R-Parity Violation and Family Symmetry *

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In this talk, we investigate the implications of R-parity violating (RPV) operators in a model with family symmetry [1]. Family symmetry can determine the form of RPV operators as well as the Yukawa matrices. We consider a concrete model with non-abelian discrete symmetry Q_6, which has only three RPV trilinear operators with no baryon number violating terms. We find that ratios of decay rates of the lepton flavor violating processes are fixed thanks to the family symmetry, predicting $BR(\tau \to 3e)/BR(\tau \to 3\mu) \sim 4m_\mu^2/m_\tau^2$.

Keywords: Supersymmetric models; Flavor symmetries; Masses and mixing.

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1. Introduction

Despite the remarkable success of the gauge sector of the Standard Model (SM), there still exist some problems in the Higgs and Yukawa sectors. The Yukawa matrices are responsible for the masses and mixings of matter fermions: quarks and leptons. The Yukawa sector in the SM can give experimentally consistent masses and mixings, because it contains more free parameters than the number of observables in general. There is no predictivity in the Yukawa sector because of this redundancy of the parameters. One of the ideas to overcome this issue is to introduce a family symmetry (flavor symmetry), which is the symmetry between generations. In this paper we consider a concrete model which is symmetric under the binary dihedral group Q_6 [2,3].

On the other hand, in the Higgs sector, the most important problem is that the Higgs boson has not been experimentally discovered yet. Discovery of the Higgs boson is expected at the Large Hadron Collider (LHC). In the SM, the Higgs mass is quadratic divergent. This problem is solved by introducing Supersymmetry (SUSY) at $O(1)$ TeV. The Minimal Supersymmetric Standard Model (MSSM) has low en-

ergy SUSY. In general it contains gauge symmetric, lepton and baryon number violating operators

$$W_{\not R} = \frac{1}{2}\lambda_{ijk}L_iL_jE_k^c + \lambda'_{ijk}L_iQ_jD_k^c + \frac{1}{2}\lambda''_{ijk}U_i^cD_j^cD_k^c + \mu_iH_uL_i \qquad (1)$$

in addition to the usual Yukawa couplings and μ-term. The asymmetric properties $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$ mean that $9 + 27 + 9 + 3 = 48$ (complex) parameters are included in this interactions. These couplings generate unacceptable processes such as Lepton Flavor Violating (LFV) processes and proton decay. The conservation of R-parity [8,9]

$$R = (-1)^{3B+L+2s}, \qquad (2)$$

where B, L and s denote baryon, lepton number and spin of the particles, respectively, is one possibility to forbid the couplings Eq.(1). From the definition, R-parity is $+1$ for all SM particles and -1 for all their superpartners. However, R-parity is not the only possible choice to forbid the interactions. Matter- or lepton- and baryon-parity [10] can be also a possibility. On the other hand, without R-parity, these coupling constants have to be strictly constrained not to conflict with experimental data. Constraints on the R-parity violating couplings have been obtained by many authors from LFV processes [11,12,13,14,15,16], neutrino mass [11,17,18,19,20], neutral meson system [21,22,23,24,25,26,27,28,29], proton decay [13,30,31], and so on [9,32,35].

Family symmetries also constrain the form of R-parity interactions [19,30,33,34] as well as the Yukawa matrices. In the model that we consider [2,3], the Q_6 family symmetry reduces the 45 trilinear couplings to three: λ, λ'_1 and λ'_2. The baryon number violating couplings $\lambda''_{ijk}U_i^cD_j^cD_k^c$ are forbidden by the symmetry in our model, so it is guaranteed by the symmetry that the R-parity violating operators do not induce proton decay.

In this paper, we study the phenomenology of the three R-parity violating interactions in the model with Q_6 family symmetry [2,3]. First, we obtain upper bounds on three coupling constants λ and $\lambda'_{1,2}$ from the experimental constraints. Next we focus on LFV processes induced by λ. The λLLE^c coupling generates the LFV decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ (m, i, j, k denote the flavor of the charged lepton) at tree level, and the Branching Ratios (BR) of the decay processes are proportional to λ^4. Therefore, the ratios of these processes are independent of λ and can be predicted unambiguously to be $BR(\tau \to eee)/BR(\tau \to \mu\mu\mu) \sim 4m_\mu^2/m_\tau^2$. It reflects the properties of the family symmetry. We introduce the Q_6 symmetric model in the next section, and derive the predictions in the Sec. 3. Sec. 4 is conclusions.

