HOLOGRAPHY AND COSMOLOGICAL SINGULARITIES

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Certain null singularities in ten dimensional supergravity have natural holographic duals in terms of Matrix Theory and generalizations of the AdS/CFT correspondence. In many situations the holographic duals appear to be well defined in regions where the supergravity develops singularities. We describe some recent progress in this area.

1. Introduction

This talk is based on work done in collaboration with Jeremy Michelson, K. Narayan and Sandip Trivedi [1], [2], [3], [4].

Space-like and null singularities pose a peculiar puzzle. At these singularities, "time" begins or ends - and it is not clear what is the meaning of this. Classic examples of such singularities are those which appear in the interior of neutral black holes and those which appear in cosmology.

It has been always suspected that near singularities usual notions of space and time break down and a consistent quantization of gravity would provide a more abstract structure which replaces space-time. However we do not know as yet what this abstract structure could be in general. In some situations, String Theory has provided concrete ideas about the nature of this structure. These are situations where gravitational physics has a tractable holographic description [5] in terms of a non-gravitational theory in lower number of space-time dimensions. In view of the spectacular success of the holographic principle in black hole physics, it is natural to explore whether this can be used to understand conceptual issues posed by singularities.

In String Theory, holography is a special case of a more general duality between open and closed strings. This duality implies that the dynamics of open strings contains the dynamics of closed strings. Since closed strings contain gravity, space-time questions can be posed in the open string theory which does not contain gravity and therefore conceptually easier. Under special circumstances, the open string theory can be truncated to its low energy limit - which is a gauge theory on a *fixed* background. In these situations, open-closed duality becomes particularly useful. The simplest example is non-critical closed string theory in two space-time dimensions.

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Here the holographic theory is gauged Matrix Quantum Mechanics [6]. The second class of examples involve string theory or M theory defined on spacetimes with a compact null direction. Then a sector of the theory with some specified momentum in this null direction is dual to a $d + 1$ dimensional gauge theory, where d depends on the number of additional (spacelike) compact directions. Using standard terminology we will call them Matrix theories [7]-[11]. Finally, the celebrated AdS/CFT correspondence [12] relates closed string theory in asymptotically anti-de-Sitter spacetimes to gauge theories living on their boundaries. In all these cases, the dynamical "bulk" spacetime (on which the closed string theory lives) is an approximation which holds in a specific regime of the gauge theory. In this regime, the closed string theory reduces to supergravity. Generically, there is no space-time interpretation, though the gauge theory may make perfect sense. This fact opens up the possibility that in regions where the bulk gravity description is singular, one may have a well defined gauge theory description and one has an answer to the question : *What replaces space-time ?*

Treating time dependent backgrounds in string theory, particularly those with singularities, has been notoriously difficult. However, some modest progress has been made recently in both worldsheet formulations as well as holographic formulations of all the three types mentioned above. The key idea in these various types of holography are similar. One looks for toy models where the space-time background on which the closed string theory is defined is singular, but the holographic gauge theory description does not appear to be problematic. Thus, the gauge theory provides the correct description of the region which would appear singular if the gravity interpretation is extrapolated beyond its regime of validity.

In the following, we will discuss recent attempts to understand cosmological singularities using Matrix theories as well as AdS/CFT correspondence [a].

2. Matrix Big Bangs

In [14], Craps et. al. considered Type IIA string theory with string coupling g_s and string length l_s, living on a flat string frame metric with a compact null direction x^- with radius R

$$ds^2 = 2dx^+dx^- + d\vec{x} \cdot d\vec{x}, \tag{1}$$

and a dilaton linearly proportional to the other null direction x^+

$$\Phi = -Qx^+, \tag{2}$$

As a supergravity solution, this background preserves half of the supersymmetries which satisfy $\Gamma^+\epsilon = 0$. For $Q > 0$, the effective string coupling $\bar{g}_s = g_s e^{-Qx^+}$ is small for $x^+ \to \infty$ and one should have a perturbative spectrum, while for $x^+ \to -\infty$ the

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[a]For discussions of cosmological singularities in the Matrix Model description of two dimensional string theory, see [13]

string theory becomes strongly coupled and the corresponding Einstein metric has a null big bang like singularity [b].

For $Q = 0$, DLCQ string theory in this background with a momentum

$$p_- = \frac{J}{R} \tag{3}$$

is dual to a $1 + 1$ dimensional $U(J)$ gauge theory - usually called Matrix String Theory [10]-[11] - living on a circle of radius $\tilde{R}$ given by

$$\tilde{R} = \frac{l_s^2}{R}; \tag{4}$$

and a Yang-Mills coupling given by

$$g_{YM} = \frac{R}{g_s l_s^2}. \tag{5}$$

The bosonic part of the gauge theory action is

$$S = \int d\tau \int_0^{2\pi\tilde{R}} d\sigma \mathrm{Tr}\{\frac{1}{2g_{YM}^2}F_{\tau\sigma}^2 + \frac{1}{2}(D_\tau X^i)^2 - \frac{1}{2}(D_\sigma X^i)^2 + \frac{g_{YM}^2}{4}[X^i, X^j]^2\} \tag{6}$$

where $X^i, i = 1 \cdots 8$ are adjoint scalars. The above relations show that when the original string coupling is small, $g_s \ll 1$, the Yang-Mills coupling is large and the theory flows to the IR. The potential term then restricts the X^i to belong to a cartan subalgebra and may be therefore chosen to be diagonal

$$X^i = \mathrm{diag}(X_1^i, X_2^i, \cdots X_J^i) \tag{7}$$

in a suitable gauge. The gauge field decouples, and one is left with $8J$ scalar fields X_n^i in $1+1$ dimensions. The boundary conditions of these fields are labelled by the conjugacy classes of the group. For example, the maximally twisted sector has

$$X_n^i(\sigma + 2\pi) = X_{n+1}^i(\sigma) \tag{8}$$

where $X_{J+1}^i \equiv X_1^i$. In this sector we therefore have 8 scalars on a circle of size $2\pi l_s^2 \frac{J}{R}$ and the action then reduces to the worldsheet action of a *single* string in a light cone gauge. As is appropriate in the light cone gauge, the spatial extent of the worldsheet is proportional to the longitudinal momentum p_-. In a similar way one has boundary conditions with cycles of smaller length - these sectors represent multiple strings. Effects of finite $g_{YM}\tilde{R}$ are now manifested as string interactions.

