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SUSY BREAKING IN GUTS

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I will describe a minimal supersymmetric SU(5) model, in which the adjoint multiplet breaks spontaneously both gauge and supersymmetry. This inevitably leads to intermediate scales, which in turn increase the GUT scale. The dimension 6 proton decay is thus suppressed, while soft terms coming from supergravity typically dominate.

1. Introduction

In the last few years there has been an increasing interest in the study of flavour structure of light fermions in the context of supersymmetric grand unified theories. The best candidate is clearly the SO(10) GUT, which automatically incorporates neutrino masses. The minimal model, which is at the same time not obviously wrong and constrained enough, has been proposed long ago [1] and recently revived [2]. After many analyses it has been ruled out due to a nontrivial interplay of the Yukawa and Higgs sectors [3]. Apart from some minor checks still to be done (for example, in all known computations the fitting has been done at the GUT scale, where the experimental errors are not known, which brings a big uncertainty in the χ^2 analyses), there are two main objections to the whole program.

The first one is our ignorance of the physics at the Planck scale. This is seen through completely arbitrary higher dimensional non-renormalizable terms in any Lagrangian. Such ignorance is obviously present also in the SM, but at the GUT scale it can be very important, since they are suppressed only by positive powers of $M_{GUT}/M_{Planck} \approx 10^{-3}$. The actual situation is even worse: the minimal model is not asimptotically free and the Landau pole is approximately only one order of magnitude above the GUT scale. Here we do not have much to say on this important but very difficult problem. We know from experiment that at least some of these unknown non-renormalizable operators must be very much suppressed. For example, the coefficient in front of the operator that mediates $d = 5$ proton decay $QQQL/M_{Planck}$ must be at least 10^{-7}. All one can do without introducing extra flavour or other symmetries is to assume that all these higher dimensional operators are absent. Ony in such a case any analyses of the fermion masses and mixings can make sense.

The second problem is our ignorance of the soft supersymmetry breaking terms. As it is well known, these terms in general contribute to fermion masses as finite corrections [4] and so are not included in the RGE evolution. They have sometimes been already used to improve the GUT relations [5], but not in the context of the minimal supersymmetric SO(10) model. Needless to say, without knowing the way supersymmetry is broken, not much can be said.

So there is an intricate interplay among supersymmetry breaking, grandunification and fermion masses, which is worth pursuing. This is the main motivation of the work presented here, which essentially summarizes the results obtained recently [6].

2. What we do not know about supersymmetry beaking

Very little is known of the supersymmetry breaking terms. Experimentally the masses of the spartners must weigh 100 GeV or more, while the absence of flavour changing neutral currents tells us that the sfermion mass matrices cannot be arbitrary, but must be close to unity, or related to the quark mass matrices. Any scheme that breaks supersymmetry must take into account these constraints.

If supersymmetry were broken in the MSSM sector (for example by a nonzero F-term of the Higgs), the sfermions would get their soft masses at tree order. The mass sum-rules would then predict a selectron lighter than the electron, clearly ruled out by experiment. So supersymmetry must be first broken in a hidden sector (hidden from the SM, it must be only weakly coupled to it) and then transmitted by a messenger to our sector, the MSSM.

The hidden sector is usually assumed to be strongly interacting, so new interactions and representations are needed. In this talk we will on the contrary assume that no new sectors is present, and the GUT breaking sector is also the SUSY breaking (hidden) sector.

What is known about the mediation of supersymmetry breaking? Unfotunately, although the Higgs doublet could do the job in principle [7] , explaining also why flavour changing neutral currents are suppressed also in the superpartner sector, the gluino mass would be too light and the stop would get a negative contribution to its mass square at 1-loop [8].

Regarding supersymmetry breaking candidates let me mention just few cases, usually the most considered in the literature (for a recent detailed review see for example [9] and references therein).

The first considered and simplest option is to take gravity as the messenger . The nice feature of it is that gravity is present anyway, so it is in a sense the minimal option. On the other side, it is just a different parametrization of the soft terms. As we already explained above, they depend on the assumptions on the choice of the Kähler potential at the Planck scale. In the case of a canonical Kähler one gets the so called constrained minimal supersymmetric model. Everything here depends on a very few parameters, but one should keep in mind that such a case is theoretically really not motivated: why should the Kähler be canonical? And if it is not, the soft terms become generically flavour dependent, contributing to the flavour changing neutral currents. Sometimes their absence is wrongly used as an evidence or argument in favour of a canonical Kähler. But since there is no extra symmetry in the limit of a canonical Kähler, the argument is wrong: all we know is that the flavour changing neutral currents must be small enough. A close to canonical Kähler is a consequence of experimentally small flavour changing neutral currents, so one should not start with a canonical Kähler and then predict some definite (small) fcnc due to running. Although allowed by data, this is just one point in a much more vast parameter space.