This talk is based on ref.[1].

2. The Model

2.1. *Group Theory of Q_6 and Assignment*

The binary dihedral group $Q_N (N = 2, 4, 6, ...)$ is a finite subgroup of $SU(2)$ and defined by the following set of $2N$ elements

$$Q_N = \left\{ 1, A, A^2, ..., A^{N-1}, B, AB, ..., A^{N-1}B \right\}, \tag{3}$$

where two dimensional representation of matrix A and B is given by

$$A = \begin{pmatrix} \cos\phi_N & \sin\phi_N \\ -\sin\phi_N & \cos\phi_N \end{pmatrix}, \ \phi_N = \frac{2\pi}{N}, \quad B = \begin{pmatrix} i & \\ & -i \end{pmatrix}. \tag{4}$$

In this note, we consider the case of $N = 6$ [a].

Q_6 group contains 2 two-dimensional irreducible representations (irreps), $\mathbf{2}_1, \mathbf{2}_2$ and 4 one-dimensional ones $\mathbf{1}_{+,0}, \mathbf{1}_{+,2}, \mathbf{1}_{-,1}, \mathbf{1}_{-,3}$, where $\mathbf{2}_1$ is pseudo real and $\mathbf{2}_2$ is real representation. In the notation of $\mathbf{1}_{\pm,n}(n = 0, 1, 2, 3)$, $\pm$ stands for the change of sign under the transformation by matrix A, and n the factor $\exp(in\pi/2)$ by B. So $\mathbf{1}_{+,0}$ and $\mathbf{1}_{+,2}$ are real representations, while $\mathbf{1}_{-,1}$ and $\mathbf{1}_{-,3}$ are complex conjugate to each other.

We consider a extension of Supersymmetric Standard Model with Q_6 family symmetry, where three generations of matter and Higgs fields are assumed to be embedded into two- and one- dimensional irreps. of Q_6 group. Assignment of Q_6 group for the quark, lepton and Higgs chiral supermultiplets are shown below in an obvious notation:

$$
\begin{aligned}
\mathbf{2}_1 & : \ Q_I, \\
\mathbf{2}_2 & : \ U_I^c, \ D_I^c, \ \hat{L}_I, \ E_I^c, \ N_I^c, \ H_I^u, \ \hat{H}_I^d, \\
\mathbf{1}_{+,0} & : \ L_3, \ E_3^c, \\
\mathbf{1}_{+,2} & : \ Q_3, \ Y, \\
\mathbf{1}_{-,1} & : \ U_3^c, \ D_3^c, \ H_3^u, \ H_3^d, \\
\mathbf{1}_{-,3} & : \ N_3^c.
\end{aligned}
\tag{5}
$$

The generation indices $I, J, ... = (1, 2)$ are applied to the Q_6 doublet, and $i, j, ... = (1, 2, 3)$ to three generations throughout the paper. Y is gauge singlet Higgs super-multiplet to give neutrino mass by seesaw mechanism. In a model without R-parity conservation, there is no distinction between lepton doublet and down type Higgs doublet, because both have the same gauge quantum numbers. In our model, L_3 and H_3^d are distinguishable because they have different Q_6 quantum number to each other, although L_I and H_I^d belong to the same irreps. Therefore, we have written these fields as $\hat{L}_I, \hat{H}_I^d$, and physical lepton doublet and down type Higgs doublet (L_I, H_I^d) should be written as linear combination of these.

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[a]In Q_8 model of ref[36], definition of Q_N is different from ours. Our Q_N is equivalent to their Q_{2N}.

2.2. *Fermion mass matrices and diagonalization*

We assume that physical Higgs fields acquire complex vacuum expectation values (VEVs) $\langle H_I^{u,d} \rangle = v_D^{u,d} e^{i\theta^{u,d}}/2$ and $\langle H_3^{u,d} \rangle = v_3^{u,d} e^{i\theta_3^{u,d}}/\sqrt{2}$ in order to avoid SUSY CP problem in soft SUSY breaking sector.