The fields in the Yang-Mills theory are the low energy degrees of freedom of open string field theory on D1 branes. Holography is realized as the metamorphosis of the fields X^i of the YM theory into transverse coordinates in ten dimensional space-time. Note that this space-time interpretation is valid only when $g_s \ll 1$. For finite g_s the Yang-Mills theory of course makes perfect sense - but there is no natural space-time interpretation of the nonabelian degrees of freedom.

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[b]For $Q < 0$ we have a time-reversed situation where the big bang is replaced by the big crunch. In this paper we will exclusively deal with $Q > 0$.

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In [14] it was argued that a similar Matrix String Theory may be written down for $Q \neq 0$. The line of reasoning which leads to this is similar to the Sen-Seiberg argument [15], but more subtle - as explained in [14]. The action is a simple modification of (6)

$$S = \int d\tau \int_0^{2\pi\tilde{R}} d\sigma \mathrm{Tr}\{\frac{e^{-Q\tau}}{2g_{YM}^2}F_{\tau\sigma}^2 + \frac{1}{2}(D_\tau X^i)^2 - \frac{1}{2}(D_\sigma X^i)^2 + \frac{g_{YM}^2}{4}e^{Q\tau}[X^i, X^j]^2\} \tag{9}$$

Since this is essentially the action of J D1 branes in the light cone gauge, τ is the same as the coordinate x^+ in the background. Thus, in the far future in light cone time, the gauge theory is strongly coupled, while near the singularity at $x^+ \to -\infty$ the gauge theory is *weakly coupled*. This means that while the theory has a nice interpretation as a space-time theory with dynamical gravity in the future, such an interpretation breaks down at $\tau \to -\infty$ - precisely the place where there is a null singularity. Here all the J^2 degrees of freedom are relevant and might "resolve" the singularity.

2.1. *IIB Big Bangs*

The Type IIB version of this background shows a richer structure [2]. The background is once again given by (1) and (2) where both x^- and x^8 are compact,

$$x^- \sim x^- + 2\pi R, \qquad x^8 \sim x^8 + 2\pi R_B \tag{10}$$

The usual DLCQ Matrix Theory logic then implies that string theory in the sector with $p_- = J/R$ and $p_8 = 0$ is described by a $SU(J)$ 2+1 dimensional Yang-Mills theory of J D2 branes [9], [10]. This gauge theory lives on a T^2 with sides

$$R_\rho = g_B \frac{l_B^2}{R} \qquad R_\sigma = \frac{l_B^2}{R} \tag{11}$$

where g_B, l_B are the string coupling and the string length of the original IIB theory. The dimensional coupling constant of the Yang-Mills is

$$G_{YM}^2 = \frac{R}{R_\sigma R_\rho} = \frac{RR_B^2}{g_B l_B^4} \tag{12}$$

We will call this theory "Matrix Membrane Theory".

The action of this matrix membrane theory is given by

$$S = \int d\tau \int_0^{2\pi R_\rho} d\sigma \int_0^{2\pi R_\sigma} d\rho\,\mathcal{L} \tag{13}$$

where

$$\mathcal{L} = \mathrm{Tr}\{\frac{1}{2}[(D_\tau X^a)^2 - (D_\sigma X^a)^2 - e^{2Q\tau}(D_\rho X^a)^2] + \frac{1}{2(G_{YM}e^{Q\tau})^2}[F_{\sigma\tau}^2 + e^{2Q\tau}(F_{\rho\tau}^2 - F_{\rho\sigma}^2)]$$
$$+ \frac{(G_{YM}e^{Q\tau})^2}{4}[X^a, X^b]^2\}, \tag{14}$$

where $X^a, a = 1 \cdots 7$ are now seven scalar fields and $F_{\mu\nu}$ denotes the gauge field strength. Note that there is a factor of $e^{Q\tau}$ with each ∂_ρ or a covariant vector component V_ρ, in addition to a factor of $e^{Q\tau}$ for each G_{YM}

For $Q = 0$ and $g_B \ll 1$ this action reproduces the worldsheet action for Type IIB strings in the light cone gauge. In this limit the commutator terms force the fields to be diagonal. The gauge field strengths can be dualized to a scalar which we will call X^8, so that we have a $2+1$ dimensional action of eight scalar fields. Finally, since for small g_B we have $R_\rho \ll R_\sigma$, the action reduces to a $1+1$ dimensional action which may be then identified with the Green-Schwarz light cone worldsheet theory. Once again sectors of boundary conditions describe upto J strings with the spatial extent of the worldsheet proportional to their longitudinal momenta.

