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$\mathbf{Z}_{12-I}$ Orbifold Compactification toward SUSY Standard Model

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We explain the orbifold compactification in string models and present a $\mathbf{Z}_{12-I}$ orbifold compactification toward supersymmetric standard models. We also point out an effective R-parity from this string construction. The VEVs of gauge singlets are chosen such that phenomenological constraints are satisfied.

Keywords: Compactification; R-parity; Supersymmetric SM

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1. Introduction

The standard model (SM) with 45(+3) chiral fields is really remarkable. The big question in particle physics is, "How does this SM arise?" In late 1980s, there were attempts for standard-like models (which has three families with the gauge group SU(3)×SU(2)×U(1)^n) [1] from the orbifold compactification of heterotic string. Recent attempts have been more ambitious but it is fair to say that a model free of any phenomenological problems has not appeared yet even though partially attractive ones have been proposed in trinification, Pati-Salam, or just SM[2]. So, searches for good supersymmetric (SUSY) standard models (SSM) are going on now vigorously. Even some string models are suggested as roots for explaining the PVLAS data [3]. In this talk, I follow the compactification route through orbifolds. Orbifolds are manifolds modded by discrete actions. A nice feature of the orbifold compactification is that it is basically a geometric one.

For an SSM, we may obtain it either directly from compactification or through an intermediate step of SUSY GUT. In $\mathbf{Z}_{12-I}$, we have constructed both kinds [4,5], and here we focus on the direct construction.

In the orbifold construction, there has been the adjoint difficulty that at the Kac-Moody level $k = 1$ there does not appear an adjoint matter representation. Thus, GUTs SU(5), SO(10), and E_6 are not good candidates toward SSMs simply because the Higgs mechanism for breaking the GUT group is not present. This prefers GUTs with factor groups such as SU(3)^3, SU(4)×SU(2)×SU(2) and SU(5)×U(1). The trinification SU(3)^3 is possible only in $\mathbf{Z}_3$ which is nice toward achieving [6] $\sin^2\theta_W = \frac{3}{8}$. In addition to the $\sin^2\theta_W$ problem, there are several problems to be explained in those models,

- Approximate R-parity for proton longevity,
- Exotics problem,
- Vectorlike pairs problem,
- Successful fit to quark and lepton masses and mixing angles,
- Strong CP problem [7,8], etc.

Among all these problems, the most difficult and urgent one to overcome is the R-parity problem. One of the nice features of SO(10) GUT is said to be that it has a scheme to introduce the R-parity. Noting that SO(10) has both spinor ($\mathcal{S}$) and vector ($\mathcal{V}$) representations, one can assign $R = -1$ for $\mathcal{S}$ and $R = +1$ for $\mathcal{V}$ and then tree level couplings respect the R-parity. However, this is true only when gauge singlets are not introduced. The gauge singlets present in string compactification may behave like a spinor or a vector and the above simple argument of SO(10) GUT is not applicable to SSMs from string compactification. Thus, the R-parity consideration is most important. Only, approximate R-parity is obtained in string models so far [9,4]. Previous SSMs from string have not obtained the R-parity problem properly.

2. Strings on Orbifolds

Orbifolds are manifolded moded by discrete actions. The simplest example is $S_1/\mathbf{Z}_2$. In string compactifications, six internal spaces are usually divided into three two tori $T^2 \otimes T^2 \otimes T^2$. Each T^2 can be moded out be a discrete action. The totality of each discrete action is given a name $\mathbf{Z}_N$ orbifold.

The simplest orbifold is moding T^1 by $\mathbf{Z}_2$, identifying two points connected by $\mathbf{Z}_2$ actions. The points which stays at the same point under the $\mathbf{Z}_2$ action is called fixed points. Pictorially, we show this $S_1/\mathbf{Z}_2$ orbifold in Fig. 1 with fixed points located at *the boundary of the fundamental region* shown as the thick arc. Another

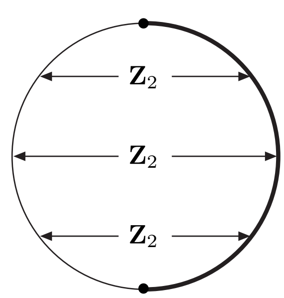

Fig. 1. The simplest orbifold $S_1/\mathbf{Z}_2$.

frequently discussed orbifold is $T^2/\mathbf{Z}_2$ shown in Fig. 2, which has four fixed points. For a $\mathbf{Z}_N$ orbifold, satisfying $\theta^N = 1$ for the consistency of the worldsheet spinors,

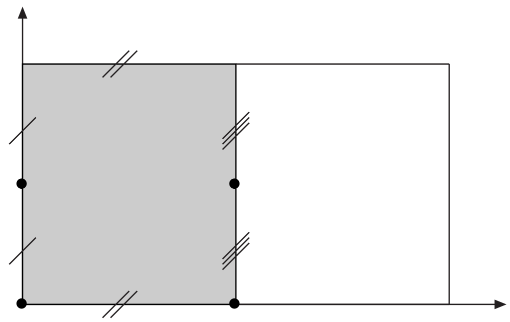

Fig. 2. The $T^2/\mathbf{Z}_2$ orbifold with two different length scales. The fundamental region is grey-colored which becomes a pillow after the identifications. The fixed points are at the boundary.

