The Standard Model of Particles and Interactions III- Towards The Standard Model

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The gauge symmetries of the Standard Model

The (Yang-Mills) action $\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is invariant under

$$\Psi(x) \rightarrow U(x)\Psi(x)$$

Abelian U(1) symmetry

$$U(x) = e^{-iq\theta(x)}$$

$$A_\mu(x) \rightarrow A_\mu + \frac{i}{q}(\partial_\mu U)U^\dagger$$

$$D_\mu\Psi = (\partial_\mu + iqA_\mu)$$

Non-abelian SU(N)

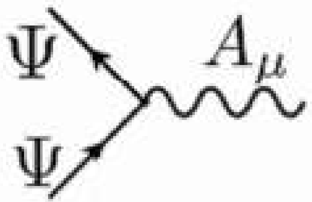
$$U(x) = e^{-ig\theta^a(x)T^a}$$

T^a: N^2-1 generators (N×N matrices) acting on

$$A_\mu(x) = A_\mu^a T^a$$

$$A_\mu(x) \rightarrow UA_\mu U^\dagger + \frac{i}{g}(\partial_\mu U)U^\dagger$$

coupling constants



infinitesimal transformation $U(x) = 1 - ig\theta^a(x)T^a + \mathcal{O}(\theta^2)$

$$A_\mu^a(x) \longrightarrow A_\mu^a + \partial_\mu\theta^a - gf^{abc}\theta^b A_\mu^c$$

$$D_\mu\Psi = (\partial_\mu + igA_\mu^a T^a)$$

More about Matter and Higgs fields

Nature is symmetric under the group of Lorentz transformations, rotations, and translations which all together form the Poincaré group.

Particles are classified by spin: scalars, fermionic spinors, vector bosons. They correspond to irreducible representations of the Poincaré group

Spinors are of two types: the fundamental (left-handed) and the anti-fundamental (right-handed). **The chirality of a spin 1/2 field refers to whether it is in the fundamental or the anti-fundamental and is therefore a label associated with a representation of the Lorentz group**

Weyl spinors

$$\Psi_L : (\frac{1}{2}, 0)$$

$$\Psi_R : (0, \frac{1}{2})$$

Dirac spinor

$$\Psi = \begin{bmatrix} \Psi_L \\ \Psi_R \end{bmatrix}$$

helicity is a physical quantity: it is the projection of the spin onto the direction of motion

for a massless particle: chirality= helicity

$SU(2)_L$

(thus we call the fundamental spinors the left-handed spinors and the anti-fundamental spinors the right-handed spinors)

The Standard model is a chiral theory: the left-handed and right-handed spinors not only transform differently under the Lorentz group but also under the EW gauge group $SU(2)_L$*$U(1)$

The left-handed fields are denoted $Q=(u_L, d_L)$ and $L=(\nu_L, e_L)$ while the right-handed fields are denoted u_R, d_R and e_R

Fermi Model

- Current-current interaction of 4 fermions

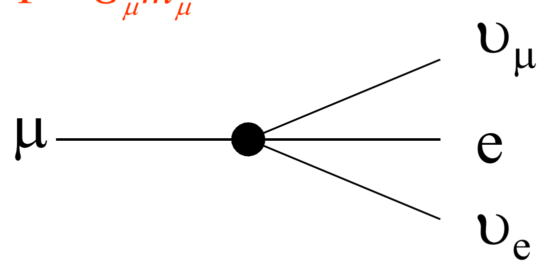
$$L_{FERMI} = -2\sqrt{2}G_F J_\rho^+ J^\rho$$

- Consider just leptonic current

$$J_\rho^{lept} = \overline{v}_e \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) e + \overline{v}_\mu \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) \mu + hc$$

- Only left-handed fermions feel charged current weak interactions (maximal P violation)
- This induces muon decay

$$\Gamma \approx G_\mu^2 m_\mu^5$$



G_F=1.16639 x 10^{-5} GeV^{-2}

This structure known since Fermi

Fermion Multiplet Structure

- Ψ_L couples to $W^{\pm}$ (cf Fermi theory)
  - Put in SU(2) doublets with weak isospin $I_3=\pm 1/2$
- Ψ_R doesn't couple to $W^{\pm}$
  - Put in SU(2) singlet with weak isospin $I=I_3=0$

What about fermion masses?