The quark mass matrices are given by

$$\mathbf{m}^u = m_t \begin{pmatrix} 0 & q_u/y_u & 0 \\ -q_u/y_u & 0 & b_u \\ 0 & b_u' & y_u^2 \end{pmatrix}, \tag{6}$$

and similarly for $\mathbf{m}^d$, where phases from VEVs have been absorbed into a part of the CKM matrix.

Many set of the parameters give observables consistent with experimental data, and one example is

$$\theta_q = \theta_3^d - \theta^d - \theta_3^u + \theta^u = -1.25, q_u = 0.0002150, b_u = 0.04440, b_u' = 0.09300,$$

$$y_u = 0.99741, q_d = 0.005040, b_d = 0.02500, b_d' = 0.7781, y_d = 0.7970. \tag{7}$$

One can easily obtain unitary matrices in explicit form which diagonalize mass matrices from these parameters. Moreover, since the CKM parameters and the quark masses are related to each other because of the family symmetry, we find that *nine* independent parameters (Eq.(7)) of the model can well describe *ten* physical observables: there is *one* prediction. An example of the prediction is $|V_{td}/V_{ts}|$, whose experimental value has been obtained from the measurement of the mass difference Δm_{B_s} of the B_s^0 meson [6]:

$$\text{Model}: |V_{td}/V_{ts}| = 0.21 - 0.23,$$

$$\text{Exp.}: |V_{td}/V_{ts}| = 0.208\,^{+0.001}_{-0.002}\,(\text{exp.})\,^{+0.008}_{-0.006}\,(\text{theo.}). \tag{8}$$

The mass matrix in the charged lepton sector is:

$$\mathbf{m}^e = \frac{1}{2} \begin{pmatrix} -Y_c^e & Y_c^e & Y_b^e \\ Y_c^e & Y_c^e & Y_b^e \\ Y_{b'}^e & Y_{b'}^e & 0 \end{pmatrix} v_D^d e^{-i\theta^d}. \tag{9}$$

One finds that U_{eL} and U_{eR} can be approximately written as

$$U_{eL} \simeq \begin{pmatrix} \epsilon_e & 1/\sqrt{2} & 1/\sqrt{2} \\ -\epsilon_e & -1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -\sqrt{2}\epsilon_e & 0 \end{pmatrix}, \ U_{eR} \simeq \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & \epsilon_\mu \\ -\epsilon_\mu & 0 & 1 \end{pmatrix} e^{i\theta^d}, \tag{10}$$

where terms of $\mathcal{O}(\epsilon^2)$ are neglected, and small parameters ϵ_e, ϵ_μ are defined as

$$\epsilon_e = \frac{m_e}{\sqrt{2}m_\mu} = 3.42 \times 10^{-3}, \ \epsilon_\mu = \frac{m_\mu}{m_\tau} = 5.94 \times 10^{-2}. \tag{11}$$

The upper-right 2×2 block of U_{eL} is the origin of maximal mixing of the atmospheric neutrino oscillation.

As for the neutrino sector, we assume that a see-saw mechanism [7] takes place. However, we do not present the details of the neutrino sector here because there is no need to know it in the following analysis. We obtain some specific predictions of our model: (i) only an inverted mass hierarchy $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$ is consistent with the experimental constraint $|\Delta m_{21}^2| < |\Delta m_{23}^2|$, (ii) the $(e, 3)$ element of the MNS matrix is given by $|U_{e3}| \simeq \epsilon_e$. See Refs. [3,4] for details.

