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Dark Matter in Brane Cosmology

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The impact of the brane cosmology on the relic density of non-relativistic stable particles in high and low reheating temperature scenarios is analyzed. We show that, in non-conventional brane cosmology, the dark matter relic density can be enhanced by many order of magnitudes.

1. Introduction

The existence of non-baryonic dark matter (DM) is one of firmed observational evidences of new physics beyond the Standard Model (SM). The most interesting candidate for this dark matter is a long lived or stable weakly interacting massive particle (WIMP) which can remain from the earliest moments of the universe in sufficient number to account for the dark matter relic density.

The standard computation of the WIMP relic density, based on the usual early universe assumptions, leads to the following fact: The WIMP relic density is inversely proportional with its annihilation cross section. However, the detection rate of this particle, which is given in terms of its elastic cross section with the nuclei in the detector, is proportional to its annihilation cross section. Therefore, in order to detect the WIMP experimentally, its cross section should be large, of $\mathcal{O}(10^{-6} - 10^{-8})\mathrm{GeV}^{-2}$. Nevertheless, in this case the WIMP relic density is quite small: $\Omega_\chi h^2 \leq 0.01$ which contradicts the recent observational bounds: $0.094 \leq \Omega_\chi h^2 \leq 0.128$ [1].

In the last few years there has been a growing interest in studying the impact of non-conventional brane cosmology on the relic abundance of dark matter [2,3,4]. Warped extra dimensions have been proposed [5] to explain the large hierarchy between the electroweak scale ($M_W \sim 10^2$ GeV) and the Planck scale ($M_{pl} \sim 10^{19}$ GeV). In this class of models, the ordinary matter is assumed to be localized on a three-dimensional subspace, called brane which is embedded in a larger space, called bulk. It has been emphasized that the brane cosmology in these models can be quite different from the standard cosmology of four dimensional universe. In particular, the derived Friedman equation of a brane embedded in five dimensional

(5D) warped geometry is given by [6]

$$H^2 = \frac{8\pi G_{(4)}}{3}\rho\left(1+\frac{\rho}{2\sigma}\right) - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}, \tag{1}$$

where $H = \dot{a}/a$ is the Hubble parameter and $a(t)$ is the scale factor, ρ is the energy density of ordinary matter on the brane while σ is the brane tension. $G_{(4)}$ refers to the 4D Newton coupling constant. Finally k stands for the curvature of our three spatial dimensional and $\mathcal{C}$ is a constant of integration known as dark-radiation. As can be seen from the above equation, $H \propto \rho$ rather than $\sqrt{\rho}$ as in the conventional cosmology. Thus, the evolution of the scale factor will be different from the standard one. This modification would be very relevant for the cosmological events, as dark matter relic abundance, that may occur during the radiation dominated phase of the early universe.

The aim of this article is to provide a detail analysis for the relic density in the context of non-conventional brane cosmology.

2. Warped geometry and brane cosmology

The warped extra dimensions scenario has been proposed by Randall and Sundrum(RS)[5]. In this model, an extra dimension compactified on a S_1/Z_2 orbifold, with two branes sitting on each orbifold fixed point, is assumed. The brane at $y = 0$ is called the Planck brane, while the other one at $y = \pi r_c$ is called the TeV or SM brane. With an appropriate tuning for cosmological constants in the bulk and on the branes, we obtain the warped metric

$$ds^2 = e^{-2\kappa|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \tag{2}$$

where $\eta_{\mu\nu} = diag(1,-1,-1,-1)$ is the usual Minkowski metric. This type of geometry is called 'non-factorizable' because the metric of the 4D subspace is y-dependent. In the simplest version of the RS model it is assumed that the SM fields live on the so-called TeV brane while gravity lives everywhere. A basic assumption of this model is that there are no large mass hierarchies present so that very roughly we expect that $k{\sim}M*$ the 5D fundamental or Planck scale. In fact, once we solve Einstein's equations and plug the solutions back into the original action and integrate over y we find that

$$M_{pl}^2 = \frac{M_*^3}{k}(1 - e^{-2\pi k r_c}) \tag{3}$$

The warp factor $e^{-2\pi k r_c}$ is a very small quantity which implies that M_{pl}, M_* and k have essentially comparable magnitudes following from the assumption that no hierarchies exist. If we calculate the Ricci curvature invariant for this 5D space we find it is a constant, i.e.,$R_5 = -20k^2$ and thus k is a measure of the constant curvature of this space. A space with constant negative curvature is called an Anti-DeSitter space Ads_5.