This story changes interestingly when $Q \neq 0$. The mass scale associated with the Kaluza Klein modes in the ρ direction is given by $M_{KK} \sim \frac{R}{g_B l_B^2}$ while the mass scale which determines the non-abelian dynamics is G_{YM} given in (12). Thus for $R_B \gg l_B$ the KK modes are much lighter than the Yang-Mills scale. In our present time-dependent context, these scales become time-dependent and it follows from the coupling and the ∂_ρ terms in (14) that the KK modes are expected to decouple much *later* than the time when the non-abelian excitations decouple. Therefore, there is a regime where we can ignore the non-abelian excitations, but cannot ignore the KK modes. In this regime, the Matrix Membrane lagrangian density is given by

$$\mathcal{L}_{diag} = \frac{1}{2}[\sum_{I=1}^{8}(\partial_\tau X^I)^2 - (\partial_\sigma X^I)^2 - e^{2Q\tau}(\partial_\rho X^I)^2] - 2\mu^2 \sum_{I=1}^{8}(X^I)^2 \qquad (15)$$

It is tempting to argue that as $\tau \to \infty$ the Kaluza-Klein modes in the ρ direction become infinitely massive, so that the theory becomes $1+1$ dimensional and exactly identical to the Green-Schwarz string action in this background. However, this is too hasty since we have a time-dependent background here and energetic arguments do not apply.

Instead, we should ask whether *any* state at an early time evolves into a state of the perturbative fundamental string - i.e. states which do not carry any momentum in the ρ direction. The modes of the field $Y^I(\rho, \sigma, \tau)$ which are positive frequency at early times are given by

$$\varphi_{m,n}^{(in)} = \{\frac{R}{8\pi^2 l_B^4 g_B}\}^{1/2} \, \Gamma(1 - i\omega_m/Q) \, e^{i(\frac{mR}{l_B^2}\sigma + \frac{nR}{g_B l_B^2}\rho)} \, J_{-i\frac{\omega_m}{Q}}(\kappa_n e^{Q\tau}) \qquad (16)$$

where

$$\omega_m^2 = \frac{m^2 R^2}{l_B^4} \qquad\qquad \kappa_n = \frac{nR}{Q g_B l_B^2} \qquad (17)$$

while those which are appropriate at late times are

$$\varphi_{m,n}^{(out)} = \{\frac{R}{16\pi l_B^4 g_B Q}\}^{1/2} \, e^{i(\frac{mR}{l_B^2}\sigma + \frac{nR}{g_B l_B^2}\rho)} \, H_{-i\frac{\omega_m}{Q}}^{(2)}(\kappa_n e^{Q\tau}) \qquad (18)$$

The problem at hand is identical to that of a bunch of two dimensional scalar field (living on τ, σ spacetime) with time dependent masses. It is well known that such time dependence leads to particle production or depletion [16], [17],[18]. Because of standard relations between the Hankel function $H_\nu^{(2)}(z)$ and the Bessel function $J_\nu(z)$ there is a non-trivial Bogoliubov transformation between these modes which imply that the vacua defined by the in and out modes are not equivalent. In fact, the out vacuum $|0>_{out}$ is a squeezed state of the "in" particles. In other words, *if we require that the final state at late times does not contain any of the KK modes, the initial state must be a squeezed state of these modes.* The occupation number of the in modes in the out state is thermal

$$_{out} < 0| \, a_{m,n}^{\dagger I,(in)} a_{m,n}^{I,(in)} |0>_{out} = \frac{1}{e^{\frac{2\pi\omega_m}{Q}} - 1} \tag{19}$$

Note that the Bogoliubov coefficients and number densitites depend only on m for all $n \neq 0$. This follows from the fact that n- dependence may be removed by shifting the time τ by $\log(\kappa_n)$. However, the modes with $n = 0$ need special treatment. Indeed, in the $n \to 0$ limit the "in" modes (16) go over to standard positive frequency modes of the form $e^{-i\omega_m\tau}$ as expected. In this limit, however, the out modes (18) contain both positive and negative frequencies. This is of course a wrong choice, since for these $n = 0$ modes there is no difference between "in" and "out" states. In fact, the "out" modes (18) have been chosen by considering an appropriate large time property for *nonzero n* and do not apply for $n = 0$. In other words, the squeezed state contains only the $n \neq 0$ modes.

The operators $a_{m,n}^I$ in fact create states of $(p.q)$ strings in the original Type IIB theory [9]. To see this, let us recall how the light cone IIB fundamental string states arise from the $n = 0$ modes of the Matrix Membrane. In this sector, the action is exactly the Green-Schwarz action. The oscillators $a_{m,0}^{\dagger I}$ defined above are in fact the world sheet oscillators and create excited states of a string. The gauge invariance of the theory allows nontrivial boundary conditions, so that m defined above can be fractional. Equivalently the boundary conditions are characterized by conjugacy classes of the gauge group. The longest cycle corresponds to a single string whose σ coordinate has an extent of $2\pi J \frac{l_B^2}{R}$ which is the same as $2\pi l_B^2 p_-$ as it should be in the light cone gauge. Shorter cycles lead to multiple strings - the sum of the lengths of the strings is always $2\pi l_B^2 p_-$, so that there could be at most J strings. Note that m is the momentum in the σ direction : a state with *net* momentum in the σ direction in fact corresponds to a fundamental IIB string wound in the x^- direction. This may be easily seen from the chain of dualities which led to the Matrix Membrane.

As shown in [9], following the arguments of [20],[19],[21], $SL(2,Z)$ transformations on the torus on which the Yang-Mills theory lives become the $SL(2,Z)$ transformations which relate (p,q) strings in the original IIB theory. In particular, the oscillators $a_{0,n}^I$ create states of a D-string.

The state $|0>_{\text{out}}$ therefore contain excited states of these (p,q) strings. The

number of such strings depends on the choice of the conjugacy classes characterizing boundary conditions. Since each (m, n) quantum number is accompanied by a partner with $(-m, -n)$ this state does not carry any F-string or D-string winding number. Finally this squeezed state contains only $n \neq 0$ modes, i.e. they do not contain the states of a pure F-string. We therefore conclude that in this toy model, the initial state has to be chosen as a special squeezed state of *unwound* (p, q) strings near the big bang to ensure that the late time spectrum contains only perturbative strings.

