Physics Beyond the Standard Model

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CTP, BUE

Outline

1. Evidence and Possible Directions Beyond the Standard Model

2. A Simple Example of a Standard Model Extension

Lecture 2: Extra Dimensions: A Window Beyond the Standard Model

1. Why Extra Dimensions?
2. Kaluza-Klein Theory
3. Large Extra Dimensions: ADD Model
4. Warped Extra Dimensions: RS Model
5. Collider Signals of Large Extra Dimension

Why Extra Dimensions

- General Relativity: why 4 dimensions?
- Possible existence of new spatial dimensions beyond the four we see have been under consideration for about eighty years already.
- The first ideas date back to the early works of Kaluza and Klein around the 1920?s, who tried to unify electromagnetism with Einstein gravity.
- Extra dimensions aim to unify the fundamental forces of the universe.
- Extra dimensions are fundamental ingredient for String Theory, since all versions of the theory are naturally and consistently formulated only in a space-time of more than four dimensions (actually 10, or 11 if there is M-theory).
- Extra dimensions offer a potential solution to the hierarchy problem.
- Extra dimensions can potentially explain cosmological inflation and the nature of dark matter and dark energy.

General relativity in 5D spacetime

- One of the first attempts to formulate a unified field theory. Introduced by Theodor Kaluza in 1921.

- The five dimensional line element is given by

$$d\hat{s}^2 = g_{MN} dx^M dx^N$$

- The five dimensional metric is assumed as,

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} - \kappa^2\phi^2 A_\mu A_\nu & -\kappa\phi^2 A_\mu \\ -\kappa\phi^2 A_\nu & -\phi^2 \end{pmatrix}$$

- $\hat{g}_{MN}$ becomes the gravitational tensor potential framed by the electromagnetic four-potential A_μ and scalar field ϕ.

- It was assumed that the metric is independent of the extra dimensional coordinate y. This assumption is known as the *cylindrical condition*: $\partial g_{MN}/\partial x_4 = 0$

- Along with the identifications $g_{44} = -\phi^2 = -1$, $\kappa = \sqrt{\frac{16\pi G}{c^4}}$,the resulting field equations $G_{MN} = 0$ are

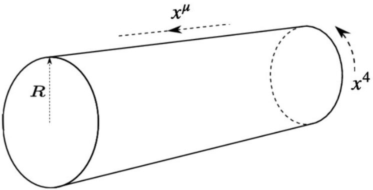
$$\begin{aligned} \tilde{G}_{\mu\nu} &= \frac{8\pi G}{c^4} T_{\mu\nu} \\ \nabla^\mu F_{\mu\nu} &= 0 \end{aligned}$$

- with $T_{\mu\nu} \equiv \frac{1}{2}(g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - F_\mu{}^\alpha F_{\nu\alpha})$. This situation is known as “Kaluza miracle”.

Compactified extra dimension

- To justify the cylinder condition, Oskar Klein assumed a microscopic, curled-up dimension. Compactified in toroidal fashion.

$$X(x^\mu, x^4) = X(x^\mu, x^4 + 2\pi R)$$



- All fields are periodic in $y = x_4$ and may be expanded in a Fourier series:

$$\begin{aligned} g_{\mu\nu}(x,y) &= \sum_{n=-\infty}^{+\infty} g_{\mu\nu n}(x) e^{in\cdot y/R} \\ A_\mu(x,y) &= \sum_{n=-\infty}^{+\infty} A_{\mu n}(x) e^{in\cdot y/R} \\ \phi(x,y) &= \sum_{n=-\infty}^{+\infty} \phi_n(x) e^{in\cdot y/R} \end{aligned}$$

with $g^*_{\mu\nu n}(x) = g_{\mu\nu -n}(x), \quad A^*_{\mu n}(x) = A_{\mu -n}(x), \quad \phi^*_n(x) = \phi_{-n}(x)$

- So, the Kaluza-Klein theory describes an infinite number of four-dimensional fields.