We consider the most general metric that preserves three dimensional rotational and translation invariance:

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{4}$$

where γ_{ij} is a maximally symmetric 3-dimensional metric with spatial curvature $k = \pm 1, 0$. In our analysis, we identify the hypersurface defined by $y = 0$ with the brane that forms our universe. The induced metric in this brane is the usual $4D$ RW metric.

The five-dimensional Einstein equations (with bulk cosmological constant Λ) are given by

$$\tilde{R}_{AB} - \frac{1}{2}\left(\tilde{R} - 2\Lambda\right)\tilde{g}_{AB} = \kappa^2_{(5)}\tilde{T}_{AB}, \tag{5}$$

where $\tilde{R}_{AB}$ is the $5D$ Ricci tensor, $\tilde{R}$ is the $5D$ scalar curvature and the constant $\kappa_{(5)}$ is related to the $5D$ Newton's constant $G_{(5)}$ and the $5D$ Plank mass $M_{(5)}$, by the relations

$$\kappa^2_{(5)} = 8\pi G_{(5)} = M^{-3}_{(5)}. \tag{6}$$

Assuming an empty bulk, the energy momentum tensor is due to the matter on the brane, which is considered to be an infinitely thin. Thus $T_A{}^B$ is given by

$$T_A{}^B = \frac{\delta(y)}{b}\mathrm{diag}\left(-\rho_\mathrm{b}, \mathrm{p_b}, \mathrm{p_b}, \mathrm{p_b}, 0\right), \tag{7}$$

where ρ_b and p_b are the total energy density and pressure on the brane, respectively. In order to have a well defined geometry, the metric ($i.e.$, the function $f \equiv n, a, b$ in Eq.(4) must be continuous across the brane localized at $y = 0$ but its derivative with respect to y can be discontinuous. Therefore, its second derivative with respect to y will contain Dirac delta function, *i.e.*,

$$f^{''} = \widehat{f''} + [f^{'}]\delta(y), \tag{8}$$

where $\widehat{f''}$ is the non-distributional part of the second derivative of f respect to y and $[f^{'}]$ is the jump in the first derivative of f across $y = 0$ which is defined as

$$[f^{'}] = f^{'}(0^+) - f^{'}(0^-). \tag{9}$$

By matching the Dirac $\delta(y)$ function in Eq.(5), one obtains the following relations, which are known as junction conditions [6]:

$$\frac{[a^{'}]}{a_0 b_0} = -\frac{\kappa^2_{(5)}\rho_b}{3}, \tag{10}$$

$$\frac{[n^{'}]}{n_0 b_0} = \frac{\kappa^2_{(5)}(2\rho_b + 3p_b)}{3}, \tag{11}$$

where the subscript 0 stands for the evaluation at $y = 0$. Using the condition Eq.(**??**) with the components $(0,0)$ and $(4,4)$ of Einstein's equations in the bulk, one finds the following equation

$$H^2 \equiv \frac{\dot{a}_0^2}{a_0^2} = \frac{\Lambda}{6} + \frac{\kappa_{(5)}^4}{36}\rho_b^2 - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}, \tag{12}$$

where $\mathcal{C}$ is a constant of integration. As can be seen from this equation, the Hubble parameter is proportional to the energy density of the brane, in contrast with the standard four-dimensional Friedmann equation where it depends on the square root of the energy density. Let us consider a brane with total energy density

$$\rho_b = \sigma + \rho, \tag{13}$$

where σ is a brane tension, constant in time, and ρ is the energy density of ordinary cosmological matter. This implies

$$H^2 = \left(\frac{\kappa_{(5)}^4\sigma^2}{36} + \frac{\Lambda}{6}\right) + \frac{\kappa_{(5)}^4}{18}\sigma\rho + \frac{\kappa_{(5)}^4}{36}\rho^2 - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}. \tag{14}$$