3. R-Parity Violation

Since Q_6 family symmetry controls the whole flavor structure of the model, the form of the R-parity violating couplings are also constrained by the family symmetry. We find that only possible trilinear couplings allowed by the symmetry can be written as

$$W_{\not R} = \lambda L_3 L_I E_I^c + \lambda_1' L_I (i\sigma^2)_{IJ} Q_3 D_J^c + \lambda_2' L_I (\sigma^1)_{IJ} Q_J D_3^c \qquad (12)$$

in the physical lepton doublet L_I. Here the superpotential $W_{\not R}$ is written in the flavor eigenstates, so the unitary matrices $U_{u(L,R)}$, $U_{d(L,R)}$ and $U_{e(L,R)}$ should appear when we rotate the fermion components into their mass eigenstates. On the other hand, these matrices do not appear from sfermion components, because we approximate that sfermions are in the mass eigenstate basis. This approximation is valid in our model, because scalar masses in the soft SUSY breaking sector have diagonal form because of non-Abelian property of the family symmetry. In the present model, there are only three R-parity violating trilinear interactions allowed by the family symmetry, and baryon number violating terms $\lambda_{ijk}'' U_i^c D_j^c D_k^c$ are forbidden by the symmetry. It should be compared with the MSSM case in which there are 45 trilinear couplings. These interactions can generate a lot of new processes which have not been observed yet such as lepton flavor violating (LFV) processes, or new contributions to already observed processes. Many authors have studied phenomenology of R-parity violation and obtained constraints on each coupling constant corresponding to each process in the MSSM case. [b]

In this section, we obtain constraints on the coupling constants $\lambda, \lambda_{1,2}'$ at the weak scale. Since the various new processes generated by the interactions $W_{\not R}$ depend only on the three coupling constants, we can predict ratios of new processes independent of λs. We will also find the ratios of the LFV processes in this section. In the following analysis, we assume that the R-parity violating couplings λ and $\lambda_{1,2}'$ are real and positive [c].

3.1. *Constraint on λ*

In this subsection, we consider the constraint on $\lambda L_3 L_I E_I^c$ operator. The most stringent constraint on λ is obtained from both $\mu \to eee$ and neutrinoless double

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[b] See Ref. [9,35] and references therein.
[c] CP violation induced by the R-parity violating trilinear couplings in the soft SUSY breaking sector has been studied in Ref. [37].
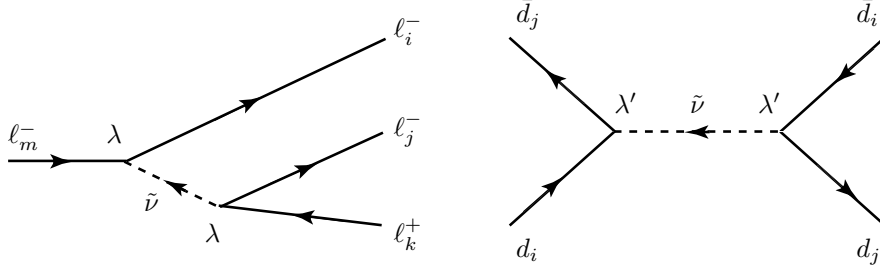
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Fig. 1.   Contributions to the decay processes $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ by coupling λ (left) and the neutral meson mixing by λ' (right).

beta decay, both the processes give similar bound. Here we explicitly show only $\mu \to eee$ process[11,12,13,14,32]. The decay process $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ is generated by tree-level t- and u- channel sneutrino exchange (Fig. 1), and its effective Lagrangian is given by

$$\mathcal{L}_{eff} = \lambda^2 A_L \left(\bar{\ell}_i P_L \ell_m\right) \left(\bar{\ell}_j P_R \ell_k\right) + \lambda^2 A_R \left(\bar{\ell}_i P_R \ell_m\right) \left(\bar{\ell}_j P_L \ell_k\right) + (i \leftrightarrow j), \quad (13)$$

where the coefficients are

$$A_L = \frac{1}{m_{\tilde{\ell}1L}^2}(U_{eR}^\dagger)_{iJ}(U_{eR})_{Jk}(U_{eL}^\dagger)_{j3}(U_{eL})_{3m}$$
$$+ \frac{1}{m_{\tilde{\ell}3L}^2}(U_{eR}^\dagger)_{iJ}(U_{eL})_{Jm}(U_{eL}^\dagger)_{jK}(U_{eR})_{Kk}, \quad (14)$$