A second option is anomaly mediation [10]. As before, since it is connected with gravity, it is always automatically there. By itself it is very predictive, so predictive that it is wrong. The point is that the sfermion masses square are roughly proportional to the beta function of the gauge coupling, so that sleptons get a negative mass due to the Abelian hypercharge. The result is also RG invariant, so running cannot change it. One needs to combine this contribution to other mediations, which brings us back to the old uncertainties.

The most popular scenario is undoubtedly the so called gauge mediation [11]. As in any model, supersymmetry must be broken in a hidden sector: let X be the field responsible for it, so that $F_X \neq 0$. As we said, it cannot be coupled to the MSSM fields directly, so one introduces extra quark-type and lepton-type vectorlike multiplets $q, \bar{q}$ and $l, \bar{l}$. Couple them to X in the superpotential as

$$W = \lambda_q X \bar{q} q + \lambda_l X \bar{l} l + W_{MSSM} \ . \tag{1}$$

These extra chiral multiplets get a tree order susy breaking masses from X and transmit this information to the MSSM sector through SM gauge interactions. This happens only at 2-loops for the sfermions and thus the sum-rules are evaded.

The nice point of this mechanism is that it predicts flavour conservation in the sfermion soft sector, since gauge interactions are flavour blind.

The bad and often hidden part of this mechanism is that there are usually no good reasons not to couple these extra quarks and leptons to the SM quarks and leptons. For example, in order not to spoil gauge coupling unification, these extra states come from full multiplets like $5 = (q, l)$ and $\bar{5} = (\bar{q}, \bar{l})$. They have the same quantum numbers as the SM quarks and leptons, and thus for example terms like

$$\Delta W = \tilde{Y} H l e^c \tag{2}$$

between the SM e^c and this extra l are possible. Flavour conservation is maintained only if $\tilde{Y} \ll 1$, which again puts just a limit on $\tilde{Y}$, and the absence of flavour changing neutral currents is used as an input to constrain model parameters, not as a prediction of the theory. In this sense it is usually not much better than supergravity.

3. The model

To see whether the above program could work, we start with a simplified (although still realistic) SU(5) model and not with the minimal SO(10) model mentioned above.

The rules of the game are:

- no extra intercations except GUT (no room for dynamical supersymmetry breaking or Fayet-Iliopoulos) and gravity;
- no singlets (no room for O'Raifeartaigh).

We will consider a minimal SU(5) model, with the adjoint 24_H needed to break the GUT symmetry to the SM one, and the $5_H - \overline{5}_H$ pair needed to give masses to the three generations of 10_F and $\overline{5}_F$ fermionic components. In the spirit of minimality we would like to see whether this minimal set-up can describe also supersymmetry breaking. In other words, is it possible to construct a minimal SU(5) model with the above Higgs and matter content, in which the same field (the adjoint) breaks spontaneously both the GUT symmetry and supersymmetry? Nonminimal models of this kind are already known. We will assume here also perturbativity (this is actually not an extra assumption: SU(5) is complicated enough, so that its nonperturbative behaviour is not well known, see however Seiberg).

It is not hard to see, that a renormalizable version is too constrained and simple to get what we want. Also, although we allow our minimum to be only local, it is simpler to stabilize it with supergravity terms than with one-loop corrections. The sugra corrections are needed anyway to cancel the cosmological constant. Both nonrenormalizability and supergravity go away from our original purpose to be predictive for the fermion masses, but are needed simplifications to start with. We will see that even in this case one can get (rather surprisingly) some nontrivial predictions and restrictions.

Any model must satisfy the following constraints:

- the total energy vanishes: $V = 0$;
- the equations of motion are satisfied: $\partial V / \partial \phi_i = 0$;
- there are no tachyonic directions: the matrix with second derivatives can have only nonnegative eigenvalues.