Then with fine tuning of brane tension, the first term in Eq.(14) vanishes if we have

$$\frac{\kappa_{(5)}^4}{36}\sigma^2 = -\frac{\Lambda}{6}. \tag{15}$$

Furthermore, fixing the value of Λ in terms of $M_{(5)}$ and M_{pl} as $\Lambda = -6M_{(5)}^6/M_{pl}^4$, one finds

$$8\pi G_{(4)} = \frac{\kappa_{(5)}^4}{6}\sigma, \tag{16}$$

which leads to the new Friedmann equation:

$$H^2 = \frac{8\pi G_{(4)}}{3}\rho\left(1 + \frac{\rho}{2\sigma}\right) - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}. \tag{17}$$

As can be seen from Eq.(17), at low energies *i.e.*, at late time, the cosmology can be reduced to the standard one, but in the early time the ρ^2 term becomes dominant, so the universe undergoes nonconventional cosmology.

3. Relic density of dark matter in brane world cosmology

In this section we compute the relic density of the WIMP (χ) within the non-conventional brane cosmology. In the standard computation for the WIMP relic density, one assumes that χ was in thermal equilibrium with the standard model particles in the early universe and decoupled when it was non-relativistic. Once the χ annihilation rate $\Gamma_\chi = \langle\sigma_\chi^{ann}\, v\rangle n_\chi$ dropped below the expansion rate of the universe, $\Gamma_\chi \leq H$, the WIMPs stop to annihilate, fall out of equilibrium and their relic density remains intact till now . The above $\langle\sigma_\chi^{ann}\, v\rangle$ refers to thermally averaged total cross section for annihilation of $\chi\chi$ into lighter particles times the relative velocity, v.

The relic density is then determined by the Boltzmann equation for the WIMP number density (n_χ) and the law of entropy conservation:

$$\frac{dn_\chi}{dt} = -3Hn_\chi - \langle\sigma_\chi^{ann}\, v\rangle \left[(n_\chi)^2 - (n_\chi^{eq})^2\right], \tag{18}$$

$$\frac{ds}{dt} = -3Hs, \tag{19}$$

where n_χ^{eq} is the WIMP equilibrium number density which, as function of temperature T, is given by $n_\chi^{eq} = g_\chi(m_\chi T/2\pi)^{3/2} e^{-m_\chi/T}$. Here m_χ and g_χ are the mass and the number of degrees of freedom of the WIMP respectively. Finally, s is the entropy density. In the standard cosmology, the Hubble parameter H is given by $H^2 = \left(8\pi/3M_{pl}^2\right)\rho$ to be compared with the expression in Eq.(17) for brane cosmology. In our analysis we will set $k = \mathcal{C} = 0$ in order to focus on the impact of the modification of the ρ dependence in H.

Let us introduce the variable $x = m_\chi/T$ and define $Y = n_\chi/s$ with $Y_{eq} = n_\chi^{eq}/s$. In this case, the Boltzmann equation is given by

$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma_\chi^{ann}\, v\rangle \left(Y^2 - Y_{eq}^2\right). \tag{20}$$

In radiation domination era, the entropy, as function of the temperature, is given by $s = \frac{2\pi^2}{45} g_{*_s}(x)\, m_\chi^3\, x^{-3} \equiv k_1 x^{-3}$, which is deduced from the fact that $s = (\rho + p)/T$ and g_{*_s} is the effective degrees of freedom for the entropy density. Therefore one finds

$$\frac{ds}{dx} = -\frac{3s}{x}, \tag{21}$$

which is the same in both cases of standard and brane cosmology. In the standard case, the following expression for the Boltzmann equation for the WIMP number density is obtained

$$\frac{dY}{dx} = -\frac{s}{Hx}\langle\sigma_\chi^{ann}\, v\rangle \left(Y^2 - Y_{eq}^2\right), \tag{22}$$

where $H(x)$ is given by $H = \sqrt{\frac{4\pi^3 g_* m_\chi^4}{45 M_{pl}^2}}\, x^{-2} = \sqrt{k_2}\, x^{-2}$ and g_* is the effective degrees of freedom for the energy density. Therefore, one obtains the following expression for the Boltzmann equation for the WIMP number density ($g_{*s} \simeq g_*$ is assumed):

$$\left(\frac{dY}{dx}\right)_s = -\sqrt{\frac{\pi g_*}{45}} M_{pl}\, m_\chi \frac{\langle\sigma_\chi^{ann}\, v\rangle}{x^2} \left(Y^2 - Y_{eq}^2\right). \tag{23}$$