we have

$$N\sum_i \phi_i = \text{even integer.} \tag{1}$$

The most widely discussed and relevant one for us in this talk is $T^2/\mathbf{Z}_3$ orbifold shown in Fig. 3. For the coordinates of three two-tori, we use the complexified

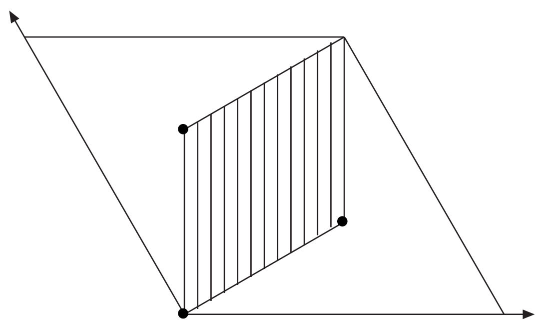

Fig. 3. The $T^2/\mathbf{Z}_3$ orbifold with 120^o rotation actions. The fundamental region is shown as slashed vertically.

coordinates,[a]

$$z \equiv (z_1, z_2, z_3) \sim \theta\cdot z = (e^{2\pi i\phi_1}z_1, e^{2\pi i\phi_2}z_2, e^{2\pi i\phi_3}z_3) \tag{2}$$

where ϕ_i denote the rotation angle in the $i^{\rm th}$ torus. Since one $T^2/\mathbf{Z}_3$ orbifold has three fixed points, the $(T^2/\mathbf{Z}_3)^3$ orbifold has 27 fixed points. There is only one way to write the *twist vector* ϕ,[b]

$$\phi = (\tfrac{2}{3}, \tfrac{1}{3}, \tfrac{1}{3}). \tag{3}$$

[a]For a more concrete discussion, see [10].
[b]For $\mathbf{Z}_6, \mathbf{Z}_8$, and $\mathbf{Z}_{12}$, there are two kinds of twists denoted by $\mathbf{Z}_{6-I}, \mathbf{Z}_{6-II}, \mathbf{Z}_{8-I}, \mathbf{Z}_{8-II}, \mathbf{Z}_{12-I}$, and $\mathbf{Z}_{12-II}$.

In quantum mechanics, any symmetry action is embedded in the quantum mechanical group space, here in particular in the $E_8 \times E_8'$ space. The simplest example called the standard embedding is the following embedding [11],

$$V = (\tfrac{2}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ 0\ 0\ 0\ 0\ 0)(0^8)'$$

which breaks E_8 down to $E_6 \times SU(3)$. Particle states are classified by untwisted and twisted sectors. Particles in the untwisted (U) sector are the closed strings in the torus. Particles in the twisted (T) sector are the closed strings only after the discrete action is taken into account. These U and T sector strings for a $\mathbf{Z}_3$ torus are shown in Fig. 4. The T strings are located at fixed points as shown in Fig. 4.

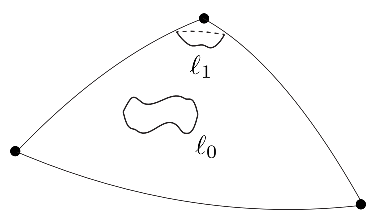

Fig. 4. The untwisted string ℓ_0 and twisted string ℓ_1 are shown.

There is additional degrees of freedom to embed the action in the group space [12], called the Wilson lines. Wilson lines are embedded in the group space, for example as $(0\ 0\ 0\ \tfrac{2}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ 0\ 0)(0^8)'$. There are consistency conditions for the shift vector V and the Wilson lines to satisfy for the modular invariance. The Wilson lines can break the group further. Without a Wilson line in the $i^{\rm th}$ torus, three fixed points of the $i^{\rm th}$ torus is not distinguishable and theory must respect an S_3 permutation symmetry [14]. The three fixed points of the $i^{\rm th}$ torus are distinguished by a Wilson line in the $i^{\rm th}$ torus, i.e. by $V + a_i, V - a_i$, and V.

The massless U strings satisfy just the modular invariance conditions with V and Wilson lines a_i. If P is a weight in the group space the massless U sector strings must satisfy, for $\mathbf{Z}_3$ for example,

$$\begin{aligned} P\cdot V &= 0, \pm\tfrac{1}{3} \text{ mod integer} \\ P\cdot a_i &= 0, \text{ mod integer} \end{aligned} \tag{4}$$

Since Wilson lines distinguish fixed points, the U sector strings are not affected by their presence except the modular invariance condition (4). Considering the untwisted sector vacuum energy, there is a masslessness spectrum condition also [11].

