^{+2.3}_{-0.9}\right)\times 10^{-2}\ . & \end{aligned} \tag{9}$$

[a]The lower uncertainty for $\sin^2\theta_{13}$ is purely formal, and correspond to the positivity constraint $\sin^2\theta_{13} \geq 0$.

With the aid of a Monte Carlo program we generated the values for each variable: for the sine square of the lepton mixing angles and for the quark parameters $A,\ \lambda,\ \bar{\rho},\ \bar{\eta}$ we took two sided Gaussian distributions with central values and standard deviations taken from eqs. (7-9). For the unknown phases we took flat random distributions in the interval $[0, 2\pi]$. We divided each variable range into short bins and counted the number of occurrences in each bin for all the variables, having run the program 10^6 times. In this way the corresponding histogram is smooth and the number of occurrences in each bin is identified with the probability density at that particular value. A comparatively high value of this probability density extending over a wide range in the variable domain means a high probability for the variable to lie in this range, therefore that such range is 'favored' by the data being used as Monte Carlo input. Conversely higher probability implies better compatibility with experimental data, while lower probability means poor or no compatibility with data.

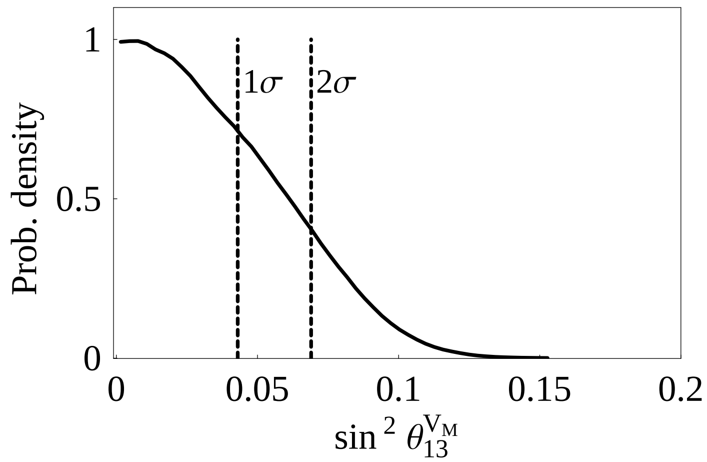

Fig. 1. The distribution, normalized to one at the maximum, of $\sin^2\theta_{13}^{V_M}$ obtained from the definition of the correlation mixing matrix V_M given in eq. (2) by using a Monte Carlo simulation of all the experimental data. We also plot the 1σ and the 2σ lines.

The range of $\tan^2\theta_{23}^{V_M}$ which is compatible with experiments at 90%CL is the interval $[0.35, 1.4]$, so that $\tan^2\theta_{23}^{V_M} = 1.0$ is consistent with data. For $\tan^2\theta_{12}^{V_M}$ we obtain a range between 0.25 and 1.1 at 90%CL and so $\tan^2\theta_{12}^{V_M} = 1.0$ (which corresponds to a Bi-Maximal matrix) only within 3σ. Moreover the value $\tan^2\theta_{12}^{V_M} = 0.5$ (which corresponds to a TriBi-Maximal matrix), is well inside the allowed range. We checked that for $\tan^2\theta_{12}^{V_M} = 0.3,$ and 0.5 the resulting distribution for $\tan^2\theta_{12}^{PMNS}$ is compatible with the experimental data. Instead maximal $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ taken together are disfavored, as the solar angle is hardly compatible with the corresponding allowed interval. We also checked that the distribution of $\tan^2\theta_{23}^{PMNS}$ for $\tan^2\theta_{23}^{V_M} \in \{0.5, 1.0, 1.4\}$ with $\tan^2\theta_{12}^{V_M} = 0.5$ are compatible with the experimental data.

In fig.1 we plot the distribution for $\sin^2\theta_{13}^{V_M}$. We see that $\sin^2\theta_{13}^{V_M} = 0$ is not only allowed by the experimental data, but also it is the preferred value. In the next section we will see that this has important consequences in the model building of flavor physics.

3. Prediction for θ_{13}^{PMNS}

In this section we investigate the consequences of a V_M correlation matrix with zero (1,3) entry on the still experimentally undetermined θ_{13}^{PMNS} mixing angle. In particular we will see that the θ_{13}^{PMNS} prediction arising from eq. (2) or, equivalently,

$$U_{PMNS} = (U_{CKM}\cdot\Omega)^{-1}\cdot V_M \tag{10}$$

is quite stable against variations in the form of V_M allowed by the data.