In brane cosmology the Hubble parameter is given by $H = (k_2 x^{-4} + k_3 x^{-8})^{1/2}$ where $k_3 = \pi^4 g_*^2 m_\chi^8/(32400 M_5^6)$. Thus, the Boltzmann equation in brane cosmology takes the form

$$\left(\frac{dY}{dx}\right)_b = -\sqrt{\frac{\pi g_*}{45}} M_{pl}\, m_\chi \left(x^4 + \frac{k_3}{k_2}\right)^{-1/2} \langle\sigma_\chi^{ann}\, v\rangle \left(Y^2 - Y_{eq}^2\right). \tag{24}$$

It is worth noticing that in the limit of $k_3 \to 0$ (*i.e.*, $\sigma \to \infty$), the above equation tends to the standard Boltzmann equation in Eq.(23). Therefore, at early times, the universe undergoes a nonstandard brane cosmology till it reaches a temperature, known as transition temperature T_t where the universe sustains the standard cosmology. This transition temperature is defined as [4]

$$\rho(T_t) = 2\sigma \quad \Rightarrow \quad T_t = 0.51 \times 10^{-9} M_5^{\frac{3}{2}} \text{ GeV}. \tag{25}$$

In order to analyze the brane cosmology effect on the WIMP relic density, one should assume that the freeze out temperature of the WIMP (T_F) is higher than the transition temperature, *i.e.*, $T_F \geq T_t$. Therefore, one finds

$$M_5 \leq 1.57 \times 10^6 \left(\frac{m_\chi}{x_F}\right)^{2/3}. \tag{26}$$

Since the WIMPs freeze out at temperature $T_F \ll m_\chi$, they are non-relativistic and therefore the averaged annihilation cross section can be expanded as follows:

$$\langle \sigma_\chi^{ann} v \rangle = a + \frac{6b}{x}, \tag{27}$$

where a describes the s-wave annihilation and b comes from both s- and p- wave annihilation. To obtain the present WIMP abundance Y_∞, we should integrate the Boltzmann equation for the WIMP number density from x_F (the decoupling temperature) to $x_\infty \simeq \infty$ (present temperature). It is important to notice that this integral must be divided to two parts from x_F to x_t where the non-conventional brane cosmology is applied and from x_t to ∞ where the universe undergoes the standard cosmology. Thus, one obtains

$$Y_{\infty b}^{-1} = \sqrt{\frac{\pi g_*}{45}} M_{pl} m_\chi \left[\int_{x_F}^{x_t} (a + \frac{6b}{x}) \left(\frac{k_3}{k_2} + x^4\right)^{-1/2} dx + \int_{x_t}^{\infty} \left(\frac{a}{x^2} + \frac{6b}{x^3}\right) dx \right]. \tag{28}$$

Here we have used the usual assumption that $Y_{eq} \ll Y$ and $Y_{x_F} \gg Y_\infty$. Evaluating the above integrals, one finds

$$\begin{aligned} Y_{\infty b}^{-1} = \sqrt{\frac{\pi g_*}{45}} M_{pl}\ m_\chi \Bigg[3\sqrt{\frac{k_2}{k_3}} b \left(\sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_F^{-2}\right) - \sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_t^{-2}\right)\right) \\ + \ a \left(\frac{1}{x}\ {}_2F_1\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{-k_3}{k_2 x^4}\right]\right)_{x_F}^{x_t} + \left(\frac{a}{x_t} + \frac{3b}{x_t^2}\right)\Bigg], \end{aligned} \tag{29}$$