2.2. *pp-wave Big Bangs*

The nonabelian degrees of freedom of Matrix String Theory or Matrix Membrane theory become important near the "singularity". In the background considered above, this theory has one length scale - given by the Yang-Mills coupling G_{YM}^{-1}. It would be worthwhile to find similar situations with an additional length scale with the hope that tuning the dimensionless ratio would allow us to go to a regime where some class of nonableian configurations become important. One such example is provided by pp-waves [2],[1]. The string frame metric (1) is now modified to [c]

$$ds^2 = 2dx^+ dx^- - 4\mu^2[(x^1)^2 + \cdots (x^6)^2](dx^+)^2 - 8\mu x^7 dx^8 dx^+ + [(dx^1)^2 + \cdots (dx^8)^2]$$
(20)

The dilaton remains the same as (2), and there is an additional 5-form field strength

$$F_{+1234} = F_{+5678} = \mu \ e^{Qx^+}$$
(21)

For $Q = 0-$, the matrix membrane action in this background has been derived in [23]. The matrix membrane action for $Q \neq 0$ now has additional terms [2]

$$\mathcal{L} = \text{Tr} \ \{ \ \frac{1}{2}[(D_\tau X^a)^2 - (D_\sigma X^a)^2 - e^{2Q\tau}(D_\rho X^a)^2] + \frac{1}{2(G_{YM}e^{Q\tau})^2}[F_{\sigma\tau}^2 + e^{2Q\tau}(F_{\rho\tau}^2 - F_{\rho\sigma}^2)]$$

$$- 2\mu^2[(X^1)^2 + \cdots (X^6)^2 + 4(X^7)^2] + \frac{(G_{YM}e^{Q\tau})^2}{4}[X^a, X^b]^2$$

$$- \frac{4\mu}{(G_{YM}e^{Q\tau})}e^{Q\tau}X^7 \ F_{\rho\sigma} - 8\mu i(G_{YM}e^{Q\tau})X^7 \ [X^5, X^6] \ \},$$
(22)

The new length scale is now μ.

Let us briefly recall the physics of this model for $Q = 0$. When the original IIB theory is weakly coupled, $g_B \ll 1$ with $\frac{\mu l_B^4}{RR_B^2} \sim O(1)$, the effective coupling constant of this YM theory is strong. Then, along the lines of the discussion in the previous subsection, the action becomes identical to the worldsheet action for Green-Schwarz string in the pp-wave background [d]. In fact, as shown in [26], integrating out the Kaluza Klein modes in the ρ direction generates string couplings with exactly the correct strength.

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[c]The coordinates used here make a space-like isometry explicit [22]
[d]The dualization required to convert the gauge field to a scalar involves a time dependent rotation [23].

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It is straightforward to see that one could rescale the fields and the coordinates to write the lagrangian $\mathcal{L}$ in the form

$$\mathcal{L} = \frac{\mu}{G_{YM}^2} \, \mathcal{L} \; (\mu = 1, G_{YM} = 1) \tag{23}$$

Therefore, in the limit $\lambda \gg 1$ the Yang-Mills theory becomes weakly coupled and nonabelian classical solutions play a significant role. These classical solutions are *fuzzy ellipsoids* discussed in [23,24] similar to fuzzy spheres in M theory and Type IIA pp-waves [25],

$$X^5 = 2\sqrt{2}\frac{\mu l_p^3}{R}J^1,$$
$$X^6 = 2\sqrt{2}\frac{\mu l_p^3}{R}J^2,$$
$$X^7 \;\; = 2\frac{\mu l_p^3}{R}J^3, \tag{24}$$

where J^a obey the SU(2) algebra, and the remaining matrices X^i vanish. These solutions have vanishing light cone energy and can be shown [23,24] to preserve all 24 supercharges of the M-theory background. In the original Type IIB description they are fuzzy D3 branes with a topology $S^2 \times S^1$ where the S^1 factor is the compact space direction.

For $Q \neq 0$ the coupling is always weak near $\tau \to -\infty$ so that these fuzzy ellipsoids proliferate. As τ increases the coupling gets stronger and one would expect that they should not be present, leaving behind only perturbative abelian degrees of freedom representing the fundamental string. This indeed happens. The size of the ellipsoids is now time dependent : with some initial size the equations of motion may be used to examine the size at later times. Numerical results [2] show that with generic initial conditions, the size oscillates with an amplitude decaying fast with time. In other words, at late times we are left with only the abelian configurations which can be now interpreted as fundamental strings. The phenomenon of production/depletion of (p, q) strings is identical to the $\mu = 0$ case described in the previous subsection.

2.3. *Issues*

The key feature of holographic models of this type is that conventional space-time is an *emergent* phenomenon in a very special regime. In matrix theories, this is the regime where the gauge theory coupling is strong so that the fields of the theory can be interpreted as space-time coordinates of a point on a fundamental string. In the toy models of cosmology described above, such an interpretation appears to be valid at late times. If we forcibly extrapolate this interpretation to early times we encounter a singularity. At this singularity, however, the holographic gauge theory is *weakly* coupled : as such a space-time interpretation is not valid in any case. Since the coupling is weak there is a good chance that we have a well defined time evolution.

There are several caveats in this general story. The success of Matrix Theory generally depends on supersymmetry. Even though the backgrounds considered have half of the supersymmetries, the matrix theory does not. One of the consequences of this is that a potential for the fields X^a could be generated which spoils the interpretation in terms of space-time coordinates. This issue has been investigated in [27] and [28]. Indeed there is a potential at one loop. However it turns out that at late times the potential vanishes fast, indicating that X^a become moduli [e].