As previously shown (see section **2**), the data favors a vanishing (1,3) entry in V_M. So in the whole following analysis we fix $\sin^2\theta_{13}^{V_M} = 0$. We allow the U_{CKM} parameters to vary, with a two-sided Gaussian distribution, within the experimental ranges given in eq. (7), while for the Ω phases in eq. (10) we take flat distributions in the interval $[0, 2\pi]$. We make Monte Carlo simulations for different values of $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ mixing angles, allowing $\tan^2\theta_{12}^{V_M}$ and $\tan^2\theta_{23}^{V_M}$ to vary respectively within the intervals $[0.3, 1.0]$ and $[0.5, 1.4]$ in consistency with the lepton and quark mixing angles (see section **2**).

From eq. (10), the parameterization of the CKM mixing matrix in eq. (5) and the definition of the phase matrix Ω we get

$$\begin{aligned}(U_{PMNS})_{13} = e^{-i\omega_1}\Big[\left(1-\frac{\lambda^2}{2}\right)\sin\theta_{13}^{V_M}e^{-i\phi^{V_M}} - \lambda\sin\theta_{23}^{V_M}\cos\theta_{13}^{V_M}\\ +A\lambda^3(-\rho+i\,\eta+1)\cos\theta_{23}^{V_M}\cos\theta_{13}^{V_M}+O(\lambda^4)\Big]\,,\end{aligned} \tag{11}$$

so that

$$\sin^2\theta_{13}^{PMNS} = \sin^2\theta_{23}^{V_M}\lambda^2 + O(\lambda^3)\,, \tag{12}$$

where we have used the fact that $\sin^2\theta_{13}^{V_M} = 0$ and $A \approx O(1)$. We see that $\sin^2\theta_{13}^{PMNS}$ does not depend on $\tan^2\theta_{12}^{V_M}$. For this reason the parameter $\sin^2\theta_{13}^{PMNS}$ needs to be studied as a function of $\tan^2\theta_{23}^{V_M}$ only. Fixing for definiteness $\tan^2\theta_{12}^{V_M} = 0.5$ and taking the three different values $\tan^2\theta_{23}^{V_M} \in \{0.5, 1.0, 1.4\}$, we computed the corresponding distributions of $\sin^2\theta_{13}^{PMNS}$. We note that these values of $\tan^2\theta_{23}^{V_M}$ practically cover the whole range consistent with the data. It is seen that the $\sin^2\theta_{13}^{PMNS}$ distributions are quite sharply peaked around maxima of 7.3°, 8.9° and 9.8°. Recalling that the shift of this maximum is effectively determined by the parameter $\tan^2\theta_{23}^{V_M}$ which was chosen to span most of its physically allowed range, it is clear that we have a stable prediction for θ_{13}^{PMNS}.

In order to better clarify this stability, we show in fig. 2 the mean and the standard deviation of $\sin^2\theta_{13}^{PMNS}$ obtained with our Monte Carlo simulation for

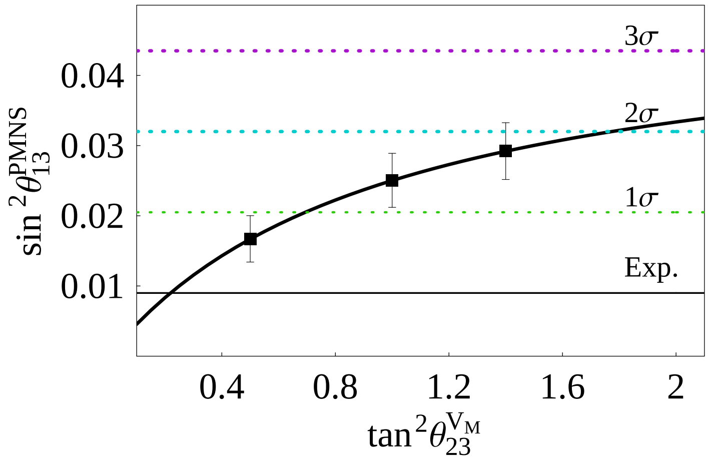

Fig. 2. The allowed values for $\sin^2\theta_{13}^{PMNS}$ as a function of $\tan^2\theta_{23}^{V_M}$ under the assumption that $\sin^2\theta_{13}^{V_M} = 0$. We report the central and 3σ values, and the approximate analytical dependence given in eq. (12). We also plot the experimental central value, the 1σ, the 2σ, and the 3σ. We fixed $\tan^2\theta_{12}^{V_M} = 0.5$ for definiteness.