The T sector strings are made closed by the orbifolding action. Thus, the masslessness spectrum condition in the T sector is different from that of the U sector [11]. The modular invariance of the theory is given by the required form for V and a_i, and in the T sector distinguishing the fixed points we do not require any more.

But in the T sector where Wilson lines do not distinguish fixed points, we require the condition similar to (4) of the U sector.

The method of obtaining orbifold models is explicitly illustrated below with a $\mathbf{Z}_{12-I}$ twist and shift vectors V and $a_3 = a_4$.

3. Model

The $\mathbf{Z}_{12-I}$ twist is

$$\phi = (\tfrac{5}{12}, \tfrac{4}{12}, \tfrac{1}{12}) \tag{5}$$

and we take the following shift vector V and Wilson line [4]

$$V = \tfrac{1}{12}(3\ 3\ 3\ 3\ 3\ 5\ 5\ 1)(3\ 9\ 0^6)' \tag{6}$$

$$a_3 = a_4 = \tfrac{1}{3}(2\ 2\ 2\ -2\ -2\ 2\ 0\ 2)(0\ 2\ 2\ 0^5)' \tag{7}$$

$$a_1 = a_2 = a_5 = a_6 = 0. \tag{8}$$

From Eq. (5), we note that (12)- and (56)-tori are truly $\mathbf{Z}_{12}$ moding while the (34)-torus is $\mathbf{Z}_3$ moding. Therefore, Wilson lines distinguishing fixed points are applicable only to the (34)-torus and that must satisfy a $\mathbf{Z}_3$ shift as shown in (7). Thus, for $\mathbf{Z}_{12-I}$ the Wilson line part is very simple, i.e. it is just distinguishing three fixed points of the (34)-torus. In this sense, $\mathbf{Z}_{12-I}$ is a very simple model. For the gauge symmetry breaking, already there are much more possibilities of breaking even without Wilson lines in $\mathbf{Z}_{12-I}$ because many integers can be assigned in the numerator of $\frac{n}{12}$. In this sense, $\mathbf{Z}_{12-I}$ is very simple and also has a simple geometrical interpretation.

The gauge group is obtained by counting massless vector multiplets which appear in the untwisted sector,

$$\begin{aligned} P\cdot V = 0,\ \text{mod integer} \\ P\cdot a_3 = 0,\ \text{mod integer} \end{aligned} \tag{9}$$

If we consider $V^{\pm} = V \pm a_3$ as the twisted sector, a similar condition with $V^{\pm}$ instead of V can give gauge groups in the corresponding twisted sectors. In fact, the gauge group obtained from (9) is the intersection of gauge groups of V and $V^{\pm}$ which is automatically incoded by the second condition of (9). This gauge group is the one obtained from the U sector vector multiplets. From (9), we obtain the following gauge group,

$$SU(3)\times SU(2)\times U(1)_Y \times U(1)^4 \times [SO(10)\times U(1)^3]'. \tag{10}$$

Embedding of the electroweak hypercharge Y is possible for two cases for which different weak mixing angles are obtained,

$$\text{Model E}: \ Y = (\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{-1}{2}\ \tfrac{-1}{2}\ 0\ 0\ 0)(0^8)',\quad \sin^2\theta_W = \tfrac{3}{8} \tag{11}$$

$$\text{Model S}: \ Y = (\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{-1}{2}\ \tfrac{-1}{2}\ 0\ 0\ 0)(0\ 0\ 1\ 0^5)',\quad \sin^2\theta_W = \tfrac{3}{14}. \tag{12}$$

For Model E, we obtain vectorlike exotics while in Model S there does not appear any exotics.[c] Below, we discuss Model E in detail.

Matter particles are obtained from the U sector and the T sector. For the U sector, the modular invariance condition for chiral matter is $P \cdot V =$ one of the entries of ϕ given in Eq. (5), denoted as U_1 if it comes from matching with the first entry, etc. In fact, the CTP conjugates also appear in one of these. The same chirality set is $\{\frac{1}{12}, \frac{4}{12}, \frac{7}{12}\}$.

The fundamental twist ϕ is the defining one and called the T_1 sector. If the twist is ϕ, then any integer (n) multiple of ϕ must be considered also. Thus, we consider T_n twist sectors up to $n = 6$.[d] Since a_3 is a $\mathbf{Z}_3$ shift 3, 6, 9 multiples of twist do not have any Wilson line. The other twist sectors, however, can have Wilson lines. Thus, the twist sectors are denoted as

$$T_1^0, T_1^{\pm}, T_2^0, T_2^{\pm}, T_3, T_4^0, T_4^{\pm}, T_5^0, T_5^{\pm}, T_6.$$

Twisted sector massless condition is given by considering the corresponding twisted sector vacuum energy, and there is a well-defined way to calculate their chiralities [10]. Since T_3, T_6, T_9 sectors do not involve Wilson lines, in addition they must satisfy in a sense an untwisted sector-like modular invariance condition similar to (9), generalized to

$$(P + kV) \cdot a_3 = 0 \text{ mod } \mathbf{Z}, \quad \text{for } k = 0, 3, 6, 9. \tag{13}$$