$$A_R = \frac{1}{m_{\tilde{\ell}1L}^2}(U_{eR}^\dagger)_{jJ}(U_{eR})_{Jm}(U_{eL}^\dagger)_{i3}(U_{eL})_{3k}$$
$$+ \frac{1}{m_{\tilde{\ell}3L}^2}(U_{eR}^\dagger)_{jJ}(U_{eL})_{Jk}(U_{eL}^\dagger)_{iK}(U_{eR})_{Km}, \quad (15)$$

with mixing matrices in Eq.(10). In our approximation, sneutrino mass is the same as that of the left-handed slepton. From this Lagrangian, the branching ratio of $\mu \to eee$ is given by,

$$BR(\mu \to eee) = \frac{4\lambda^4}{64G_F^2} \left[|A_L|^2 + |A_R|^2\right] BR(\mu \to e\bar{\nu}_e\nu_\mu). \quad (16)$$

The requirement that this branching ratio should not exceed the experimental bound $BR(\mu \to eee)^{exp} < 1.0 \times 10^{-12}$ provides a constraint on λ

$$\lambda < 1.4 \times 10^{-2} \left(\frac{m_{\tilde{\ell}L}}{100\text{GeV}}\right), \quad (17)$$

where we have assumed $m_{\tilde{\ell}1L} = m_{\tilde{\ell}3L} \equiv m_{\tilde{\ell}L}$ in order to forbid the contribution to FCNC processes from the soft scalar mass terms.

3.2. *Constraints on λ'_1 and λ'_2*

Constraints on $\lambda'_{1,2}$ are obtained from neutral meson mixings [21,22,23,24,25,26,32], and on the products $\lambda\lambda'_1$ and $\lambda\lambda'_2$ from leptonic decays of neutral mesons [23,27,28,29,32]. Since both processes are generated at tree level, these give the most stringent bounds on $\lambda'_{1,2}$. Although $\mu - e$ conversion in nuclei [14,15] is also generated at tree level by $\lambda'_{1,2}$, bounds from this process are weaker than those from neutral meson system.

The neutral meson mixing is generated at the tree level through the exchange of a sneutrino in both s- and t- channels (Fig. 1). For $K^0 - \bar{K}^0$ mixing, the effective Hamiltonian is obtained as

$$\mathcal{H}_{eff} = \frac{\Lambda'_{I21}\Lambda'^*_{I12}}{m^2_{\tilde{\ell}1L}}(\bar{d}_R s_L)(\bar{d}_L s_R), \tag{18}$$

where

$$\Lambda'_{Ijk} = \lambda'_1(U^\dagger_{dR})_{jJ}(U_{dL})_{3k}(i\sigma^2)_{IJ} + \lambda'_2(U^\dagger_{dR})_{j3}(U_{dL})_{Jk}(\sigma^1)_{IJ}. \tag{19}$$

We require that these additional contributions to the mass difference of neutral K meson are smaller than its experimental value: The assumptions $\theta^d_1 - \theta^d_3 = 0$ and $\lambda'_1 = \lambda'_2$ lead to the most stringent constraints on $\lambda'_{1,2}$:

$$\lambda'_1 = \lambda'_2 < 3.1 \times 10^{-3}\left(\frac{m_{\tilde{\ell}L}}{100\text{GeV}}\right). \tag{20}$$

From leptonic decays of neutral Kaon, we get constraints on $\lambda\lambda'_{1,2}$, which are

$$\lambda\lambda'_1 < 5.4 \times 10^{-7}\left(\frac{m_{\tilde{\ell}L}}{100\text{GeV}}\right)^2 \tag{21}$$

from $K_L \to \mu^\mp e^\pm$, and

$$\lambda\lambda'_2 < 1.1 \times 10^{-8}\left(\frac{m_{\tilde{\ell}L}}{100\text{GeV}}\right)^2 \tag{22}$$

from $K_L \to e^- e^+$.