the three chosen values of $\tan^2\theta_{23}^{V_M}$. In addition we plot the analytic dependence of $\sin^2\theta_{13}^{PMNS}$ given by eq. (12) with the central value of λ, the best fit point of $\sin^2\theta_{13}^{PMNS}$ and its 1σ, 2σ and 3σ from standard analysis. Our prediction for θ_{13}^{PMNS} then follows from the experimental data on λ ,A, ρ, η, $\tan^2\theta_{12}^{PMNS}$ and $\tan^2\theta_{23}^{PMNS}$ and the values of $\tan^2\theta_{12}^{V_M}$, $\tan^2\theta_{23}^{V_M}$ are taken in the intervals $[0.3, 1.0]$, $[0.5, 1.4]$ respectively, as allowed by the data. For a vanishing $(1, 3)$ entry of the matrix V_M we finally find θ_{13}^{PMNS} in the interval $[7°, 10°]$.

To conclude this section we note that another prediction for a small θ_{13}^{PMNS} has recently been derived [63]

$$\theta_{13}^{PMNS} = 9° + O(sin^3\theta_{12}^{CKM}). \tag{13}$$

This follows from an assumed Bi-maximality of a matrix relating Dirac to Majorana neutrino states together with the assumption that neutrino mixing is described by the CKM matrix at the grand unification scale. Our approach on the other hand is free from any *ad hoc* assumptions. We show that it is a zero texture of the V_M correlation matrix, namely $V_{M_{13}} = 0$, together with all the experimental values of the quark and lepton mixing angles, that predicts $\theta_{13}^{PMNS} = (9\pm\frac{1}{2})°$. More importantly, in sec. **2** we show that the vanishing of this entry is favored by the data. Condition $V_{M_{13}} = 0$ is compatible with V_M being Bi-Maximal (i.e. with two angles of 45° and a vanishing one), TriBi-Maximal (i.e. with one angle of 45°, one with $\tan^2\theta = 0.5$ and a third vanishing one) or of any other form. Furthermore we make use of a phase matrix Ω, see eq. (2), that takes account of the mismatch between the quark and lepton phases and consider Majorana phases in the U_{PMNS} matrix with a flat random distribution.

4. CP violating invariants in the lepton sector

In this section we investigate the consequences of a V_M correlation matrix with a zero $(1,3)$ entry on the undetermined CP violating parameters in the lepton sector. There are two kind of invariants parameterizing CP violating effect. The Jarlskog invariant J that parametrizes the effects related to the Dirac phase, and the two invariants S_1 and S_2 that parametrize the effects related to the Majorana phases. The J invariant describes all CP breaking observables in neutrino oscillations. It is the equivalent of the Jarlskog invariant in the quark sector. It is given by

$$J = Im\{U_{\nu_e\nu_1}U_{\nu_\mu\nu_2}U^*_{\nu_e\nu_2}U^*_{\nu_\mu\nu_1}\}\,. \tag{14}$$

In the parametrization of eq. (7) one has

$$J = \frac{1}{8}\sin 2\theta_{12}\ \sin 2\theta_{23}\ \sin 2\theta_{13}\ \cos\theta_{13}\ \sin\phi\,. \tag{15}$$

Then we have the two invariants S_1 and S_2 that are related to the Majorana phases. They are

$$\begin{aligned} S_1 &= Im\{U_{\nu_e\nu_1}U^*_{\nu_e\nu_3}\} \\ S_2 &= Im\{U_{\nu_e\nu_2}U^*_{\nu_e\nu_3}\} \end{aligned} \tag{16}$$

In the parametrization of eq. (7) we have

$$\begin{aligned} S_1 &= \frac{1}{2}\cos\theta_{12}\sin 2\theta_{13}\sin(\phi+\phi_1) \\ S_2 &= \frac{1}{2}\sin\theta_{12}\sin 2\theta_{13}\sin(\phi+\phi_2) \end{aligned} \tag{17}$$

The two Majorana phases appear in S_1 and S_2 but not in J.