- Fermion mass term:

$$L = m\overline{\Psi}\Psi = m\left(\overline{\Psi}_L\Psi_L + \overline{\Psi}_R\Psi_R\right)$$

← Forbidden by SU(2)xU(1) gauge invariance

- Left-handed fermions are SU(2) doublets

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

- Scalar couplings to fermions:

$$L_d = -\lambda_d \overline{Q}_L \Phi d_R + h.c.$$

- Effective Higgs-fermion coupling

$$L_d = -\lambda_d \frac{1}{\sqrt{2}} (\overline{u}_L, \overline{d}_L) \begin{pmatrix} 0 \\ v+h \end{pmatrix} d_R + h.c.$$

- Mass term for down quark:

$$\boxed{\lambda_d = -\frac{M_d\sqrt{2}}{v}}$$

Fermion Masses, 2

- M_u from $\Phi_c = i\tau_2\Phi^*$

$$\Phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$$

$$\lambda_u = -\frac{M_u\sqrt{2}}{v}$$

$$L = -\lambda_u \overline{Q}_L \Phi^* u_R + hc$$

- For 3 generations, α, β=1,2,3 (flavor indices)

$$L_Y = -\frac{(v+h)}{\sqrt{2}} \sum_{\alpha,\beta} \left(\lambda_u^{\alpha\beta} \overline{u}_L^\alpha u_R^\beta + \lambda_d^{\alpha\beta} \overline{d}_L^\alpha d_R^\beta \right) + h.c.$$

Fermion masses, 3

- Unitary matrices diagonalize mass matrices

$$u_L^\alpha = U_u^{\alpha\beta} u_L^{m\beta} \qquad d_L^\alpha = U_d^{\alpha\beta} d_L^{m\beta}$$
$$u_R^\alpha = V_u^{\alpha\beta} u_R^{m\beta} \qquad d_R^\alpha = V_d^{\alpha\beta} d_R^{m\beta}$$

  – Yukawa couplings are ***diagonal*** in mass basis
  – Neutral currents remain flavor diagonal

- Charged current:

$$J^{+\mu} = \frac{1}{\sqrt{2}} \bar{u}_L^\alpha \gamma^\mu d_L^\alpha = \frac{1}{\sqrt{2}} \bar{u}_L^{m\alpha} \gamma^\mu (U_u^+ V_d)_{\alpha\beta} d_L^{\beta m}$$

CKM matrix

Abelian Higgs Model

- Why are the W and Z boson masses non-zero?
- U(1) gauge theory with single spin-1gauge field, A_μ

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

- U(1) local gauge invariance:

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)$$

- Mass term for A would look like:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

- Mass term violates local gauge invariance
- We understand why $M_A = 0$

Gauge invariance is guiding principle

Abelian Higgs Model, 2

- Add complex scalar field, φ, with charge –e:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| D_\mu \phi \right|^2 - V(\phi)$$

- Where

$$D_\mu = \partial_\mu - ieA_\mu \qquad\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda \left(|\phi|^2 \right)^2$$

- L is invariant under local U(1) transformations:

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \eta(x)$$

$$\phi(x) \rightarrow e^{-ie\eta(x)} \phi(x)$$

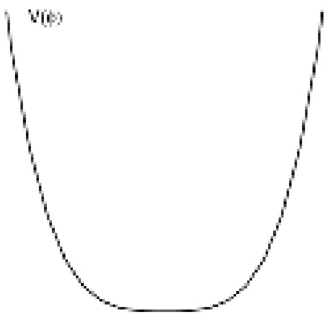
Abelian Higgs Model, 3

- Case 1: $\mu^2 > 0$
  - QED with M_A=0 and $m_\varphi=\mu$
  - Unique minimum at φ=0

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| D_\mu \phi \right|^2 - V(\phi)$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda \left(|\phi|^2 \right)^2$$