3.3. *Predictions for Lepton Flavor Violating processes*

Although many processes can be generated by the R-parity violating interactions, we focus on the LFV decays $\ell^-_m \to \ell^-_i \ell^-_j \ell^+_k$, where $m = \mu$ or τ, in this subsection. As mentioned in the subsection **3.1**, the operator $\lambda L_3 L_I E^c_I$ generates the decays $\ell^-_m \to \ell^-_i \ell^-_j \ell^+_k$ at tree level when $\lambda \neq 0$. The other two operators in Eq.(12), $\lambda'_{1,2} L Q D^c$, also generate the similar decay processes at one loop level through photon penguin diagrams shown in Fig. 2, but we found that the bounds on $\lambda'_{1,2}$ are stronger than that on λ in the previous subsections. So we neglect contributions from $\lambda'_{1,2}$ operators to the decays $\ell^-_m \to \ell^-_i \ell^-_j \ell^+_k$. Moreover, flavor changing Z boson decay $Z \to \ell^-_i \ell^+_j$ induced by the R-parity violating bilinear terms can contribute to $\ell^-_m \to \ell^-_i \ell^-_j \ell^+_k$ processes. Since branching ratios of these decays are propotional to $\sin^2\xi$, their effects are also negligible [20]. Besides these R-parity violating contributions, there are two other contributions to these processes by Higgs bosons. Since

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the charged leptons couple to the neutral Higgs bosons, these Yukawa interactions generate LFV processes at one loop level [38]. However, these effects are enhanced only when $\tan\beta$ is large. So, we assume that these are negligible because $\tan\beta$ is small enough. Moreover, since there are three generations of both up and down type Higgs doublet in this model, LFV processes mediated by the neutral Higgs bosons are generated at tree level (Fig. 2). However, the branching ratio of the $\mu \to eee$ from these effects is $BR \sim 10^{-16}$ when the neutral Higgs boson mass is 100GeV because of the smallness of the Yukawa couplings. So, these contributions can also be negligible compared to those from λLLE^c couplings unless $\lambda < 10^{-3}$. Therefore, we can approximate that $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ processes are induced at tree level only by λ. In this approximation, the ratios of these processes are independent of λ, but depend on the mixing matrices $U_{eL(R)}$ which reflect the flavor structure of the model. Therefore we find some predictions of LFV decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ in our model.

From the branching ratio Eq.(16), we can easily find the ratios of processes in the approximation that all scalar masses are equal:[d]

$$\frac{BR(\tau \to eee)}{BR(\tau \to \mu\mu\mu)} \simeq \frac{4\epsilon_\mu^2}{1+\epsilon_\mu^2} = 0.014, \tag{23}$$

$$\frac{BR(\tau \to \mu\mu e)}{BR(\tau \to \mu\mu\mu)} \simeq \frac{1-\epsilon_\mu^2+2\epsilon_e^2}{1+\epsilon_\mu^2-2\epsilon_e^2} = 0.99, \tag{24}$$

$$\frac{BR(\mu \to eee)}{BR(\tau \to eee)} \simeq \frac{\tau_\mu}{\tau_\tau}\epsilon_\mu^5 \frac{\epsilon_e^2}{2\epsilon_\mu^2+\epsilon_e^2} = 0.0093, \tag{25}$$

where small parameters $\epsilon_{e,\mu}$ are given in Eq.(11) and $\tau_\mu(\tau_\tau)$ stand for the lifetime of the $\mu(\tau)$ lepton. Also, $BR(\tau \to \mu\mu e)$ means $BR(\tau^- \to \mu^-\mu^-e^+)$, and similar for the other processes. One can obtain the ratios of other combinations from the

[d]From the conditions to suppress $\mu \to e + \gamma$ process from the scalar mass terms, slepton masses are required to be degenerated with mass differences of order $10^{-13,5}$.
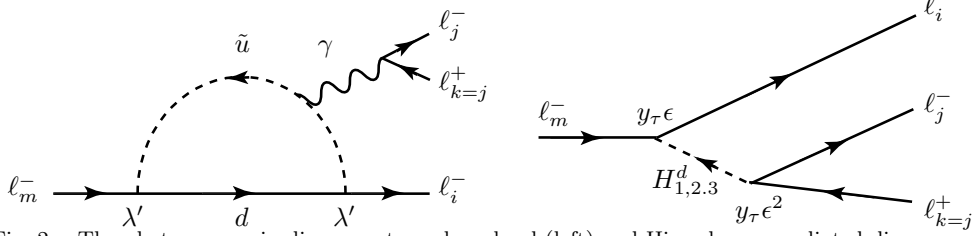
Fig. 2.   The photon penguin diagram at one loop level (left) and Higgs boson mediated diagram at tree level (right) to the decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_{k=j}^+$. These contributions can be negligible compared to the tree level processes induced by the coupling λ.