where ${}_2F_1[a, b, c, z]$ is the Hypergeometric function, which is a solution of the hypergeometric differential equation: $z(1-z)y'' + [c - (a+b+1)z]y' - aby = 0$. As can be seen from Eq.(29) that for $x_t = x_F$ the expression of $Y_{\infty b}^{-1}$ coincides with the standard known result for $Y_{\infty\ s}^{-1}$, namely $Y_{\infty s}^{-1} = \sqrt{\frac{\pi g_*}{45}}\ M_{pl}\ m_\chi \left(\frac{a}{x_F} + \frac{3b}{x_F^2}\right)$. The relic abundance of the WIMP is given by

$$\Omega_\chi h^2 = \frac{\rho_\chi}{\rho_c/h^2} = 2.9 \times 10^8\ Y_\infty \left(\frac{m_\chi}{\text{GeV}}\right), \tag{30}$$

where the critical density ρ_c is given by $\rho_c \simeq 10^{-5}h^2$ GeVcm^{-3} and h is the Hubble constant, $h \simeq 0.7$. Furthermore, the ρ_χ is defined as $\rho_\chi = m_\chi s_0 Y_\infty$ where $s_0 \simeq 2900$ cm^{-3} is the present entropy density. As in the standard scenario, the relic density of the WIMP is inversely proportional to its annihilation cross section. However, unlike the standard case, it depends explicitly on WIMP mass since $k_3/k_2 \propto m_\chi^4$. In standard cosmology x_F is given by

$$x_F = \ln \frac{0.0765\ c\ m_\chi M_{pl} g_\chi (a + 6b/x_F)}{\sqrt{x_F g_*(x_F)}}, \tag{31}$$

which can be solved iteratively to determine the value of x_F. It turns out that for $m_\chi \sim \mathcal{O}(100)$ GeV, $x_F \sim \mathcal{O}(25)$. In brane cosmology, one can easily show that x_F is obtained by iterative solution of

$$x_F = \ln \frac{0.0765\ c\ m_\chi M_{pl} g_\chi x_F^{3/2}(a + 6b/x_F)}{\sqrt{g_*(x_F)(\frac{k_3}{k_2} + x_F^4)}}. \tag{32}$$

In this case, one finds that x_F is smaller than the above value obtained within the standard cosmology. Also, it turns out that x_F is sensitive to the scale M_5. For example with $M_5 \sim 10^6$ one gets $x_F \sim \mathcal{O}(7)$. Let us introduce the factor $R = (\Omega_\chi h^2)_b/(\Omega_\chi h^2)_s$ that measures the enhancement/suppression in the relic abundance due to the brane cosmology. From Eq.(30), one finds

$$R = \frac{\frac{a}{x_F} + \frac{3b}{x_F^2}}{3\sqrt{\frac{k_2}{k_3}}b\left(\sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}}x_F^{-2}\right) - \sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}}x_t^{-2}\right)\right) + a\left(\frac{1}{x}\ {}_2F_1\left[\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{-k_3}{k_2 x^4}\right]\right)_{x_F}^{x_t} + \frac{a}{x_t} + \frac{3b}{x_t^2}}. \tag{33}$$

This expression is different from that obtained by Okada and Seto [4], which is

$$R = (\Omega_\chi h^2)_b/(\Omega_\chi h^2)_s \simeq 0.54(x_t/x_d(s)) \tag{34}$$

The reason is that in doing the integration to get $Y_{\infty b}$, we divide the integration into two parts, from x_F to x_t where the non-conventional brane cosmology is applied and from x_t to ∞ where the universe undergoes the standard cosmology, where as for Okada and Seto, they have done the integration in one step. This factor could be larger or smaller than one depending on the values of the annihilation cross section parameters a and b and also on the masses m_χ and M_5. In order to analyze the impact of the non-conventional brane cosmology on the relic density result, we consider, as an example, the lightest supersymmetric particle (LSP), which is one of the most interesting candidates for the WIMP. As is well known, in most of the parameter space of the supersymmetric models the LSP is mainly pure Bino. Therefore, it is mainly annihilated into lepton pairs through $t-$channel exchange of right-handed sleptons. The p-wave dominant cross section is given by [7]