- The equations of motion corresponding to the above action are,

$$
\begin{aligned}
(\partial^\mu \partial_\mu - \partial^y \partial_y) g_{\mu\nu}(x,y) &= (\partial^\mu \partial_\mu + \frac{n^2}{R^2}) g_{\mu\nu n}(x) = 0 \\
(\partial^\mu \partial_\mu - \partial^y \partial_y) A_\mu(x,y) &= (\partial^\mu \partial_\mu + \frac{n^2}{R^2}) A_{\mu n}(x) = 0 \\
(\partial^\mu \partial_\mu - \partial^y \partial_y) \phi(x,y) &= (\partial^\mu \partial_\mu + \frac{n^2}{R^2}) \phi_n(x) \quad = 0
\end{aligned}
$$

- Comparing these with the standard Klein-Gordon equation, we get 'mass' corresponding to these fields as,

$$m_n \sim \frac{n}{R}$$

where n is the mode of excitation.

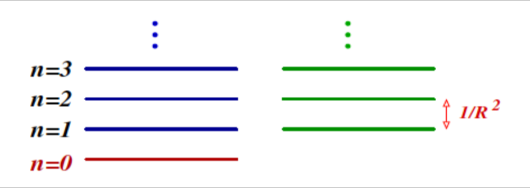
- In four dimensions we see all these excited states with mass or momentum $\sim O(n/R)$. Since we want to unify the electromagnetic interactions with gravity, the natural radius of compactification will be the Planck length: $R = \dfrac{1}{M_p}$, where the Planck mass $M_p \sim 10^{18}\, GeV$.

- The resulting action of the scalar field (called dilaton) is given by

$$S = \int d^4x \left\{ \frac{1}{2} \partial_\mu \phi^{(0)} \partial^\mu \phi^{(0)} + \sum_{n=1}^{\infty} \left[\partial_\mu \phi^{(n)\,\dagger} \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \phi^{(n)\,\dagger} \phi^{(n)} \right] \right\}.$$

- From the 4D point of view that the action describes an (infinite) series of particles (Kaluza-Klein tower) with masses $m_{(n)} = n/R$.

- If the field $\Phi(x^\mu, y)$ has a 5D mass m_0, then the 4D Kaluza-Klein particles will have masses, $m^2_{(n)} = m_0^2 + n^2/R^2$.



KK mass spectrum for a field on the circle.

- In $5D$, the gauge field $A_M(x^\mu, y)$ has the following Fourier decomposition along the compact dimension,

$$A_M(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_n A_M^{(n)}(x^\mu) e^{i\frac{n}{R}y}.$$

- The action of $5D$ gauge field becomes

$$\begin{aligned} S &= \int d^4x\, dy \Big[-\frac{1}{4} F_{MN} F^{MN} \Big] \\ &= \int d^4x \Big\{ \Big(-\frac{1}{4} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \frac{1}{2} \partial_\mu A_5^{(0)} \partial^\mu A_5^{(0)} \Big) + \sum_{n\geq 1} 2 \Big(-\frac{1}{4} F^{(-n)}_{\mu\nu} F^{(n)\mu\nu} + \frac{1}{2} \frac{n^2}{R^2} A_\mu^{(-n)} A_\mu^{(n)} \Big) \Big\}. \end{aligned}$$

- We can see that zero modes contain a 4D gauge field and a real scalar.

Graviton Spectrum

- The Kaluza-Klein metric has 15 independent components. 5 separate conditions to fix the gauge using harmonic gauge can be imposed as follows:

$$\partial_M g^M_N - \frac{1}{2}\partial_N g^M_M = 0.$$

- This brings down the number of degrees of freedom to 10. However, this is not yet a complete gauge fixing, the gauge transformations

$$g_{MN} \longrightarrow g_{MN} + \partial_M \epsilon_N + \partial_N \epsilon_M$$

with $\Box \epsilon_\nu = 0$ are still allowed.

- This means another 5 conditions can be imposed which results in only 5 independent degrees of freedom. Whereas in four dimensions we have only 2 degrees of freedom for a massless graviton.

- This implies that from four dimensional point of view a higher dimensional graviton will contain particles other than just ordinary four dimensional graviton.

- From 5D Einstein Hilbert action: $\hat{S} = \frac{1}{2\hat{k}^2}\int d^5\hat{x}\sqrt{-\hat{g}}\hat{R}$, one can write the four dimensional action for the $n = 0$ modes as,

$$S = \frac{1}{2k^2}\int dx^4\sqrt{-g}[R - \frac{1}{4}e^{-\sqrt{3}\phi}F_{\mu\nu 0}F_0^{\mu\nu} - \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0]$$

Matching process

- A general guideline for a higher-dimensional theory is to check that its law energy limit matches our physics.