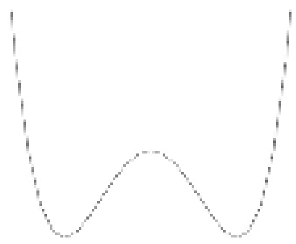


$\lambda > 0$

Abelian Higgs Model, 4

- Case 2: $\mu^2 < 0$

$$V(\phi) = -|\mu^2||\phi|^2 + \lambda(|\phi|^2)^2$$



- Minimum energy state at:

$$<\phi> = \sqrt{-\frac{\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Vacuum breaks U(1) symmetry

Aside: What fixes sign (μ^2)?

Abelian Higgs Model, 5

- Rewrite $\phi \equiv \frac{1}{\sqrt{2}} e^{i\frac{\chi}{v}} (v+h)$

χ and h are the 2 degrees of freedom of the complex Higgs field

- L becomes:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - evA_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A^\mu A_\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h + 2\mu^2 h^2\right) + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + (h, \chi \cdot \text{int } eraction)$$

- Theory now has:
  - Photon of mass M_A=ev
  - Scalar field h with mass-squared $-2\mu^2 > 0$
  - Massless scalar field χ *(Goldstone Boson)*

Abelian Higgs Model, 6

- What about mixed χ-A propagator?
  - Remove by gauge transformation

$$A'_\mu \equiv A_\mu - \frac{1}{ev}\partial_\mu \chi$$

- χ field disappears
  - We say that it has been *eaten* to give the photon mass
  - χ field called Goldstone boson
  - *This is Abelian Higgs Mechanism*
  - This gauge (unitary) contains only physical particles

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^2v^2}{2}A'^{\mu}A'_\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h\right) - V(h)$$

Higgs Mechanism summarized

Spontaneous breaking of a gauge theory by a non-zero VEV of a scalar field results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson

The (adhoc) Higgs Mechanism (a model without dynamics)

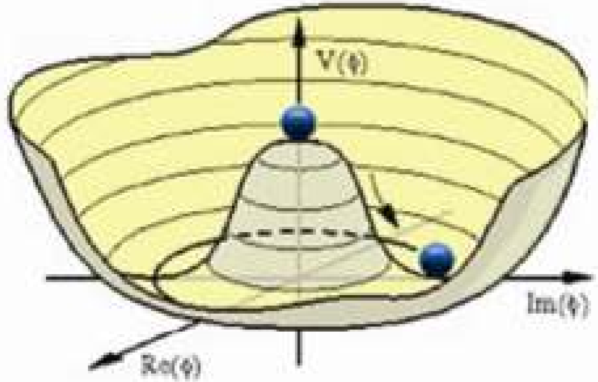
EW symmetry breaking is described by the condensation of a scalar field

$$\Phi = \begin{bmatrix} \phi^+ \\ \frac{v + H + i\varphi_Z}{\sqrt{2}} \end{bmatrix}$$

Background value, Higgs medium

Higgs boson: excitation of the higgs medium

The Higgs selects a vacuum state by developing a non zero background value. When it does so, it gives mass to SM particles it couples to.



$$V(\Phi) = \frac{\mu^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2$$

Why is μ^2 negative ?

the puzzle:

We do not know what makes the Higgs condensate.
We ARRANGE the Higgs potential so that the Higgs condensates but this is just a parametrization that we are unable to explain dynamically.

The gauge symmetries of the Standard Model

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{-iY g' \alpha_Y} \psi,$$

$$B'_\mu = B_\mu + \partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi = (\partial_\mu + i g' Y B_\mu)\, \psi$$

Gauge Group $SU(2)_L$ acts on the two components of a doublet Ψ_L=(u_L,d_L) or (ν_L,e_L)

$$\Psi_L \to e^{-i g T^a \alpha^a} \psi_L \quad U = e^{-i g T^a \alpha^a} \quad T^a = \sigma^a/2$$

Pauli matrices

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \ldots, 3$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = -i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$D_\mu \psi_L = (\partial_\mu + i g W^a_\mu T^a)\, \psi_L$$