$$b \simeq 8\pi\alpha_1^2 \frac{1}{m_\chi^2}\frac{1}{(1 + x_{\tilde{l}_R})^2}, \tag{35}$$

where $x_{\tilde{l}_R} = m^2_{\tilde{l}_R}/m^2_\chi$ and α_1 is the coupling constant for the $U(1)_Y$ interaction. Thus, for $m_\chi \sim m_{\tilde{l}_R} \sim 100$ GeV, one finds $b \simeq \mathcal{O}(10^{-8})$ GeV^{-2}, which in the standard cosmology scenario leads to $\Omega_\chi h^2 \geq 0.1$.

As advocated above, in brane cosmology the relic density $(\Omega_\chi h^2)_b$ is quite sensitive to the value of the fundamental scale M_5 which should satisfy the upper bound given in Eq.(26). Therefore, with $m_\chi \simeq \mathcal{O}(100)$ GeV and $x_F \simeq \mathcal{O}(10)$, one finds

$$M_5 < 10^7. \tag{36}$$

Furthermore, the fact that the transition process from non-conventional cosmology to convention cosmology should take place above the nucleosynthesis era (*i.e.*, $T_t > 1$ MeV) impose the following lower bound on M_5:

$$M_5 \geq 1.2 \times 10^4. \tag{37}$$

In this case the resulting relic density $(\Omega_\chi h^2)_b$ may exceed the WMAP results (at 95% confidence level) [1]

$$\Omega_\chi h^2 = 0.1126^{+0.0161}_{-0.0181}. \tag{38}$$

Moreover for $M_5 \geq 5 \times 10^6$, the ratio R becomes less than one and a small suppression for $(\Omega_\chi h^2)_s$ can be obtained. This brane enhancement or suppression for the dark matter relic density could be favored or disfavored based on the value of the relic abundance in the standard scenario. If $(\Omega_\chi h^2)_s$ is already larger than the observational limit, as in the case of bino-like particle, then a suppression effect would be favored and hence M_5 is constrained to be larger than 5×10^6 GeV. However, for wino- or Higgsino-like particle where the standard computation usually leads to very small relic density, the enhancement effect will be favored and the constraint on M_5 can be relaxed a bit [4]. In general, it is remarkable that in this scenario the dark matter relic density imposes a stringent constraint on the fundamental scale M_5.

4. Low reheating and DM relic abundance in brane cosmology

In the standard computation for the DM relic density that we have adopted in the previous section, it was assumed that the reheating temperature T_{RH} is much higher than the WIMP freeze-out temperature *i.e.*, $T_{RH} \gg T_F$. In this case, the reheating epoch has no impact on the final result of the relic density. However, it is well known that the only constraint on T_{RH} is $T_{RH} \gtrsim 1$ MeV in order not to spoil the successful predictions of the big bang neucleosynthesis. Therefore, in principle it is possible to have a cosmological scenario with low reheating such that $T_{RH} < T_F$. In this case the predictions of the relic abundance of the WIMP are modified as emphasized in [7] and recently in [8]. As in the previous section and to emphasize the effect of the brane cosmology, we assume that the WIMP freeze-out temperature is larger than the transition temperature which is also larger than the reheating temperature, *i.e.*, $T_F > T_t > T_{RH}$. Within a low reheating temperature scenario, the relic density depends on whether WIMPs are never in chemical equilibrium either before or after reheating or they reach chemical equilibrium but they freeze-out before the completion of the reheat process. The resultant relic density in these two scenarios are quite different and are also different from the one derived by the standard computation with a large reheating temperature.