As show in sec. **2**, the data favors a vanishing $(1,3)$ entry in the correlation matrix V_M [58]. So in the whole analysis we fix $\sin^2\theta_{13}^{V_M} = 0$. Moreover $\tan^2\theta_{12}^{V_M}$ and $\tan^2\theta_{23}^{V_M}$ are allowed to vary respectively within the intervals $[0.3, 1.0]$ and $[0.5, 1.4]$. We allow the U^{CKM} parameters to vary, with a two-sided Gaussian distribution, within the experimental ranges given in eq. (7). For the Ω phases in eq. (2) we take flat distributions in the interval $[0, 2\pi]$. We make Monte Carlo simulations for different values of $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ mixing angles, allowing $\tan^2\theta_{12}^{V_M}$ and $\tan^2\theta_{23}^{V_M}$ to vary respectively within their allowed intervals, in consistency with the lepton and quark mixing angles. From eq. (15), by using the fact that θ_{13} is small and that θ_{23} is maximal, we get

$$J \approx \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{13}\sin\phi$$

This expression tells us that the J parameter is within the range $|J| < 0.042$. However there is a non trivial correlation between J and θ_{12}^{PMNS}. Because the CKM is given by the experimental data, and $(V_M)_{13}$ is fixed to be zero, the phase ϕ and the θ_{13}^{PMNS} angle are almost fixed as a function of θ_{12}^{PMNS}.

In figs. 3-5 we report the result of our simulation for J. We plot the correlation between the J invariant and $\sin^2\theta_{12}^{PMNS}$ for V_M Bi-Maximal (fig. 3), TriBi-Maximal

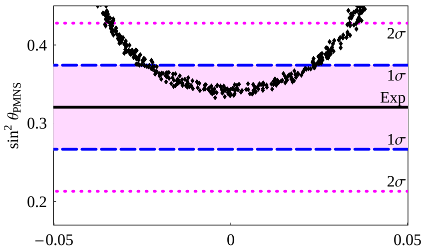

Fig. 3. The correlation between the Dirac CP violating parameter J and $\sin^2\theta_{12}^{PMNS}$ for V_M Bi-Maximal. We also plot the experimental central value, the 1σ, and the 2σ for $\sin^2\theta_{12}^{PMNS}$.

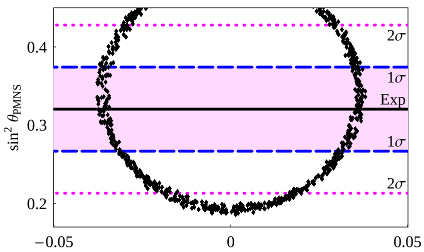

Fig. 4. The correlation between the Dirac CP violating parameter J and $\sin^2\theta_{12}^{PMNS}$ for V_M TriBi-Maximal.

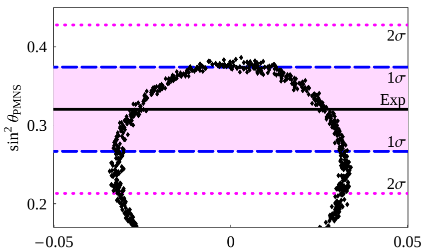

Fig. 5. The correlation between the Dirac CP violating parameter J and $\sin^2\theta_{12}^{PMNS}$ for V_M such that $\tan^2\theta_{12}^{V_M} = 0.4$.

(fig. 4), and V_M with $\tan^2\theta_{12}^{V_M} = 0.4$ (fig. 5). First of all, from fig. 3, we see that the *solar* mixing angle θ_{12}^{PMNS} is constrained to have $\sin^2\theta_{12}^{PMNS} > 0.36$ for V_M Bi-Maximal. From figs. 3-5 we see the correlation between the structure of V_M and the CP violating invariant J. In particular, for V_M Bi-Maximal J is close to zero. For V_M TriBi-Maximal $|J|$ is around its maximum value 0.042. Finally for V_M such that $\tan^2\theta_{12}^{V_M} = 0.4$ we get that J can be any value between -0.04 and 0.04. We also see that a better determination of the $\sin^2\theta_{12}^{PMNS}$ could give a stronger prediction for the J invariant in the case of V_M TriBi-maximal.