Gauge Group $SU(3)_c$ q=(q_1,q_2,q_3) (the three color degrees of freedom)

$$q \to e^{-i g_s T^a \alpha^a} q \quad U = e^{-i g_s T^a \alpha^a} \quad \left[T^a, T^b\right] = i f^{abc} T^c$$

(3×3) Gell-Man matrices

$$G^a_\mu T^a \to U G^a_\mu T^a U^{-1} + \frac{i}{g_s} \partial_\mu U U^{-1}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu, \quad a = 1, \ldots, 8$$

$$D_\mu q = (\partial_\mu + i g_s G^a_\mu T^a)\, q$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The gauge symmetries of the Standard Model

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{-iY g' \alpha_Y} \psi,$$

$$B'_\mu = B_\mu + \partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi = (\partial_\mu + i g' Y B_\mu) \psi$$

$$\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

all Standard Model fermions carry U(1) charge

Gauge Group $SU(2)_L$

$$\Psi_L \to e^{-i g T^a \alpha^a} \psi_L \quad U = e^{-i g T^a \alpha^a}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g \epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \ldots, 3$$

$$D_\mu \psi_L = (\partial_\mu + i g W^a_\mu T^a) \psi_L$$

$\Psi_L = (u_L, d_L)$ or (ν_L, e_L)

only left-handed fermions charged under it -> chiral interactions

Gauge Group $SU(3)_c$

$$q \to e^{-i g_s T^a \alpha^a} q \quad U = e^{-i g_s T^a \alpha^a}$$

$$G^a_\mu T^a \to U G^a_\mu T^a U^{-1} + \frac{i}{g_s} \partial_\mu U U^{-1}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu, \quad a = 1, \ldots, 8$$

$$D_\mu q = (\partial_\mu + i g_s G^a_\mu T^a) q$$

$q = (q_1, q_2, q_3)$

all quarks transform under it -> vector-like interactions

The Lagrangian of the Standard Model

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

describe massless gauge bosons

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{quarks}} i\bar{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\overline{\psi_L}\gamma^\mu D_\mu \psi_L + \sum_{\psi_R} i\overline{\psi_R}\gamma^\mu D_\mu \psi_R$$

describe massless fermions and their interactions with gauge bosons

$$D_\mu \psi_R = \left[\partial_\mu + ig' Y B_\mu\right] \psi_R$$

only left-handed fermions ($\sum_{\psi_L}$)

all fermions carrying a $U(1)_Y$ charge i.e. all Standard Model fermions ($\sum_{\psi_R}$)

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger D_\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2$$

⟶ gives mass to EW gauge bosons $\frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^+ W^{-\mu}$

$$D_\mu \Phi = \left[\partial_\mu + i\frac{g}{\sqrt{2}}\left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right) + i\frac{g}{2}\tau_3 W_\mu^3 + i\frac{g'}{2}B_\mu\right]\Phi$$

: covariant derivative of the Higgs

H charged under $SU(2) \times U(1)_Y$

$$\mathcal{L}_{\text{Yukawa}} = -Y_l \overline{L}\,\Phi\,\ell_R - Y_d \overline{Q}\,\Phi\, d_R - Y_u \overline{Q}\,\tilde{\Phi}\, u_R + \text{h.c.}$$

⟶ gives mass to fermions

responsible for electroweak symmetry breaking!

$$SU(3) \times SU(2)_L \times U(1)_Y \longrightarrow SU(3) \times U(1)_{em}$$

8 massless gluons — **3 massive gauge bosons W^+ W^- Z_0** — **8 massless gluons** — **1 massless photon γ**

remaining unbroken symmetry

The W and Z bosons interact with the Higgs medium, the γ doesn't.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$SU(3)_c$ $\quad$ $SU(2)_L$ $\quad$ $U(1)_Y$