Let us start our analysis with the case of non-equilibrium production and freeze-out. In this case, at early times the number density n_χ is much smaller than n_χ^{eq} and the Boltzmann equation (22) can be written as

$$\frac{dY}{dx} = 0.02095 \left(\frac{g_\chi}{g_{*s}}\right)^2 \frac{s}{Hx} \langle \sigma_\chi^{ann}\ v \rangle\ x^3\ e^{-2x}. \tag{39}$$

Although this equation is valid only at early times, it can be approximately integrated in the full range of x, namely from $x = 0$ to $x = \infty$ due to the exponential suppression in the right hand side. Thus for the standard cosmology (where $H \propto x^{-2}$), one can easily integrate this equation and obtains

$$Y_{\infty s} = 0.02095 \sqrt{\frac{\pi}{45}} g_\chi^2 g_*^{-3/2} M_{pl}\ m_\chi \left(\frac{a}{4} + 3b\right). \tag{40}$$

The Y_∞ is related to the mass density of χ particle today as follows, at reheating we have $\rho_\chi(T_{RH}) = m_\chi n_\chi(T_{RH}) = \frac{2\pi^2}{45} g_{*s}(T_{RH}) m_\chi Y_\infty T_{RH}^3$. After the reheating the universe is radiation dominated and the following relation is satisfied [7]:

$$\frac{\rho_\chi(T_{now})}{\rho_R(T_{now})} = \frac{T_{RH}}{T_{now}} \frac{\rho_\chi(T_{RH})}{\rho_R(T_{RH})}. \tag{41}$$

Therefore, in this case $\Omega_\chi h^2$ is proportional to the annihilation cross section instead of being inversely proportional as in case of high reheating temperature.

Now we consider this scenario of low reheating with non-equilibrium production and freeze-out in brane cosmology. The Boltzmann equation is still given by Eq.(39), but with $H = (k_2 x^{-4} + k_3 x^{-8})^{1/2}$ in the range of $x \in [0, x_t]$ and with the usual Hubble parameter $H = \sqrt{k_2} x^{-2}$ between x_t and $x = \infty$. Integrating this equation one finds

$$Y_{\infty b} \simeq 0.02095 \times 10^{-6} \sqrt{\frac{\pi}{45}} g_\chi^2 g_*^{-3/2} M_{pl}\ m_\chi \left(9.46\ a + 37.8\ b\right). \tag{42}$$

Here we have used $m_\chi \sim 100$ GeV and $M_5 \sim 10^6$ GeV, as an example, to do the integration numerically. However, we have checked the result of $Y_{\infty b}$ for different values of m_χ and M_5. It turns out that $Y_{\infty b}$ is diminished significantly for $M_5 \lesssim 10^6$. As can be observed from equations (40) and (42) that this non-equilibrium scenario produces a very suppressed relic density, particularly in brane cosmology. Furthermore, the assumption that $n_\chi \ll n_\chi^{eq}$ impose a sever constraint on the annihilation cross section. Thus, one can conclude that within this scenario, it is not possible to account for the dark matter experimental results.

Now we turn to the second scenario in which the annihilation cross section of the WIMP is large and hence it reaches the chemical equilibrium before reheating.

In this case, the computation of the relic density $\Omega_\chi h^2$ is very close to the standard case with high reheating temperature. At the early times *i.e.*, when $T > T_F$, the WIMP's are very close to equilibrium. So, as usual, one can use the variable $\Delta(x) = Y(x) - Y^{eq}(x)$ to write the Boltzmann equation as

$$\Delta' = -(Y^{eq})' - f(x)\Delta\left(2Y^{eq} + \Delta\right), \tag{43}$$

where $f(x)$ is given by $f(x) = \sqrt{\frac{\pi g_*}{45}} M_{pl} m_\chi \left[\frac{a}{x^2} + \frac{6b}{x^3}\right]$. Then by neglecting Δ' and Δ^2 , one obtains

$$\Delta \simeq -\frac{(Y^{eq})'}{2f(x)Y^{eq}}. \tag{44}$$

At late time, when $T < T_F$ one gets $Y \gg Y^{eq}$ and hence we can use the approximation $Y^2 - (Y^{eq})^2 \simeq Y^2$ in Eq.(23) and integrate from T_F down to T_{RH} to determine $Y(T_{RH})$. In the standard cosmology, one finds

$$\frac{1}{Y(x_{RH})}\Big|_s = \frac{1}{Y(x_F)} - \sqrt{\frac{\pi g_*}{45}} M_{pl} m_\chi \left[\frac{a}{x} + \frac{3b}{x^2}\right]_{x_F}^{x_{RH}}, \tag{45}$$