Similar results hold for S_1 and S_2 (the plots have similar shapes). The expressions in eqs. (17) give us the range for these invariants:

$$|S_1| < 0.14 \qquad |S_2| < 0.11 \tag{18}$$

We obtain that for V_M Bi-Maximal the Majorana CP invariant S_1 is close to zero, for V_M TriBi-Maximal S_1 is around 0.13. Finally for V_M such that $\tan^2\theta_{12}^{V_M}=0.4$ we obtain that S_1 can be any value between -0.14 and 0.14. We see that also in this case a better determination of the θ_{12}^{PMNS} mixing angle will give a stronger constraint for the S_1 and S_2 invariant for V_M TriBi-Maximal. As for J, these correlations of S_1 and S_2 with respect to θ_{12}^{PMNS} are predictions of any theoretical model that gives a relation of the type $V_M = U^{CKM}\,\Omega\,U^{PMNS}$ with $(V_M)_{13}=0$. In the next section we will show how to construct an explicit model that predict $(V_M)_{13}=0$.

5. $l_i \to l_j\gamma$

In this section we compute the effect of non diagonal neutrino mass on $l_i \to l_j\gamma$ in SUSY theories with non trivial Quark-Lepton complementarity an a flavor symmetry. The correlation matrix $V_M = U_{CKM}U_{PMNS}$ is such that its $(1,3)$ entry, as preferred by the actual experimental data, is zero. We obtain a clear prediction for the contribution to $l_i \to l_j\gamma$. There are three cases. They depend on the spectrum of the Dirac neutrino mass matrix and the low energy neutrino. We may have: 1) hierarchical Dirac neutrino eigenvalues (in this case we have very hierarchical right-handed neutrino masses); 2) degenerate Dirac neutrino eigenvalues, with non degenerate low energy neutrino masses (in this case the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum); 3) degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum (that implies right-handed neutrino close to be degenerate). For each of these cases we have different contributions to $l_i \to l_j\gamma$. We will show that only when Dirac neutrino eigenvalues are degenerate and low energy neutrino masses are not degenerate, then the explicit form of V_M plays an important role.

The contribution at first order approximation to the process $l_i \to l_j\gamma$ in SUSY models is given by

$$BR(l_i\to l_j\gamma) \propto \frac{\Gamma(l_i\to e\nu\nu)}{\Gamma(l_i)}\frac{\alpha^3}{G_f m_s^8 v_u^4}\tan^2\beta\left(\frac{3m_0+A_0}{8\pi^2}\right)^2\left|\left(\tilde{M}_D L \tilde{M}_D^\dagger\right)_{ij}\right|^2$$

where m_0 is the universal scalar mass, A_0 is the universal trilinear coupling parameter, $\tan\beta$ is the ratio of the vacuum expectation values of the up and down Higgs doublets, and m_s is a typical mass of superparticles with [73] $m_s^8 \approx 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$, where $M_{1/2}$ is the gaugino mass. The matrix $L_{ij} = \mathbf{1}_{ij}\log M_x/M_i$ takes into account the RGE effects on the Majorana right-handed neutrino masses. In fact the eq. (19) is computed in the base where the Yukawa of the charged lepton and the Majorana neutrino mass are diagonal. Eq. (19) is valid in the base where right-handed Majorana neutrino mass matrix, charged lepton mass matrix and weak gauge interactions are diagonal. The experimental limit for the branching ratio of $\mu\to e\gamma$ is 1.2×10^{-11} at 90% of confidence level [74] and it could go down to 10^{-14} as proposed by MEG collaboration.

In supergravity theories if the effective Lagrangian is defined at a scale higher than the Grand Unification scale, then the matter fields have to respect the underlying gauge and flavor symmetry. Hence, we expect quark-lepton correlations among entries of the sfermion mass matrices. In other words, the quark-lepton unification seeps also into the SUSY breaking soft sector [75]. In general we do not get strongly renormalization effects on flavor violating quantities from the heavy neutrino scale to the electroweak scale because the absence of flavor violation. In fact the remaining flavor violation related to the low energy neutrino sector gives negligible contribution with the exception of the case with high degenerated neutrino and $\tan\beta > 40$ [69,65].