An important question relates to backreaction. Sometimes null singularities of the type described here are unstable under perturbations. In the past, orbifold singularities of this type have been investigated as possibly consistent backgrounds for *perturbative string theory*. However, it was soon found that these null singularities turn spacelike under small perturbations - large curvatures develop invalidating the use of perturbative string theory [31]. In our case, the significance of such an instability, if present, is rather different. Here the string theory is in any case strongly coupled near the singularity and there is no question of a perturbative description. Rather the correct description is provided by a weakly coupled Yang-Mills theory. The question now is to find out the meaning of a bulk instability in the gauge theory. It remains to be seen if this causes any problem even though the coupling is weak. This issue is particularly significant for variations of this model based on null branes [30].

Perhaps the most important question is about continuation through the singularity. Even though the holographic theory is weakly coupled near the null singularity, the hamiltonian expressed in terms of the conjugate momenta have a singular behavior as one approaches this region - and it is not clear whether there is an unambiguous prescription to continue back in time beyond this point. Recently [32] has put forward an interesting proposal to address this issue.

3. Null Singularities in the AdS/CFT correspondence

In many respects the AdS/CFT correspondence is a more controlled example of the holographic principle. In its simplest setting, the correspondence implies IIB string theory on $AdS_5 \times S^5$ with a constant 5-form flux is dual to $3+1$ dimensional $N = 4$ supersymmetric $SU(N)$ Yang-Mills theory which lives on the boundary of AdS_5. If R_{AdS} denotes the radius of the S^5 as well as the curvature length scale of AdS_5 and g_s denotes the string coupling, the coupling constant g_{YM} and the rank of the gauge group N of the Yang Mills theory are related by

$$\frac{R_{AdS}^4}{l_s^4} = 4\pi g_{YM}^2 N \qquad\qquad g_s = g_{YM}^2 \qquad\qquad (25)$$

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[e]In [28] it is claimed that the potential in fact vanishes. However it turns out that the quantity which is computed in this paper is a time averaged potential rather than the time dependent potential [29]

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This immediately implies that the gauge theory describes *classical* string theory in the 't Hooft limit

$$N \to \infty \qquad g_{YM} \to 0 \qquad g_{YM}^2 N = \text{finite} \tag{26}$$

The low energy limit of the closed string theory - supergravity - is a good approximation only in the strong coupling regime $g_{YM}^2 N \gg 1$. For small $g_{YM}^2 N$ supergravity and hence conventional space-time is not a good description of the gauge theory dynamics. Finite N corrections correspond to string loop effects.

There have been several approaches to cosmological singularities by finding appropriate modifications of the AdS solutions which correspond to deformations of the Yang-Mills theory or to states in the theory [33]. We will discuss one approach developed in [3,4,34] [f]. The hope is similar to that in the Matrix Theory approach. The idea is to find bulk solutions which have cosmological singularities where the usual notions of space-time break down, while the gauge theory description remains tractable.

In the following we will recount the main points in [3,4].

3.1. *The Supergravity Background and the Conjecture*

The usual $AdS_5 \times S^5$ solution is given by the Einstein frame metric in Poincare coordinates

$$ds^2 = (\frac{r^2}{R_{AdS}^2})\eta_{\mu\nu}dx^\mu dx^\nu + (\frac{R_{AdS}^2}{r^2})dr^2 + R_{AdS}^2 d\Omega_5^2 \tag{27}$$

and a 5-form field strength and dilaton Φ

$$F_{(5)} = R_{AdS}^4(\omega_5 + *_{10}\omega_5) \qquad \Phi = \text{constant} \tag{28}$$

This has maximal supersymmetry.

We consider supergravity solutions which are *non-normalizable* deformations of this,

$$ds^2 = (\frac{r^2}{R_{AdS}^2})\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu + (\frac{R_{AdS}^2}{r^2})dr^2 + R_{AdS}^2 d\Omega_5^2 \ ,$$
$$\Phi = \Phi(x)$$
$$F_{(5)} = R_{AdS}^4(\omega_5 + *_{10}\omega_5) \tag{29}$$

The equations of motion then imply that the Ricci tensor $\tilde{R}_{\mu\nu}$ constructed from the metric $\tilde{g}_{\mu\nu}(x)$ must obey the equation

$$\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\Phi, \tag{30}$$

while the dilaton must satisfy

$$\partial_\mu(\sqrt{-\det(\tilde{g})} \ \tilde{g}^{\mu\nu}\partial_\nu\Phi) = 0. \tag{31}$$

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[f]See [35] for an interesting approach to find signatures of space-like singularities inside AdS black holes in the CFT

It turns out that this solution is the near-horizon limit of the geometry produced by three branes whose worldvolume metric is given by $\tilde{g}_{\mu\nu}(x)$.

For generic $\tilde{g}_{\mu\nu}(x)$ this solution will have curvature singularities at the Poincare horizon at $r = 0$. This does not happen when $\tilde{g}_{\mu\nu}(x)$ and $\Phi(x)$ are functions of a null coordinate x^+. We will therefore restrict our attention to such solutions. Such solutions retain half of the supersymmetries with parameters ϵ satisfying $\Gamma^+\epsilon = 0$. Furthermore, for reasons which will become clear soon, we will consider brane metrics which are conformal to flat space

$$\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu = e^{f(x^+)} \left[-2dx^+dx^- + dx_1^2 + dx_2^2\right] \tag{32}$$

The equations of motion (30) then require that the dilaton is also a function of x^+ alone. The dilaton equation (31) is automatically satisfied, while (30) simplifies to

$$\frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\partial_+\Phi)^2. \tag{33}$$

where prime denotes derivative with respect to x^+.