If it is assumed that there is no entropy production for $T < T_{RH}$, then there is no WIMP production for temperature below the reheating temperature. Thus, the present value of Y is given by $Y(x_{RH})$ up to an overall correction due to the fact that the reheating process is not complete at T_{RH} [7]. Here, two comments are in order: *i*) As mentioned above, $Y(x_F)^{-1} \simeq Y_{eq}(x_F)^{-1}$ which is of order $\mathcal{O}(10^9)$, so its contribution to $Y(x_{RH})^{-1}$ in Eq.(45) can be neglected respected to the second term. *ii*) Since $T_{RH} < T_F$ (*i.e.*, $x_{RH} > x_F$), one can approximate Eq.(45) and finds that the relic abundance $\Omega_\chi h^2$ is very close to the one obtained by using the standard calculation with high reheating temperature, namely

$$\Omega_\chi h^2 \sim 1.1 \times 10^{-11} \left(\frac{a}{x_F} + \frac{3b}{x_F^2}\right)^{-1}, \tag{46}$$

which implies that unless annihilation cross sections are quite small ($\lesssim 10^{-8}$), one gets, as usual, very small relic density.

In brane cosmology, Eq.(24) describes the correct Boltzmann equation that should be used. Also, as in the previous scenario, one has to integrate this equation from x_F to x_t using brane cosmology feature, and from x_t to x_{RH} using the standard cosmology feature. In this respect, one finds

$$\begin{aligned}\frac{1}{Y(x_{RH})}\Big|_b = \frac{1}{Y(x_F)} - \sqrt{\frac{\pi g_*}{45}} M_{pl} m_\chi \Bigg[3\sqrt{\frac{k_2}{k_3}} b \left(\sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_F^{-2}\right) - \sinh^{-1}\left(\sqrt{\frac{k_3}{k_2}} x_t^{-2}\right)\right) \\ + a\left(\frac{1}{x}\,{}_2F_1\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{-k_3}{k_2 x^4}\right]\right)_{x_F}^{x_t} + \left(\frac{a}{x} + \frac{3b}{x^2}\right)_{x_t}^{x_{RH}} \Bigg].\end{aligned} \tag{47}$$

One can easily check that the three parts of the second term in the above equation are of the same order and give the dominant contribution to $Y(x_{RH})^{-1}$. In this case, one can show that the relic density (for $m_\chi = 100$ GeV and $M_5 = 10^6$ GeV) is given by

$$\Omega_\chi h^2 \sim 1.1 \times 10^{-7} \left(95.2\ a - 4.12\ b\right)^{-1}. \tag{48}$$

From this equation it can be easily seen that even with large annihilation cross section $\mathcal{O}(10^{-6} - 10^{-8})$, we are able to obtain the cosmologically interesting value $\Omega_\chi h^2 \sim 0.1$. This implies that the scenario of brane cosmology with low reheating termperature and chemical equilibrium WIMPs is the most interesting model for dark matter. It provide an interesting possibility for having dark matter with large cross section (hence their detection would be possible in future DM experiments) with suitable relic aboundanc.

5. Conclusions

In this article we review the analysis of the relic abundance of cold dark matter in brane cosmology. We have studied the impact of brane cosmology on the cold dark matter relic density. We investigated this effect in two different scenarios, namely when the reheating temperature is higher or lower than the freeze-out temperature. We showed that with high reheating temperature, the relic density is enhanced with many order of magnitude for$M_5 \leq 10^6$. This imposes one of the strongest constraints on the scale of large extra dimensions. In case of low reheating temperature, we have considered the possibility that WIMPs are in chemical equilibrium or non-equilibrium, which depends on the value of their annihilation cross section. We emphasized that if WIMPs are in chemical non-equilibrium, then their relic density is very small and they can not account for the observational limits. While in case WIMPs reach chemical equilibrium before reheating, we showed that the relic density is enhanced by two order of magnitudes than the standard thermal scenario result. This enhancement can be considered as an interesting possibility for accommodating dark matter with large cross section, which is favored by the detection rate experiments.

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