Let be M_R the Majorana mass matrix for the right neutrino and M_D the Dirac mass matrix. Under the assumption that the low energy neutrino masses are given by the see-saw of Type I we have that the light neutrino mass matrix is given by

$$M_\nu = M_D \frac{1}{M_R} M_D^T \,. \tag{19}$$

The lepton mixing matrix is

$$U_{PMNS} = U_l^\dagger U_\nu = U_l^\dagger U_0 V_M \,. \tag{20}$$

where U_l, U_ν and U_0 diagonalize on the left respectively the charged lepton, M_ν and M_D. The mixing matrix V_M is here defined to verify the equality $U_\nu \equiv U_0 V_M$ and is such that

$$V_M M_\nu^\Delta V_M^T \equiv \mathcal{C} = M_D^\Delta V_0^\dagger \frac{1}{M_R} V_0^\star M_D^\Delta \,, \tag{21}$$

In the quark sector we introduce Y_u and Y_d to be the Yukawa matrices for up and down sectors. They can be diagonalized by

$$Y_u = U_u Y_u^\Delta V_u^\dagger \text{ and } Y_d = U_d Y_d^\Delta V_d^\dagger \,, \tag{22}$$

where the Y^Δ are diagonal and the Us and Vs are unitary matrices. Then the quark mixing matrix is given by

$$U_{CKM} = U_u^\dagger U_d \,. \tag{23}$$

Notice that if there is a flavor symmetry that constrains the Yukawa couplings in such a way that the diagonalizing unitary matrices are fixed then the entries of Y_l can still be very different from the entries of Y_d^T. However both Yukawa matrices are diagonalized by the same mixing matrices. This is exactly the case in the presence of an A_4 discrete flavor symmetry dinamically broken into Z_3 [6,10] and can be partially true in the case of S_3 softly broken into S_2 [4]. In this case we have

$$Y_l \approx Y_d^T \to U_l \simeq V_d^\star \,.$$

In the same way, if we call Y_ν the Yukawa coupling that will generate the Dirac neutrino mass matrix M_D, we have also the relation

$$Y_\nu \approx Y_u^T \to U_0 \simeq V_u^\star \,. \tag{24}$$

This relation, together with the previous one, implies

$$U_{PMNS} \simeq V_d^T V_u^\star V_M \,.$$

If the Yukawa matrices are diagonalized by similar matrix on the left and on the right, for example in minimal renormalizable $SO(10)$ with only small contributions from the antisymmetric representations such as **120** or more important in models where the diagonalization is strongly constrained by the flavor symmetry, the previous relationship translates into a relation between U_{PMNS}, U_{CKM} and V_M. In fact we have

$$Y_u \simeq Y_u^T \rightarrow V_u^\star = U_u \text{ and } Y_d \simeq Y_d^T \rightarrow V_d^\star = U_d \,.$$

Finally we get that V_M satisfies eq. (2). The form of V_M can be obtained under some assumptions about the flavor structure of the theory. Some flavor models give for example a correlation V_M with $(V_M)_{13} = 0$. As a consequence of the from of the non trivial Quark-Lepton complementarity there are some predictions for the model. For example the prediction for θ_{13}^{PMNS} of sec. **3** [58] and the correlations between CP violating phases and the mixing angle θ_{12} of sec. **4** [5].

5.1. $\tilde{M}_D$ *in non trivial Quark-Lepton complementarity*

Let us investigate the value of Dirac neutrino mass matrix $\tilde{M}_D$ in the base where right-handed Majorana neutrino mass matrix, charged leptons mass matrix and weak gauge interactions are diagonal. We define the unitary matrix V_R by the diagonalization of M_R

$$V_R M_R^\Delta V_R^T = M_R \,. \tag{25}$$

and we obtain

$$\tilde{M}_D = U_l^\dagger M_D V_R^\star \,. \tag{26}$$

We want now to related this $\tilde{M}_D$ matrix to the CKM mixing matrix by using the previous result. First of all we rewrite this matrix as

$$\begin{aligned} \tilde{M}_D &= U_l^\dagger M_D V_R^\star \\ &= U_l^\dagger U_0 M_D^\Delta V_0^\dagger V_R^\star \,. \end{aligned} \tag{27}$$

Then we notice that the matrix $V_0^\dagger V_R^\star$ is related via the $\mathcal{C}$ matrix to the diagonal low energy neutrino mass matrix m_{low}^Δ and to V_M. In fact we have

$$\begin{aligned} V_M m_{low}^\Delta V_M^T &= \mathcal{C} \\ &= M_D^\Delta V_0^\dagger \frac{1}{M_R} V_0^\star M_D^\Delta \\ &= M_D^\Delta V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star M_D^\Delta \end{aligned} \tag{28}$$

where we used the inverse of eq. (25)

$$V_R^\star \frac{1}{M_R^\Delta} V_R^\dagger = \frac{1}{M_R} \,. \tag{29}$$

We multiple on the left and on the right both sides of eq. (28) by $1/M_D^\Delta$ and we get

$$V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star = \frac{1}{M_D^\Delta} V_M m_{low}^\Delta V_M^T \frac{1}{M_D^\Delta}\,. \tag{30}$$