The calculation of spectrum is not enough. We have to find out the chirality by considering the right movers. Also, there can be some linear combinations of localized fields whose multiplicities must be calculated. After all these considerations, we obtain a complete massless spectrum. For the method, refer to [10,5,4]. The SM particles are listed in Table 1. Summarizing the standard charge partcles except extra neutral singlets

$$\begin{array}{l} U: \ Q(U_1), L(U_1), u^c(U_3), d^c(U_3), \nu^c(U_3), e^c(U_3), H_u(U_2), H_d(U_2) \\ T_4^0: \ 2\{Q, L, u^c, d^c, \nu^c, e^c, \overline{D}, D, H_u, H_d\}, \ \overline{D}, H_d \\ T_6: \ 3\{\overline{D}, D\}, \ 2\{H_u, H_d\} \\ T_3: \ \overline{D}, H_d, \ 3 \cdot \mathbf{10}' \\ T_9: \ 2D, 2H_u \end{array} \tag{14}$$

Note that we can assign the 3rd family in the U sector and the first and second families in the T_4^0 sector. It has been shown that Yukawa couplings with appropriate neutral singlets can make vectorlike pairs massive.

The study of exotic particles is very tricky, but in our model these are known to be vectorlike and made massive by choosing appropriate VEVs of gauge singlets [4].

[c]Note added: Another exotics free model [13] also has the hypercharge invaded by the hidden sector. At present, I do not know any exotics free SSM without the invasion of the electroweak hypercharge by the hidden sector.

[d]For $6 < n \leq 11$, they provide the CTP conjugates of those appearing in $1 \leq n < 6$ except in T_3 and T_9. T_3 and T_6 states can contain CTP conjugates also.

Table 1. The SM particles.

Visible states	SM notation	Γ	Γ'
$(\underline{++-};\underline{+-};+++)(0^8)'$	$Q(U_1)$	-1	$+1$
$(\underline{+--};--;+++)(0^8)'$	$d^c(U_3)$	-1	$+1$
$(\underline{+--};++;+--)(0^8)'$	$u^c(U_3)$	-1	-3
$(---;\underline{+-};+--)(0^8)'$	$L(U_1)$	-1	-3
$(+++;--;-+-)(0^8)'$	$e^c(U_3)$	$+5$	$+5$
$(+++;++;+++)(0^8)'$	$\nu^c(U_3)$	-1	$+1$
$(0\ 0\ 0;\underline{-1\ 0};-1\ 0\ 0)(0^8)'$	$H_u(U_2)$	$+2$	$+2$
$(0\ 0\ 0;\underline{1\ 0};0\ 0\ 1)(0^8)'$	$H_d(U_2)$	-4	-2
$(\underline{++-};\underline{+-};\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot Q(T_4^0)$	$+1$	$+1$
$(\underline{+--};--;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot d^c(T_4^0)$	$+1$	$+1$
$(\underline{+--};++;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot u^c(T_4^0)$	-3	-3
$(---;\underline{+-};\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot L(T_4^0)$	-3	-3
$(+++;--;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot e^c(T_4^0)$	$+5$	$+5$
$(+++;++;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot \nu^c(T_4^0)$	$+1$	$+1$
$(\underline{1,0,0};0\ 0;\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$3\cdot \overline{D}_{1/3}(T_4^0)$	$\boxed{+2}$	$\boxed{+2}$
$(\underline{-1,0,0};0\ 0;\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$2\cdot D_{-1/3}(T_4^0)$	$\boxed{-2}$	$\boxed{-2}$
$(0,0,0;\underline{-1\ 0};\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$2\cdot H_u(T_4^0)$	$+2$	$+2$
$(0,0,0;\underline{1\ 0};\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$3\cdot H_d(T_4^0)$	-2	-2
$(\underline{1,0,0};0\ 0;0^3)(\frac{-1}{2}\ \frac{1}{2}\ 0;0^5)'$	$3\cdot \overline{D}_{1/3}(T_6)$	$\boxed{+2}$	$\boxed{+2}$
$(\underline{-1,0,0};0\ 0;0^3)(\frac{1}{2}\ \frac{-1}{2}\ 0;0^5)'$	$3\cdot D_{-1/3}(T_6)$	$\boxed{-2}$	$\boxed{-2}$
$(0,0,0;\underline{-1\ 0};0^3)(\frac{-1}{2}\ \frac{1}{2}\ 0;0^5)'$	$2\cdot H_u(T_6)$	$+2$	$+2$
$(0,0,0;\underline{1\ 0};0^3)(\frac{1}{2}\ \frac{-1}{2}\ 0;0^5)'$	$2\cdot H_d(T_6)$	-2	-2
$(\underline{\frac{3}{4}\frac{-1}{4}\frac{-1}{4}};\frac{-1}{4}\frac{-1}{4};\frac{1}{4}\frac{1}{4}\frac{1}{4})(\frac{3}{4}\frac{1}{4}\ 0;0^5)'$	$\overline{D}_{1/3}(T_3)$	1	$\boxed{+2}$
$(\underline{\frac{-3}{4}\frac{1}{4}\frac{1}{4}};\frac{1}{4}\frac{1}{4};\frac{-1}{4}\frac{-1}{4}\frac{-1}{4})(\frac{-3}{4}\frac{-1}{4}\ 0;0^5)'$	$2\cdot D_{-1/3}(T_9)$	-1	$\boxed{-2}$
$(\frac{1}{4}\frac{1}{4}\frac{1}{4};\underline{\frac{-3}{4}\frac{1}{4}};\frac{-1}{4}\frac{-1}{4}\frac{-1}{4})(\frac{1}{4}\frac{3}{4}\ 0;0^5)'$	$2\cdot H_u(T_9)$	$+4$	$+3$
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4};\underline{\frac{3}{4}\frac{-1}{4}};\frac{1}{4}\frac{1}{4}\frac{1}{4})(\frac{-1}{4}\frac{-3}{4}\ 0;0^5)'$	$H_d(T_3)$	-4	-3