We will limit ourselves to the canonical Kähler potential [a]: the whole procedure can be easily generalized. The theory is thus specified with the superpotential. The SU(5) model with an adjoint and the minimal number of terms with all the above characteristics is

$$W = W_0 + m\, Tr\, 24_H^2 + \lambda\, Tr\, 24_H^3 + \frac{a_1}{M_{Pl}} Tr\, 24_H^4 + \frac{a_2}{M_{Pl}} \left(Tr\, 24_H^2 \right)^2 , \qquad (3)$$

[a] For a noncanonical Kähler one could get susy breaking even with only renormalizable terms in the superpotential.

with the constants subject to [12,16]

$$m = \frac{3\sqrt{3}\eta}{16\pi}\frac{m_{3/2}M_{Pl}}{M_{GUT}} + \mathcal{O}(m_{3/2}) \ , \tag{4}$$

$$\lambda = \frac{\sqrt{10}\eta}{8\pi}\frac{m_{3/2}M_{Pl}}{M_{GUT}^2} + \mathcal{O}(\frac{m_{3/2}}{M_{GUT}}) \ , \tag{5}$$

$$\frac{7}{30}a_1 + a_2 = \frac{\sqrt{3}\eta}{32\pi}\frac{m_{3/2}M_{Pl}^2}{M_{GUT}^3} + \mathcal{O}(\frac{m_{3/2}M_{Pl}}{M_{GUT}^2}) \ , \tag{6}$$

where η is an arbitrary phase ($|\eta| = 1$), $m_{3/2}$ is the gravitino mass and the GUT scale M_{GUT} is the vev of the SM singlet in $\langle 24_H\rangle = diag(2,2,2,-3,-3)M_{GUT}/\sqrt{30}$.

Notice that

- all the terms in the superpotential for the SM singlet are of similar order close to the minimum M_{GUT};
- although the gravitino mass is undetermined before the RGE, it is expected to be close to TeV (see however the discussion below). In this case the mass parameter (4) is much smaller than the typical scale M_{GUT}, and the dimensionless combination (5) and (6) are much smaller than one;
- the last constraint (6) on the combination of a_i allow in one limit small a_i themselves or, in the opposite limit, a fine-tuning of two large numbers. The two limits predict different mass spectra and we will consider these two and othe rpossibilities in the next section.

4. The RGE analyses

The weak triplet and colour octets in 24_H have the supersymmetric mass

$$m_3 = \frac{8}{3}a_1\frac{M_{GUT}^2}{M_{Pl}} + \left[-14\left(\frac{\sqrt{3}}{8\pi}m_{3/2}\frac{M_{Pl}}{M_{GUT}}\right) + \mathcal{O}(m_{3/2})\right] \ , \tag{7}$$

$$m_8 = \frac{2}{3}a_1\frac{M_{GUT}^2}{M_{Pl}} + \left[16\left(\frac{\sqrt{3}}{8\pi}m_{3/2}\frac{M_{Pl}}{M_{GUT}}\right) + \mathcal{O}(m_{3/2})\right] \ , \tag{8}$$

For $a_1 \gg m_{3/2}M_{Pl}^2/M_{GUT}^3$ one immediately has the relation $m_3 = 4m_8$ and both suppressed with respect to M_{GUT}.

Allowing as intermediate scales triplet and octet masses m_3 and m_8 one gets from the RGE [13]

$$M_{GUT} = M_{GUT}^0\left(\frac{M_{GUT}^0}{(m_3 m_8)^{1/2}}\right)^{1/2} \ , \tag{9}$$

$$m_T = m_T^0\left(\frac{m_3}{m_8}\right)^{5/2} \ , \tag{10}$$

with $M^0_{GUT} = m^0_T \approx 10^{16}$ GeV, and assuming that M_{GUT} is the largest scale. With not too small a_1 the triplet mass automatically increases and so does the GUT scale. This is welcome in light of the usual difficulties with the dimension 5 proton decay.

It has to be stressed however that the relation between the triplet and octet mass is a consequence of choosing the minimal model with terms only up to $24^4_H/M_{Pl}$: higher corrections would allow more freedom. On the other side the prediction of a higher M_{GUT} is robust in low energy supersymmetry. This imply that generically these minimal models of GUT and SUSY breaking have an automatic suppression of $d = 6$ proton decay rates. This should be stressed: the automatic appearence of intermediate scales increases the GUT scale.