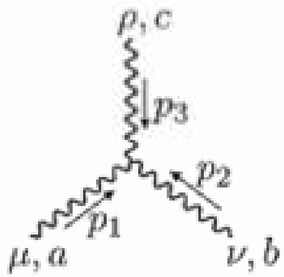
$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu \qquad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

in mass eigen state basis

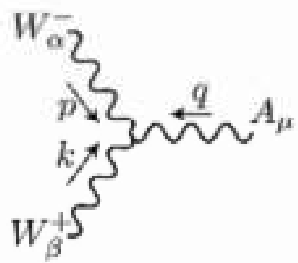
$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \qquad Z_\mu = W^3_\mu \cos\theta_W + B_\mu \sin\theta_W$$

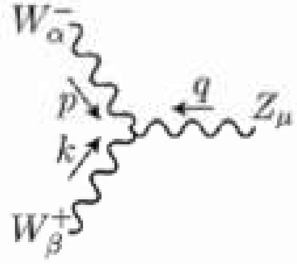
$$A_\mu = -W^3_\mu \sin\theta_W + B_\mu \cos\theta_W$$

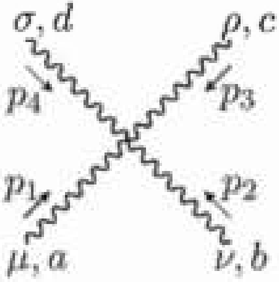
$$\cos\theta_W = g/\sqrt{g^2+g'^2} \qquad \sin\theta_W = g'/\sqrt{g^2+g'^2}$$