There exist eight U(1) gauge symmetries whose charges are denoted as

$$Y = (\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{-1}{2}\ \tfrac{-1}{2}\ 0^3)(0^8)' \tag{15}$$

$$B-L = (\tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3}\ 0\ 0\ 0^3)(0^8)' \tag{16}$$

$$Q_1 = (0^5\ 2\ 0\ 0)(0^8)' \tag{17}$$

$$Q_2 = (0^5\ 0\ 2\ 0)(0^8)' \tag{18}$$

$$Q_3 = (0^5\ 0\ 0\ 2)(0^8)' \tag{19}$$

$$\begin{aligned} Q_4 &= (0^8)(2\ 0\ 0\ 0^5)' \\ Q_5 &= (0^8)(0\ 2\ 0\ 0^5)' \\ Q_6 &= (0^8)(0\ 0\ 2\ 0^5)' \end{aligned} \tag{20}$$

One linear combination of the above charges is the $\mathrm{U}(1)_X$ charge of the flipped SU(5),

$$X = (2\ 2\ 2\ 2\ 2\ 0\ 0\ 0)(0^8)' \tag{21}$$

There exists an anomalous $\mathrm{U}(1)_{\rm anom}$ with

$$Q_{\rm anom} = Q_1 + Q_2 + Q_3 + Q_4 - Q_5 + 6X. \tag{22}$$

Toward embedding a $\mathbf{Z}_2$ matter parity P in an anomaly free U(1), we choose $\mathrm{U}(1)_\Gamma$ where

$$\Gamma = X + \tfrac{1}{4}(Q_4 + Q_5) - (Q_2 + Q_3) + 6(B - L). \tag{23}$$

4. Phenomenology

Except the gauge interactions, phenomenology results from Yukawa couplings including the nonrenormalizable terms. The Yukawa coupling structure respects

- Gauge symmetries,
- Lorentz symmetry, in particular from the internal coordinates the H-momentum conservation,
- The modular invariance conditions for the $T_k^{m_f}$ sector

$$\sum_z k(z) = 0 \bmod 12 \tag{24}$$

$$\sum_z [km_f](z) = 0 \bmod 3 \tag{25}$$

- The modular invariance requires the sum of H-momenta being $(-1, 1, 1)$ mod (12, 3, 12).[e]

For $\mathbf{Z}_{12-I}$, the H-momenta are given by

$$\begin{array}{l} U_1 : (-1, 0, 0), \quad U_2 : (0, 1, 0), \quad U_3 : (0, 0, 1), \\ T_1 : (\frac{-7}{12}, \frac{4}{12}, \frac{1}{12}), \quad T_2 : (\frac{-1}{6}, \frac{4}{6}, \frac{1}{6}), \quad T_3 : (\frac{-3}{4}, 0, \frac{1}{4}), \\ T_4 : (\frac{-1}{3}, \frac{1}{3}, \frac{1}{3}), \quad \left\{T_5 : (\frac{1}{12}, \frac{-4}{12}, \frac{-7}{12})\right\}, \quad T_6 : (\frac{-1}{2}, 0, \frac{1}{2}), \\ T_7 : (\frac{-1}{12}, \frac{4}{12}, \frac{7}{12}), \quad T_9 : (\frac{-1}{4}, 0, \frac{3}{4}). \end{array} \tag{26}$$

For example, $T_2T_4T_6$ has the H-momentum (−1, 1, 1); thus the coupling is allowed if the other conditions are satisfied. The 3rd family quark masses arise from $Q(U_1)u^c(U_3)H_u(U_2)$ and $Q(U_1)d^c(U_3)H_d(U_2)$ which are cubic. But we need much more couplings to make this model phenomenologically successful. Since there are O(100) chiral fields, a computer search may be necessary. We have shown [4] that if neutral singlets are allowed to get GUT scale VEVs then all the needed phenomenology can be met, in particular vectorlike exotics and non-exotic vectorlike pairs obtain large masses. Also, supersymmetric conditions the F-flatness and D-flatness have to be checked in a specific model for given singlet VEVs [4]. But the R-parity needs a special treatment on which we will comment shortly.