5. Mediation

In the above example essentially the SM hidden sector has been presented, and made responsible for both SUSY and GUT breaking. This information must then be transmitted to the SM multiplets. Who mediates it? The candidates are as usual gravity, the gauge fields, the weak triplet and colour octet chiral multiplets and the Higgs triplet and doublets from $5_H - \overline{5}_H$. As we will see, the main contribution will come from gravity. This is an outcome of the model, and no further assumption is needed to obtain it.

Let's start with the sfermion masses. The typical supergravity mediated contribution to the soft mass terms for the sfermions is by definition

$$m_{\tilde{f}}^{gravity} \approx m_{3/2} \; . \tag{11}$$

This should be compared with the gauge boson X and Y contributions: they come at 2-loops and are (after taking the square root)

$$m_{\tilde{f}}^{X,Y} \approx \frac{\alpha}{4\pi}\frac{M_{Pl}}{M_{GUT}} m_{3/2} \; , \tag{12}$$

thus suppressed by the loop factor and increased by the lower mass of the mediator. The fact that M_{GUT} is not much lower than M_{Pl} makes the gravity contribution dominant. Thus, although the idea of the heavy gauge boson contribution [14,15] works nice in principle, it fails in a well-defined and minimal model as ours.

Next, one can consider the triplet and octet chiral superfields. The sfermion mass is (at 2-loops again)

$$m_{\tilde{f}}^{3,8} \approx \frac{\alpha}{4\pi}\frac{M_{Pl}}{m_{3,8}(\sigma)}\frac{\partial m_{3,8}(\sigma)}{\partial\sigma}\bigg|_{\sigma=M_{GUT}} m_{3/2} \; . \tag{13}$$

In principle there is some freedom to make this contribution as large as possible. In fact $m_{3,8}(\sigma)$ can be a polinomial in σ such that the mass is small, but the first derivative is large close to the minimum. For example, imagine

$$m(\sigma) = a\sigma^2 + b\sigma + c \tag{14}$$

(this is exactly what comes out from (3)). Then one needs at the same time $aM_{GUT}^2 + bM_{GUT} + c \approx 0$ and $2aM_{GUT} + b \neq 0$. In the simplified model presented above, the only nontrivial contribution comes from the quartic terms, proportional to a_1. So there is no room for any further adjustment. Although in models with higher powers of the adjoint this is in principle possible, there is an obvious limitation of the GUT scale being below the Planck one, which gives a lower limit on $m_{3,8}$, see (9). A rough estimate tells us that these terms will be typically subdominant with respect to gravity, although some limiting cases could be borderline.

Finally, the Higgs doublet [7]. One gets a contribution to the Lagrangian of the form

$$\int d^2\theta m_H(\sigma)\overline{H}H + h.c. \tag{15}$$

A fine-tuning is needed to first make the μ term small (this is the usual fine-tuning to account for the doublet-triplet splitting)

$$\mu = m_H(M_{GUT}) \, , \tag{16}$$

while another fine-tuning is needed to make the off-diagonal mass term small, the so-called B-term:

$$B = m_{3/2} M_{Pl} \frac{\partial m_H}{\partial \sigma}(\sigma)\bigg|_{\sigma = M_{GUT}} \, , \tag{17}$$

which must be smaller than μ^2. We have here enough free parameters to fine-tune as we like, since these papameters come from a different sector than (3). What forbids this contribution to dominate is phenomenology. A 1-loop correction to the right stop mass would for example be [8]

$$m_{\tilde{t}^c}^2 \approx -\frac{y_t^2}{(2\pi)^2} B \, , \tag{18}$$

which is negative and can be dangerously large for large B. Thus the sfermion masses at 2-loops are

$$m_{\tilde{f}}^{Higgs} \approx \frac{\alpha}{4\pi}\frac{B}{\mu} \tag{19}$$

and thus for sure cannot dominate over gravity, although, strictly speaking, their contribution can be present.

In short, supergravity dominates, which means, that all the ignorance of the supergravity generated soft terms is still present and cannot be simply ignored or assumed to be smaller than some other contribution. Here, in a minimal and explicit example, there is simply no other contribution.