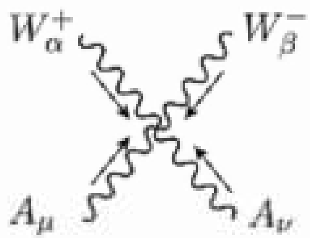
three gauge boson vertex

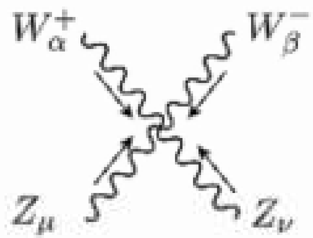


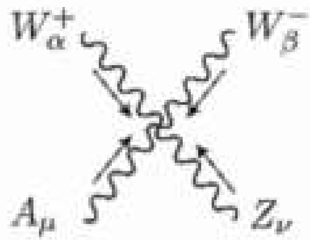


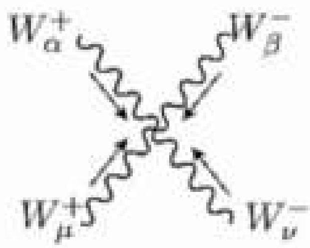


four gauge boson vertex





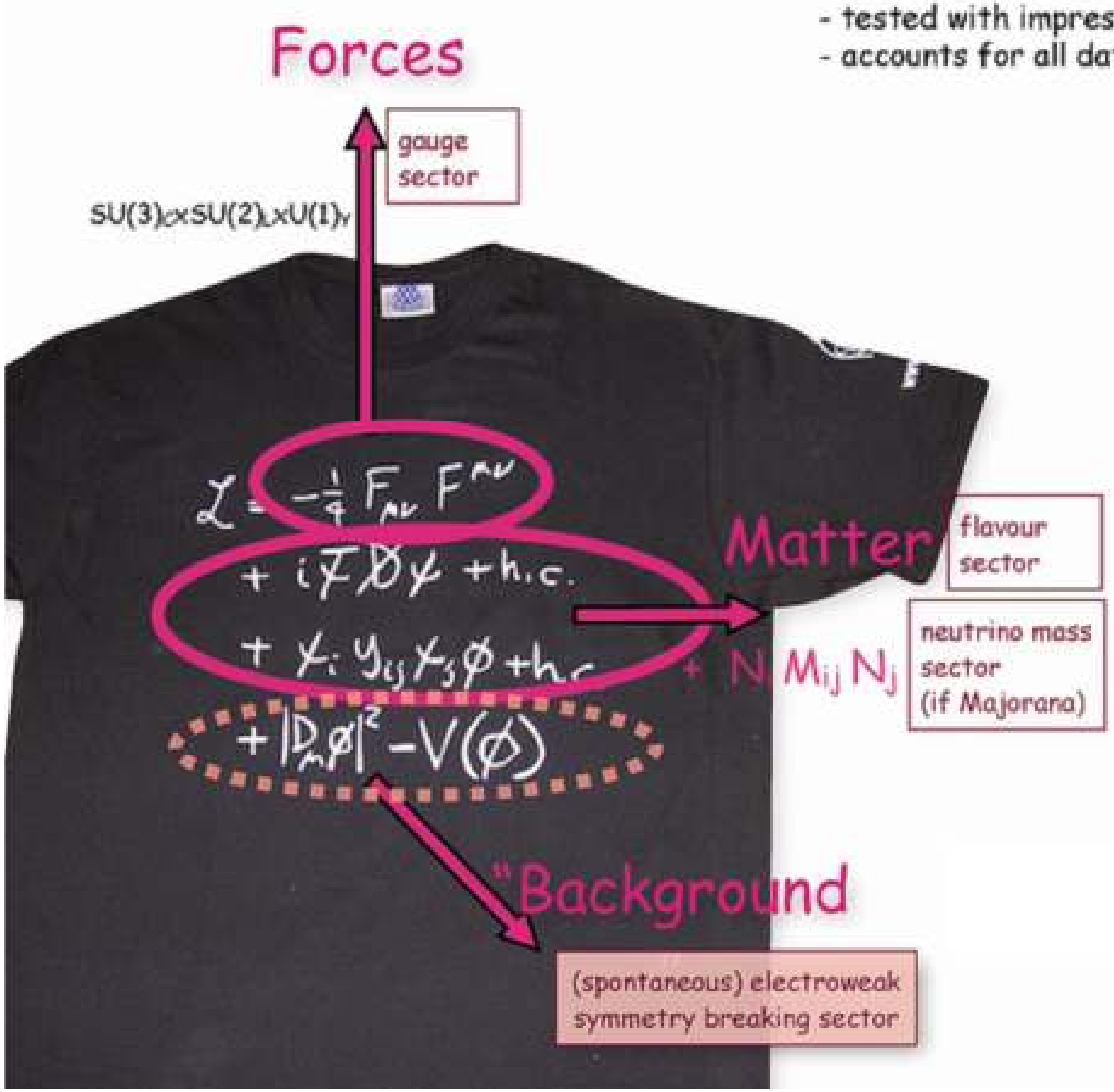




no such interactions for photon!

The Standard Model of Particle Physics

- one century to develop it
- tested with impressive precision
- accounts for all data in experimental particle physics



Finally the Higgs has been found in 2012, which is the last missing piece in the Standard Model.

Field	$SU(3)$	$SU(2)_L$	T^3	$\frac{Y}{2}$	$Q = T^3 + \frac{Y}{2}$
g^a_μ (gluons)	$\mathbf{8}$	$\mathbf{1}$	0	0	0
$(W^\pm_\mu, W^0_\mu)$	$\mathbf{1}$	$\mathbf{3}$	$(\pm 1, 0)$	0	$(\pm 1, 0)$
B^0_μ	$\mathbf{1}$	$\mathbf{1}$	0	0	0
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
u_R	$\mathbf{3}$	$\mathbf{1}$	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	$\mathbf{3}$	$\mathbf{1}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e_R	$\mathbf{1}$	$\mathbf{1}$	0	-1	-1
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\Phi^c = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

Lots still not understood!

- How to calculate predictions for the hard questions in QCD?
- What happens at nearby energies to allow the force couplings to unify at much higher energy? SUSY?
- What causes the fermions to have the observed mass pattern?
- What about neutrinos

Lots still not understood!

- What gives the universe matter excess over antimatter?
- What particles make up most of the (dark) mass of the universe?
- Where did the "dark energy" come from?
- What about gravity?

References

In preparing this presentation I used the following lectures and presentations

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2. The future of particle physics, S. F. King, 2004.
3. Sumer school lectures by Geraldine Servant , 2012.
4. Introduction to the Standard Model, Sally Dawson, TASI, 2006