Before discussing the R-parity, we point out that now it is possible to study an approximate global symmetry. In fact, in the $\mathbf{Z}_{12-I}$ flipped SU(5) model [5], we studied an approximate Peccei-Quinn symmetry [8]. We studied $\mathrm{U}(1)_{\mathrm{anom}} \times \mathrm{U}(1)_{\mathrm{global}}$ where $\mathrm{U}(1)_{\mathrm{global}}$ is an approximate one. We find that the QCD axion is possible and

[e] An extensive discussion is given in [10].

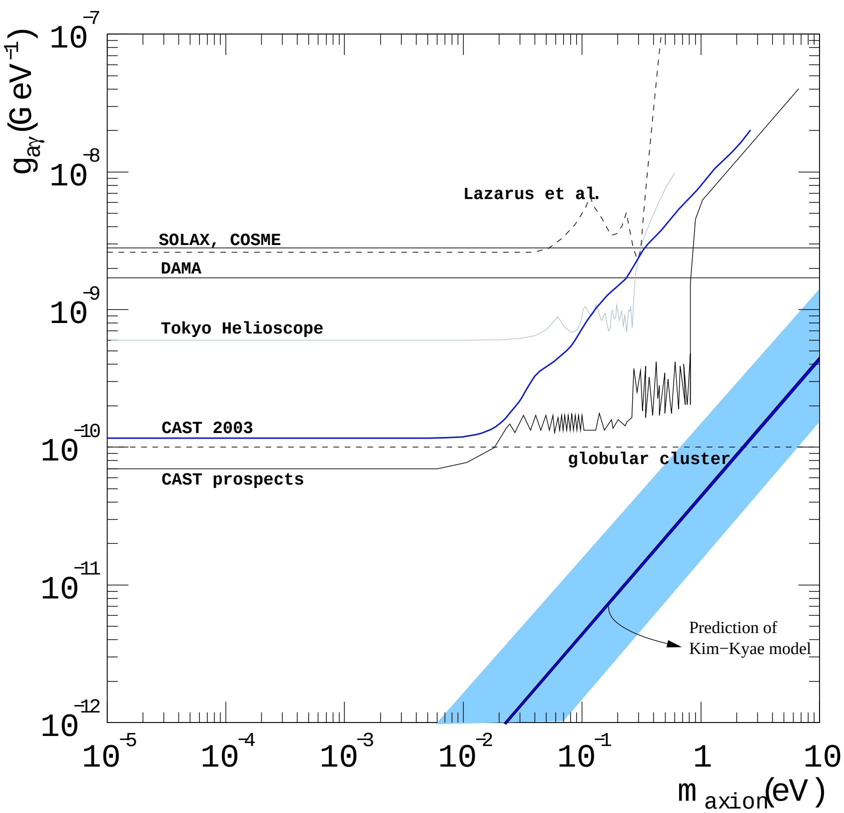

Fig. 5. The CAST 2003 data compared to the $\mathbf{Z}_{12}$ model given by the thick line. The band is the 20 % theoretical error of Ref. [15].

obtained the axion-photon-photon coupling for the first time from string compactification,

$$c_{a\gamma\gamma} = \overline{c}_{a\gamma\gamma} - 1.93 \simeq -0.26 \qquad (27)$$

where –1.93 appears from the QCD chiral symmetry breaking. However, the decay constant turns out to be at the GUT scale and hence it is very difficult to observe the very light axion from the cavity experiments even though we invoke the anthropic principle for the misalignment problem. In Fig. 5, the above axion-photon-photon coupling strength is compared to the recent solar axion search from CAST [15].

This kind of study on approximate symmetries can be done in a specific model. In particular, the R-parity must be studied in a specific model as performed in the flipped SU(5) [9]. A probable failure in obtaining an R-parity is that one needs many singlet VEVs for other phenomenological reasons.

5. Effective *R*-parity

The approximate R-parity in Ref. [8] was obtained by studying Yukawa couplings up to dimension 7. But a better way is to embed the R-parity or matter parity[f] in an anomaly free U(1) gauge group. A parity is a $\mathbf{Z}_2$ operation. Some VEVs of the U(1) charge carrying fields can break U(1) down to $\mathbf{Z}_N$ if the field carries N units of the fundamental U(1) charge. If we normalize the smallest nonvanishing U(1) charge as ± 1, then a VEV of U(1) charge N field breaks U(1)$\to$ $\mathbf{Z}_N$. So if only even integer U(1) charge fields, including $Q = 2$, are given VEVs then the final discrete group is $\mathbf{Z}_2$, and we succeed in obtaining a matter parity. However, if some phenomenological reasons dictate some $Q = \pm 1$ fields develop GUT scale VEVs, we do not obtain such a matter parity. In this case, we can resort only to an approximate matter parity.