The conjecture is that string theory in this null background is dual to $3 + 1$ dimensional $N = 4$ Yang-Mills theory which lives on a background space-time given by $\tilde{g}_{\mu\nu}(x)$ and a x^+ dependent coupling

$$g_{YM}(x^+) = e^{\Phi(x^+)/2}\sqrt{g_s} \tag{34}$$

The bosonic part of the action is

$$S = \int d^4x \, \text{Tr}\{\frac{1}{4}e^{-\Phi}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\chi^I)(D^\mu\chi^I) + \frac{1}{4}[\chi^I,\chi^J]^2\} \tag{35}$$

where $\chi^I, I = 1, \cdots 6$ are the adjoint scalars.

There are several pieces of evidence for the validity of this conjecture. First, when $f \ll 1$, we also have $\Phi \ll 1$. In that case, the solution represents small non-normalizable metric and dilaton deformations of standard $AdS_5 \times S^5$. AdS/CFT correspondence then implies that the dual gauge theory is deformed by operators which are dual to these modes, viz. the energy momentum tensor $T_{\mu\nu}$ and $\text{Tr}F^2$ respectively. This is evident from the action (35). Secondly, we may consider the action of a single probe D3 brane in the background and examine the way $f(x^+)$ and $\Phi(x^+)$ sppear - it is easy to check that this is consistent with (35). Finally, as noted above, our solution is the near-horizon limit of the asymptotically flat geometry of a stack of D3 branes with a curved brane wiorldvolume [g].

We will consider solutions which are $AdS_5 \times S^5$ in asymptotic null time $x^+ \to \pm\infty$, but develop null singularities for some value of x^+ which may be chosen to be at $x^+ = 0$. However, we will require that $g_s \ll 1$ *and the effective string coupling* $e^\Phi g_s$ *remains weak for all* x^+.. This latter feature distinguishes our solution from

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[g]The full supergravity solution is given in [3].

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some others in the literature [h]. A nice example of such a solution is

$$e^{f(x^+)} = \tanh^2 x^+ \qquad e^{\Phi} = g_s \left| \tanh \frac{x^+}{2} \right|^{\sqrt{8}}. \tag{36}$$

At $x^+ = 0$ all local curvature invariants are bounded. However this point may be reached in a finite physical time. For example the affine parameter λ along a geodesic $x^+(\lambda)$ with all other coordinates constant is given by

$$\lambda = x^+ - \tanh \ x^+ \tag{37}$$

Thus $x^+ = 0$ can be reached in a finite affine parameter. Furthermore, it turns out that tidal forces between neighboring geodesics diverge at this point. Therefore $x^+ = 0$ is a genuine null singularity.

   Consider the solution as a time evolution in light cone time x^+. At $x^+ \to -\infty$ the Yang-Mills coupling approaches $\sqrt{g_s}$ exponentially. In the dual Yang-Mills theory, we will always work in the 't Hooft limit $g_s \to 0, N \to \infty$ with $g_s N$ finite and large. Therefore, according to the usual AdS/CFT correspondence, the ground state of the theory is dual to *supergravity* in $AdS_5 \times S^5$ as stated above. This vacuum evolves in time according to the Yang-Mills hamiltonian whose effective coupling *decreases*. The dual description of this time evolution is the supergravity solution described above. Supergravity, however, makes sense only when the Yang-Mills coupling is large. Thus as we approach $x^+ \to 0$ the coupling approaches zero and the supergravity description becomes meaningless. The singularity therefore appears at a place where we expect a space-time interpretation of the gauge theory to break down.

3.2. *Some Properties of the Gauge Theory Dual*

As emphasized above, one of the salient features of our toy model is that the gauge theory is weakly coupled at the "singularity", pretty much like the Matrix Theory examples given above. This is reassuring, since one would hope that weakly coupled gauge theory makes sense and provides the alternative structure which replaces dynamical bulk space time in this region. However, it is precisely at this point that the nondynamical spacetime of the gauge theory shrinks to zero size !

   Normally this would be a disaster since a gauge theory on a zero size space-time would be singular even if it is weakly coupled. What saves the day is the fact that this particular gauge theory is Weyl invariant. This is evident at the classical level - the factor $e^{f(x^+)}$ does not appear in the classical action. If the coupling was constant the theory would have been conformally invariant (in the sense of invariance under conformal diffeomorphisms) as well. Here the x^+ dependence of the coupling breaks

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[h]These include orbifold models, backgrounds with time dependent warping, models based on tachyon condensation. References to the original literature can be found in [3]. Some of these topics are reviewed in [36].

these conformal symmetries but retains Weyl invariance at least classically. This means that at the classical level our gauge theory simply does not see the shrinking Weyl factor.

Usually Weyl invariance of quantum field theories is broken at the quantum level by anomalies. Our gauge theory is a special case of $N = 4$ Yang-Mills theory coupled to nondynamical conformal supergravity, where only the metric and the dilaton fields of the background supergravity are turned on. The Weyl anomaly of this theory has been worked out a while ago with the result [37,38,39].

$$< T_\mu^\mu > = -\frac{N^2}{64\pi^2}\{2(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2) + 4\left[2(R^{\mu\nu} - \frac{1}{3}Rg^{\mu\nu})\partial_\mu\Phi\partial_\nu\Phi + (\nabla^2\Phi)^2\right]\} \tag{38}$$

In fact, it turns out that the *operator* T_μ^μ involves only scalars made out of conformal supergravity fields [i]. In our case, the only nonvanishing components of the Riemann are R_{+i+i} with $i = 1, 2$ and the only nonvanishing component of $\partial_\mu\Phi$ is $\partial_+\Phi$. Since there are no nonvanishing components with a contravariant $+$ index, we cannot form a scalar by contracting these tensors. Thereofore the Weyl anomaly vanishes for our null background. This implies that correlation functions of dressed conformal operators are equal to those in a *flat* metric with the same x^+ dependent coupling,

$$\langle\prod_a e^{f(x_a^+)\Delta_a} \, \mathcal{O}_a(x^a)\rangle_{e^f\eta_{\mu\nu},\Phi(x^+)} = \langle\prod_a \mathcal{O}_a(x^a)\rangle_{\eta_{\mu\nu},\Phi(x^+)} \tag{39}$$

where Δ_a is the conformal dimension of the operator $\mathcal{O}_a$. In other words, the shrinking conformal factor is invisible to these observables at the quantum level.