Once we computed the $V_0^\dagger V_R^\star$ matrix form eq. (30), by using eq. (27), we get

$$\begin{aligned}\tilde{M}_D &= U_l^\dagger U_0 M_D^\Delta V_0^\dagger V_R^\star \\ &= U_{PMNS} V_M^\dagger M_D^\Delta V_0^\dagger V_R^\star \\ &= \Omega^\dagger U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^\star\,,\end{aligned} \tag{31}$$

where in the last line we used the relations in eq. (20) and (2).

5.2. Fully *determination of* $V_0^\dagger V_R^\star$ *and* M_R^Δ

Eq. (31) is the equivalent of the general formula [76] in presence of non trivial Quark-Lepton complementarity. We observe that the main modification is the presence of $U_{CKM}^\dagger$ instead of U_{PMNS} thanks to the fact the these matrices are related each other through V_M as shown in the relation of eq. (2). Let us now compute the $V_0^\dagger V_R^\star$ matrix in a general scenario.

In the following we use the experimental constraint from [58] that says us that $(V_M)_{13}$ is zero and the allowed ranges for $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ are [58]

$$\tan^2\theta_{12}^{V_M} \in [0.3, 1.0] \qquad \text{and} \qquad \tan^2\theta_{23}^{V_M} \in [0.5, 1.4]\,. \tag{32}$$

Let us denote with m_i the complex low energy neutrino masses obtained after the see-saw ($m_{low}^\Delta = \{m_1, m_2, m_3\}$), and with M_i the eigenvalues of the Dirac neutrino mass matrix M_D ($M_D^\Delta = \{M_1, M_2, M_3\}$). We get

$$V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^\dagger V_0^\star = \begin{pmatrix} \frac{(m_1 c_{12}^2 + m_2 s_{12}^2)}{M_1^2} & \frac{-(m_1-m_2)c_{12}c_{23}s_{12}}{M_1 M_2} & \frac{(m_1-m_2)c_{12}s_{12}s_{23}}{M_1 M_3} \\ \frac{(m_2-m_1)c_{12}c_{23}s_{12}}{M_1 M_2} & \frac{(m_1 s_{12}^2 c_{23}^2 + m_2 c_{12}^2 c_{23}^2 + m_3 s_{23}^2)}{M_2^2} & \frac{s_{23}c_{23}(m_3 - m_2 c_{12}^2 - m_1 s_{12}^2)}{M_2 M_3} \\ \frac{(m_1-m_2)c_{12}s_{12}s_{23}}{M_1 M_3} & \frac{s_{23}c_{23}(m_3 - m_2 c_{12}^2 - m_1 s_{12}^2)}{M_2 M_3} & \frac{s_{23}^2(m_1 s_{12}^2 + m_2 c_{12}^2) + m_3 c_{23}^2}{M_3^2} \end{pmatrix}. \tag{33}$$

Eq. (33) is general and must be specified depending on the explicit form of V_M. We have three cases [64]:

(1) hierarchical Dirac neutrino eigenvalues (very hierarchical right-handed neutrino masses, $V_0^\dagger V_R^\star \simeq I$) where we get the usual ratios

$$BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = \lambda^6 : \lambda^4 : 1 \propto M_3^4 \lambda^4 \hat{L}\,;$$

(2) degenerate Dirac neutrino eigenvalues, with non degenerate low energy neutrino masses (the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum, $V_0^\dagger V_R^\star \simeq V_M$) where we get

$$BR(\mu \to e\gamma) = \tan^2\theta_{23}^{V_M} BR(\tau \to e\gamma) = f(\theta_{12}^{V_M}, \theta_{23}^{V_M}) BR(\tau \to \mu\gamma) \propto M_3^4 \hat{L}$$

with $f(\theta_{12}^{V_M}, \theta_{23}^{V_M})$ of order one [64];

(3) degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum (right-handed neutrino close to be degenerate, $V_0^\dagger V_R^\star \simeq I$) where we have

$$BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = 1 : \lambda^4 : \lambda^2 \propto M_3^4 \lambda^{10} \hat{L}\,.$$

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