In the E_8 group space, the weights are divided into two classes, the vector type $\mathcal{V}$ and the spinor type $\mathcal{S}$. In the U sector, the vector type has the form of P such as

$$P = (\pm 1\ 0\ 0\ \ \pm 1\ 0\ 0\ 0), \tag{28}$$

while the spinor type has the form of P such as

$$P = (\pm\tfrac{1}{2}\ \pm\tfrac{1}{2}\ \pm\tfrac{1}{2}\ \pm\tfrac{1}{2}\ \pm\tfrac{1}{2}\ \pm\tfrac{1}{2}\ \pm\tfrac{1}{2}\ \pm\tfrac{1}{2}). \tag{29}$$

Below we will represent $\pm\frac{1}{2}$ simply as $\pm$. In the T sector, $P + kV_{0,\pm}$ are considered to see whether some components are of the vector type or of the spinor type. If we pick up the $U(1)_X$ charge of the flipped SU(5)

$$X = (2\ 2\ 2\ 2\ 2\ 0\ 0\ 0)(0^8)', \tag{30}$$

the vector type weight (28) has an even number for X eigenvalue while the spinor type weight (29) has an odd number for X eigenvalue. This results because we have an odd number of entries of 2 in Eq. (30). So this $U(1)_X$ can be a good mother for the matter parity. If we consider three entries of 2, it is three times $U(1)_{\rm B-L}$ of Eq. (16) which can be another good mother group for the matter parity. Q_1, Q_2, and Q_3 can do the same job. Out of these five, only four U(1)s are considered to be independent.

Note that all SM particles of Table 1 are of $\mathcal{S}$ type and some needed Higgs doublets are of $\mathcal{V}$ type. So if all the needed VEVs of gauge singlets are of $\mathcal{V}$ type, then we achieve introducing an exact matter parity in that model. However, if some $\mathcal{S}$ singlet(s) is required to have a GUT scale VEV(s), then the matter parity introduced is not exact. The charge of the hypothetical mother $U(1)_\Gamma$ group is given in Eq. (23) which has an odd number of 2 entries.

In Table 2, we list gauge singlets with $U(1)_\Gamma$ charges. Here in the Γ column, singlets having odd Γ charges are boxed. We have to check whether phenomenology

[f] Strictly speaking, we work with the matter parity since gauginos are not considered, but this matter parity can be properly extended to become an R-parity. So we use both words without distinction.

Table 2. Left-handed electromagnetically neutral $SO(10)'$ singlets. There is only one untwisted sector singlet S_0. To have a definition of parity, S_{15}, S_{16}, S_{18}, S_{20}, and S_{23} should not develop VEVs.

Visible states	SM notation	$B-L$	X	Γ	Γ'	Label
$(0\ 0\ 0; 0\ 0; 1\ 0\ -1)(0^8)'$	$\mathbf{1_0}(U_2)$	0	0	+2	0	S_0
$(0^5; \frac{-2}{3}\frac{-2}{3}\frac{-1}{3})(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$\mathbf{1_0}(T_2^0)$	0	0	+2	0	S_1
$(0^5; \frac{-2}{3}\frac{1}{3}\frac{2}{3})(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$\mathbf{1_0}(T_2^0)$	0	0	−2	0	S_2
$(0^5; \frac{1}{3}\frac{-2}{3}\frac{2}{3})(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$\mathbf{1_0}(T_2^0)$	0	0	0	0	S_3
$(0^5; \frac{1}{3}\frac{1}{3}\frac{-1}{3})(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_2^0)$	0	0	0	0	S_4
$(0^5; \frac{1}{3}\frac{1}{3}\frac{-1}{3})(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_2^0)$	0	0	0	0	S_5
$(0^5; \frac{2}{3}\frac{2}{3}\frac{-2}{3})(0^8)'$	$2\cdot\mathbf{1_0}(T_4^0)$	0	0	0	0	S_6
$(0^5; \frac{-1}{3}\frac{-1}{3}\frac{-2}{3})(0^8)'$	$7\cdot\mathbf{1_0}(T_4^0)$	0	0	+2	0	S_7
$(0^5; \frac{-1}{3}\frac{2}{3}\frac{1}{3})(0^8)'$	$6\cdot\mathbf{1_0}(T_4^0)$	0	0	−2	0	S_8
$(0^5; \frac{2}{3}\frac{-1}{3}\frac{1}{3})(0^8)'$	$6\cdot\mathbf{1_0}(T_4^0)$	0	0	0	0	S_9
$(0^5; 1\ 0\ 0)(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	0	0	S_{10}
$(0^5; -1\ 0\ 0)(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	0	0	S_{11}
$(0^5; 0\ 0\ 1)(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	−2	0	S_{12}
$(0^5; 0\ 0\ -1)(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	+2	0	S_{13}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{5}{12}\frac{1}{12})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	0	$\boxed{+1}$	S_{14}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{5}{12}\frac{1}{12})(\frac{-3}{4}\frac{-1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	$\boxed{-1}$	0	S_{15}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{-5}{12})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	$\boxed{+1}$	0	S_{16}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{-5}{12})(\frac{-3}{4}\frac{-1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	0	$\boxed{-1}$	S_{17}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-7}{12}\frac{5}{12}\frac{1}{12})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	$\boxed{-1}$	0	S_{18}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-7}{12}\frac{5}{12}\frac{1}{12})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	0	$\boxed{+1}$	S_{19}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{-7}{12}\frac{1}{12})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	$\boxed{+1}$	0	S_{20}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{-7}{12}\frac{1}{12})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	+2	$\boxed{+1}$	S_{21}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{7}{12})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	−2	$\boxed{-1}$	S_{22}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{7}{12})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	$\boxed{-1}$	0	S_{23}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-3}{4}\frac{1}{4}\frac{1}{4})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_3)$	$-\frac{1}{2}$	$\frac{5}{2}$	−2	$\boxed{-1}$	S_{24}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{3}{4}\frac{-1}{4}\frac{-1}{4})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_9)$	$\frac{1}{2}$	$-\frac{5}{2}$	+2	$\boxed{+1}$	S_{25}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{1}{4}\frac{1}{4}\frac{-3}{4})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_3)$	$-\frac{1}{2}$	$\frac{5}{2}$	0	$\boxed{-1}$	S_{26}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-1}{4}\frac{-1}{4}\frac{3}{4})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_9)$	$\frac{1}{2}$	$-\frac{5}{2}$	0	$\boxed{+1}$	S_{27}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-1}{4}\frac{-1}{4}\frac{-1}{4})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_3)$	$\frac{1}{2}$	$-\frac{5}{2}$	+2	$\boxed{+1}$	S_{28}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{1}{4}\frac{1}{4}\frac{1}{4})(\frac{-3}{4}\frac{-1}{4}\ 0; 0^5)'$	$3\cdot\mathbf{1_0}(T_9)$	$-\frac{1}{2}$	$\frac{5}{2}$	−2	$\boxed{-1}$	S_{29}