We are therefore left with a gauge theory on flat space with a x^+ dependent coupling. The coupling, however, appears as a overall factor in the gauge field term. Generally, this would imply that the propagator of canonically normalized fields would be unconventional. This could be a danger since the derivatives of $\Phi(x^+)$ diverge at $x^+ = 0$. Luckily this does not happen either. To see this, fix a light cone gauge [40]

$$A_- = 0 \tag{40}$$

The fields A_+ are then determined in terms of the transverse components by a constraint equation which turns out to be identical to that for the standard $N = 4$ theory *by virtue of the fact that the coupling depends only on x^+*,

$$\frac{1}{2}\partial_- A_+ = \partial_i A_i + \frac{i}{\partial_-}[A_i, \partial_- A_i] \tag{41}$$

Let us now define new fields $\bar{A}_i, \bar{A}_+$ as follows

$$\bar{A}_i(x) = e^{-\Phi(x^+)/2}A_i(x) \quad \bar{A}_+(x) = e^{-\Phi(x^+)/2}A_+(x) \tag{42}$$

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[i]We are grateful to A. Tseytlin for a correspondence about this point

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Since Φ is a function of x^+ alone, the equation (41) is identical with the replacement $A_i \to \bar{A}_i$, $A_+ \to A_+$. In terms of these new fields it may be easily checked that upto terms which are quadratic in the fields,

$$e^{-\Phi(x^+)}\mathrm{Tr}F^2 = \mathrm{Tr}\bar{F}^2 - \frac{1}{2}\partial_- \left[(\partial_+\Phi)\bar{A}^i\bar{A}_i\right] \tag{43}$$

where $\bar{F}$ is the field strength constructed out of $\bar{A}$. Since the additional term is a total derivative, it does not contribute to the action. *This means that the quadratic terms in the action are identical to that in the light cone gauge action for standard $N = 4$ theory.* The factors of e^Φ and its derivatives appear only in the interaction terms of the $\bar{A}$ fields. Since the coupling $e^{\Phi/2}$ approaches zero at the singularity and is small and bounded everywhere else, one might expect that the correlation functions of the fields $\bar{A}_\mu$ are well behaved.

Generically, time dependent backgrounds lead to particle production. An initial vacuum state typically evolves into a squeezed state of particle - antiparticle pairs. In our null background, however, such processes do not occur. The argument relies on the fact that in light front quantization the states are labelled by k_-, k_1, k_2 where $0 \le k_- \le \infty$ and $-\infty \le k_1, k_2 \le \infty$. Since the background depends only on x^+, the momentum along x^-, k_- is conserved. The fock vacuum of the theory has $k_- = k_i = 0$. It is then clear that this state cannot evolve into a state containing particles with nonzero k_- since there are no states with negative k_-. The $k_- = 0$ sector, however, may cause problems with this argument.

3.3. *The worldsheet theory*

Since the effective 't Hooft coupling of the Yang-Mills theory becomes small at the singularity, it is natural to expect that stringy effects are large. At the same time, in the large N limit string loop effects should be small as well. It is of interest to investigate whether worldhseet string theory could make sense in this background. Unfortunately, because of the presence of RR flux, we do not have a tractable worldsheet formulation of the full worldsheet theory. However a look at the bosonic part of the action in a physical gauge makes it clear that stringy effects are important near the singularity. For this purpose, let us write the metric in slightly different coordinates

$$ds^2 = \frac{1}{y^2}\left[e^{f(x^+)}\{2dx^+dx^- + d\vec{x}^2\} + d\vec{y}^2\right] \tag{44}$$

where $\vec{y} = (y^1 \cdots y^6)$. Fixing the light cone gauge $x^+ = \tau$ following [41] the bosonic part of the action becomes

$$S = \frac{1}{2}\int d\sigma d\tau \left[(\partial_\tau\vec{x})^2 + e^{-f(\tau)}(\partial_\tau\vec{y})^2 - \frac{1}{y^4}e^{2f(\tau)}e^{\Phi(\tau)}(\partial_\sigma\vec{x})^2 - \frac{1}{y^4}e^{f(\tau)}e^{\Phi(\tau)}(\partial_\sigma\vec{y})^2\right], \tag{45}$$

Since both e^f and e^Φ vanish at $\tau = 0$, the spatial gradient terms become small here, which implies that stringy modes are not suppressed. It is not clear as yet whether the worldsheet theory is non-singular.