Another issue are the gaugino masses. In supergravity they can be present adding a term

$$\delta L = \int d^2\theta \frac{c}{M_{Pl}} Tr\left(24_H W^\alpha W_\alpha\right) + h.c. \tag{20}$$

This contributes both to gaugino (λ) masses as

$$m_\lambda \approx c m_{3/2} Tr\left(Y\lambda\lambda\right) \tag{21}$$

(Y is the properly normalized hypercharge operator) and to the correction of the inverse gauge coupling

$$\Delta\left(\frac{1}{g^2}\right) = c\frac{M_{GUT}}{M_{Pl}} Tr\left(Y F^{\mu\nu} F_{\mu\nu}\right) . \tag{22}$$

If the parameter c is small, then the RGE are still valid as above, but the supergravity contribution to the gaugino masses is much smaller than the sfermion masse (suppressed by c). If in the opposite case $c = \mathcal{O}(1)$, then the RGE relations (9) and (10) should be generalized to

$$M_{GUT} = M_{GUT}^0 \left(\frac{M_{GUT}^0}{(m_3 m_8)^{1/2}}\right)^{1/2} \exp\left(-\frac{5}{24}\frac{(4\pi)^2}{\sqrt{30}}\frac{M_{GUT}}{M_{Pl}}c\right) , \tag{23}$$

$$m_T = m_T^0 \left(\frac{m_3}{m_8}\right)^{5/2} \exp\left(\frac{65}{12}\frac{(4\pi)^2}{\sqrt{30}}\frac{M_{GUT}}{M_{Pl}}c\right) . \tag{24}$$

The increase in M_{GUT} is still robust, while the triplet mass becomes very much dependent on the choice of this dimensionless parameter c. It is interesting that the long standing problem of a too low colour triplet mass is here extremely easy to solve!

The other contributions to the gaugino mass can be easily important due to this c dependence of the supergravity contribution. The model is clearly not predictive in this sector.

6. Conclusions

We have seen that a specified minimal SU(5) supersymmetric model can be fully realistic. It can lead to a spontaneous breaking of supersymmetry on top of the usual GUT breaking. A generic feature of such models is the automatic appearence of intermediate states: they cannot be avoided, since the renormalizable terms only are not reach enough to get a supersymmetry breaking minimum.

The price one has to pay is some fine-tuning. There are the usual μ and B terms to be of the order of the electroweak scale, but this is similar in most susy GUT models (as is the zero cosmological constant, of course). What is new is the fine-tuning needed to get a very small singlet scalar component of 24_H, i.e. the one that gets a vev and breaks susy (and it is the sueprpartner of the goldstino). In fact this mass is of order $m_{3/2}$, while the masses of the colour octet and weak triplt that lived in the same 24_H have much larger masses (below M_{GUT}, but still much larger than TeV).

Is this SUSY and GUT breaking minimum stable? The answer is no, at least usually not. This can be seen for example from the constant term W_0 in (3). At the origin the potential becomes negative, so one needs to check how stable the previous local minimum is. As usual the lifetime is proportional to

$$\exp\left(-const\frac{(\Delta\phi)^4}{\Delta V}\right), \tag{25}$$

where $\Delta\phi$ is the distance between the two minima (the local and the global ones), while ΔV is the energy barrier between them. One can easily check that typically the minima are quite far away from each other and thus suppress the decay.

We have assumed so far that supersymmetry is broken close to M_W. One can easily generalize the RGE's to a different Λ_{MSSM}

$$M_{GUT} = M_{GUT}^0\left(\frac{m_3^0 m_8^0}{m_3 m_8}\right)^{1/4}\left(\frac{\Lambda_{MSSM}^0}{\Lambda_{MSSM}}\right)^{1/3}, \tag{26}$$

$$m_T = m_T^0\left(\frac{m_3}{m_8}\right)^{5/2}\left(\frac{\Lambda_{MSSM}}{\Lambda_{MSSM}^0}\right)^{5/6}, \tag{27}$$

Interestingly enough, there are solutions also for increased SUSY breaking scales. For example, even in the limiting (nonsupersymmetric) case of $\Lambda_{MSSM} = M_{GUT} = m_T$ one gets a solution for m_3 and m_8. It corresponds to the solution found recently in [16].

The above exercise is unfortunately not very illuminating as the sfermion masses are concerned. It does not shed any light on the flavour problem of the soft terms, since supergravity terms dominate. The hope for the future is to be able to construct a model with a gauge intermediate scale, where supersymmetry breaking takes place and dominates over gravity.

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