needs definitely one of these to develop a large VEV or not. If we can choose a vacuum where it is not necessary for any of these to develop a large VEV, then we can introduce an exact matter parity.

But to make all exotics heavy, we need $\langle S_{15}\rangle \neq 0$ and $\langle S_{23}\rangle \neq 0$. So an exact R-parity is not introduced in the model. However, we note that even if we obtained an exact R-parity, proton still decays by dimension-5 operators

$$QQQL,\ u^cu^cd^ce^c.$$

So an exact R-parity conservation requirement can be considered as an over-requirement with respect to the proton longevity problem. An approximate R-parity with proton lifetime comparable to that coming from dimension-5 operators is a good enough parity. However, if one requires an absolutely stable lightest SUSY particle (LSP), the exact R-parity might be a good requirement. However, here also the LSP with its lifetime much larger than the age of universe can be considered as a good dark matter candidate.

The dimension-4 operator $u^c d^c d^c$ is particularly problematic if there exists a tree level lepton number violating operator also. In our model, $u^c d^c d^c$ carries $\Gamma = -3$, so to have a dimension-4 $u^c d^c d^c$ coupling we need composite singlets carrying $\Gamma = 3$: $\prod_i S_i$

$$\underbrace{u^c d^c d^c}_{\Gamma=-3} \underbrace{\langle \prod_i S_i \rangle}_{\Gamma=+3} \tag{31}$$

To obtain $\Gamma = +3$ from $\prod_i S_i$, we need for example $S_0 S_0 S_{15}$ where S_0 is a U_2 field and S_{15} is a T_1^0 field. Other singlets having nonvanishing VEVs must belong to $S_1 - S_{13}$. We can check that with these combinations we cannot satisfy the modular invariance condition $\sum$twists $= 0$ mod 12. This implies that in the vacuum we choose the coupling $u^c d^c d^c$ does not appear at all. However, the matter parity or the R-parity is broken by heavy intermediate states, which is nothing but those appearing in GUT models. So we estimate that the lightest neutralino lifetime is around 10^{22} years through R-parity violating interactions, considering heavy particles, which is long enough to be considered as a dark matter candidate.

6. Conclusion

We reviewed very briefly the orbifold compactification. As a definite acceptable example, we constructed a $\mathbf{Z}_{12-I}$ orbifold model with the SM gauge group and three families in which we achieve

- All exotics are removed by singlet VEVs,
- The 3rd family appears in the U sector,
- The weak mixing angle is $\sin^2\theta_W = \frac{3}{8}$, and
- An effective R-parity without $u^c d^c d^c$ coupling can be introduced. Still the neutralino can be a dark matter candidate.

In another vacuum, we can align the hypercharge so that no exotics appear but in this case the weak mixing angle is $\sin^2\theta_W = \frac{3}{14}$.

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