3.4. *Penrose Limits and Matrix Theory*

To learn a little more about the string theory in the bulk it is useful to consider the Penrose limit of our background. For this purpose it is convenient to rewrite the metric as

$$ds^2 = r^2[-dt^2 + dq^2 + e^{F(z^+)}(dx_2^2 + dx_3^2)] + \frac{dr^2}{r^2} + d\psi^2 + \sin^2\psi d\Omega_4^2, \qquad (46)$$

where we have used the affine parameter z^+ defined by $z^+ = \int^{x^+} dx\, e^{f(x)}$ along a null geodesic instead of x^+, and the function $F(z^+)$ is defined by $F(z^+) = f(x^+(z^+))$. The coordinates q, t are defined by

$$z^+ = \frac{1}{\sqrt{2}}(q + t), \qquad x^- = \frac{1}{\sqrt{2}}(t - q). \qquad (47)$$

Now we zoom on a null geodesic given by

$$r = \sin\, U, \qquad t = -\cot\, U, \qquad \psi = U, \qquad (48)$$

After the usual scaling associated with a Penrose limit and a complicated coordinate transformation the Einstein frame metric is given by [3]

$$ds^2 = 2dU dV - [H(U)\vec{X}^2 + \vec{Y}^2](dU)^2 + d\vec{X}^2 + d\vec{Y}^2. \qquad (49)$$

In the Penrose limit, the coordinate U is related to the coordinate z^+ by $z^+ - \frac{1}{\sqrt{2}}\cot\, U$ and the function $H(U)$ is determined in terms of $F(z^+)$ by

$$H(U) = 1 - \frac{[1 + 2(z^+)^2]^2}{4}\left[\frac{d^2F}{(dz^+)^2} + \frac{1}{2}\left(\frac{dF}{dz^+}\right)^2\right] = 1 + \frac{[1 + 2(z^+)^2]^2}{8}\left(\frac{d\Phi}{dz^+}\right)^2, \qquad (50)$$

In terms of these coordinates, the singularity appears at $U = Pi/2$. Near this point,

$$H(U) \sim \frac{1}{(U - \frac{\pi}{2})^2}, \qquad e^{\Phi(U)} \sim (U - \frac{\pi}{2})^{\frac{\sqrt{8}}{3}}. \qquad (51)$$

Thus the Penrose limit of our original space-time is singular as well. In fact, it turns out that the pp-wave is singular if and only if the pre-Penrose limit original spacetime is singular [42].

    The pp-wave space-time has space-like and null isometries. In a way similar to the null dilaton cosmologies in the previous section, one may write down a matrix membrane theory for such a background which has a compact null direction $x^- \sim x^- + 2\pi R$ and $x^8 \sim x^8 + 2\pi R_B$. The resulting $2 + 1$ dimensional Yang-Mills action is

$$\mathcal{L} = \mathrm{Tr}\frac{1}{2}\{[(D_\tau\chi^\alpha)^2 - e^{\Phi(\tau)}(D_\sigma\chi^\alpha)^2 - e^{-\Phi(\tau)}(D_\rho\chi^\alpha)^2]$$
$$+ \frac{1}{G_{YM}^2}[e^{\Phi(\tau)}F_{\sigma\tau}^2 + e^{-\Phi(\tau)}F_{\rho\tau}^2 - F_{\rho\sigma}^2]$$
$$- H(\tau)[(\chi^1)^2 + (\chi^2)^2] - (\chi^3)^2\cdots(\chi^6)^2 - 4(\chi^7)^2$$
$$+ \frac{G_{YM}^2}{2}[\chi^\alpha, \chi^\beta]^2 + 2iG_{YM}\chi^7[\chi^5, \chi^6] + \frac{4}{G_{YM}}\chi^7 F_{\sigma\rho}\}, \quad (52)$$

Unlike the matrix membrane in the linear null dilaton discussed in the previous section (i) the Yang-Mills coupling of this model is independent of τ, (ii) both ∂_ρ and ∂_σ have time-dependent factors. In the IR, the fields in the theory become commuting and may be chosen to be diagonal and the extent of the ρ direction shrinks to zero size. The lagrangian then reduces to the light cone gauge Green-Schwarz worldsheet lagrangian for the fundamental string in the relevant pp-wave background. An analysis similar to that in section 2.1 now shows that excited modes of both D-strings and fundamental strings are now produced by the time dependent background.

It would be interesting to analyze worldsheet string theory in this time dependent pp-wave. Backgrounds with similar singularities in the *string frame* metric have been studied earlier [43], [42] . In that case the worldsheet equations of motion are solvable in terms of special functions and certain statements about the validity of string theory could be made. In our background the worldsheet action is quadratic in the fields, but the equations of motion are not readily solvable. Our analysis of the Matrix Theory seems to indicate that nonperturbative physics becomes important. Nevertheless some insight from the worldsheet theory could be valuable.

3.5. *Issues*

The toy model of null cosmology described in this section might provide an interesting way to resolve a null singularity. In the asymptotic past (in light cone time) the gauge theory has a valid space-time interpretation in terms of supergravity. As we approach $x^+ = 0$, the 't Hooft coupling approaches zero, and the space-time description breaks down. Our investigations *suggest* that the weakly coupled gauge theory remains controlled and it is this description which should be used to approach and even continue past the singularity. Our analysis is not detailed enough to decide whether such a continuation is indeed possible.

Some of our conclusions were based on a treatment in the light cone gauge and in light front quantization. Sometimes light front quantization leads to subtleties with the zero longitudinal momentum mode. These subtleties could give rise to infra-red effects which have to be interpreted suitably.

As in the Matrix Theory models described in the previous section, backreaction due to an initially smooth perturbation which corresponds to a nice *normalizable* initial wavefunction of the gauge theory is an interesting question. Note that unlike the Matrix theory example, the bulk string coupling is small near the singularity, though stringy effects are large. If such a perturbation results in a curvature singularity in the bulk, perturbative string theory will certainly break down here. However, the main idea here is use the gauge theory description in this region. It remains to be seen whether this causes any problem for the gauge theory inspite of being weakly coupled. h These and other questions are currently under investigation.

4. Acknowledgements

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