branching ratios listed below:

$$BR(\mu \to eee) \propto \tau_\mu \epsilon_e^2 \left[2(1 - 2\epsilon_\mu^2) m_{\tilde{\ell}1L}^{-4} + \frac{1}{2} m_{\tilde{\ell}3L}^{-4} - 2(1 - \epsilon_\mu^2) m_{\tilde{\ell}1L}^{-2} m_{\tilde{\ell}3L}^{-2} \right], \quad (26)$$

$$BR(\tau \to eee) \propto \tau_\tau \left[\epsilon_\mu^2 \left(1 - \epsilon_\mu^2 - 4\epsilon_e^2 \right) m_{\tilde{\ell}1L}^{-4} + \frac{1}{2} \epsilon_e^2 m_{\tilde{\ell}3L}^{-4} \right], \quad (27)$$

$$BR(\tau \to \mu\mu\mu) \propto \tau_\tau \left[\frac{1}{4} \left(1 + \epsilon_\mu^2 - 2\epsilon_e^2 - 4\epsilon_e^2\epsilon_\mu^2 \right) m_{\tilde{\ell}3L}^{-4} - 2\epsilon_e^2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-2} m_{\tilde{\ell}3L}^{-2} \right], \quad (28)$$

$$BR(\tau \to ee\mu) \propto \tau_\tau \epsilon_e^2 \left[2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-4} + (\frac{1}{2} - \epsilon_\mu^2) m_{\tilde{\ell}3L}^{-4} - 2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-2} m_{\tilde{\ell}3L}^{-2} \right], \quad (29)$$

$$BR(\tau \to \mu\mu e) \propto \tau_\tau \frac{1}{4} \left(1 - \epsilon_\mu^2 + 2\epsilon_e^2 - 4\epsilon_e^2\epsilon_\mu^2 + \frac{1}{4}\epsilon_\mu^4 \right) m_{\tilde{\ell}3L}^{-4}, \quad (30)$$

$$BR(\tau \to \mu ee) \propto \tau_\tau \left[\epsilon_e^2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-4} + \frac{1}{8} \left(1 - 2\epsilon_\mu^2 + 6\epsilon_e^2\epsilon_\mu^2 + \frac{3}{2}\epsilon_\mu^4 \right) m_{\tilde{\ell}3L}^{-4} \right], \quad (31)$$

$$BR(\tau \to \mu e\mu) \propto \tau_\tau \left[\frac{1}{8} \left(1 - 4\epsilon_e^2 + 6\epsilon_e^2\epsilon_\mu^2 - \frac{3}{4}\epsilon_\mu^4 \right) m_{\tilde{\ell}3L}^{-4} + 2\epsilon_e^2\epsilon_\mu^2 m_{\tilde{\ell}1L}^{-2} m_{\tilde{\ell}3L}^{-2} \right], (32)$$

where the common factor is not shown explicitly.

4. Conclusion

We have considered the properties of R-parity violating operators in a SUSY model with non-Abelian discrete Q_6 family symmetry. The family symmetry can reduce the number of parameters in the Yukawa sector, and explain the fermion masses and mixings between generations. It can also reduce the number of R-parity violating couplings and determine the form of those. Only three trilinear couplings are allowed, and the baryon number violating operators are forbidden by the symmetry in our model. We derived upper bounds on these couplings: $\lambda < O(10^{-2})$, $\lambda'_{1,2} < O(10^{-3})$, and obtained the predictions on the ratios of the LFV decays $\ell_m^- \to \ell_i^- \ell_j^- \ell_k^+$ which do not depend unknown parameters. The results reflect the properties of the family symmetry because these predictions contain the mixing matrices of the charged lepton sector which is written by masses of the charged

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leptons. Our predictions can be testable at future experiments because the superB factory [39] or LHC [40] will have the sensitivity $BR \sim 10^{-(8-9)}$ for LFV τ decays.

Acknowledgments

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