- Assumed a theory with higher dimensions compactified into circles of radii $r_n = R$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - r_{(n)}^2 d\Omega_n{}^2$$

- If both gravity and gauge fields propagate in higher dimensions, we need to match

$$S_{HE}^{n+4} = -M_*^{n+2} \int d^{n+4}x \sqrt{g^{n+4}} R^{4+n}$$

$$S_{GF}^{4+n} = -\int d^{4+n}x \frac{1}{4 g_*{}^2} F_{MN} F^{MN} \sqrt{g^{4+n}}$$

- Form matching Hilbert action

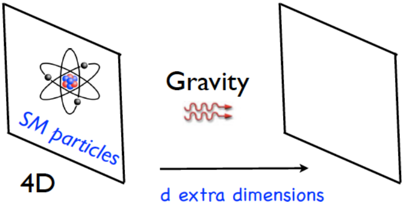
$$-M_*^{n+2} \int d\Omega_n R^n \int d^4x \sqrt{g^{(4)}} R^{(4)} = -M_{Pl}{}^2 \int d^4x \sqrt{g^{(4)}} R^{(4)} \Longrightarrow \boxed{M_{Pl}^2 = M_*^{2+n} V_n(R)}.$$

The volume of extra dimension space: $V_n(R) = \int d\Omega_n R^n = \frac{\pi^{\frac{n}{2}}}{\Gamma(n/2+1)} R^n$ implies that $R \sim \frac{M_{Pl}{}^{\frac{2}{n}}}{M_*{}^{1+\frac{2}{n}}}$

- From matching the gauge field action, we get $1 \approx R^n M_*{}^n$

- Combining these relations yields $R \sim \frac{1}{M_{Pl}} = l_{Pl} \approx 10^{-32}$ cm.

Brane-world models



- If SM fields are localized to a four-dimensional brane. The only restriction on the radius would be

$$R \sim \frac{{M_{Pl}}^{\frac{2}{n}}}{{M_*}^{1+\frac{2}{n}}}$$

- In 1998 Arkani-Hamed, Dimopoulos and Dvali (ADD) realize that extra dimensions could explain the weakness of gravity: $G_N \ll G_F$.

- For m_{EW} is the fundamental Planck scale and choose R such that the observed mass scale is M_{pl}

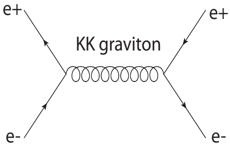
$$R \sim 10^{\frac{30}{n}-17} cm \times (\frac{1\,TeV}{m_{EW}})^{1+\frac{2}{n}}$$

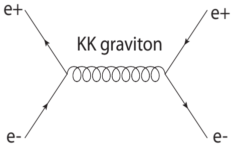
- Two extra dimensions $(n = 2) \rightarrow R \sim 100\mu m$. Deviation from Newton's law would be accommodated by the experimental limit on gravity.

Experimental Constraints and Tests of Large Extra Dimensions

- TeV cutoff: Precision electroweak tests and high energy collisions are sensitive to higher-dimensional operators suppressed by the TeV scale.

  Some operators can be induced by KK graviton exchanges. For example $e^+e^- \rightarrow e^+e^-$ through KK gravitons induces an operator,





- Light degrees of freedom: They can appear as missing energies at colliders and rare decays of unstable particles. They also affect astrophysics (*e.g.*, star cooling) and cosmology (*e.g.*, expansion rate of the universe).

  KK graviton loop gives rise to a relevant contribution to $g-2$. It gives stringent constraint on m_{KK}.

- Long-lived KK gravitons: They can affect astrophysics (diffuse γ-ray background from late decays of long-lved particles) and cosmology (over-closure of the universe).

Warped extra dimensions

- Warped space-times, the metric warps exponentially along the extra dimension

$$ds^2 = f(y) g_{\mu\nu}(x) dx^\mu dx^\nu + g_{ab}(y) dy^a dy^b$$

- Assuming the following
  - An S^1 symmetry:

$$y \to y + 2\pi R$$

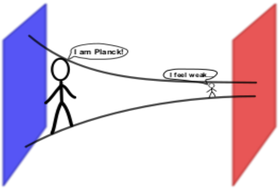
  - A $\mathbb{Z}_2$ symmetry

$$y \to -y$$

- A warp factor satisfying the field equations and the previous assumptions is $\boxed{f(y) = e^{-2k|y|}}$
- An important feature of this model is that it can only admit an AdS space: $k = \sqrt{-\Lambda\kappa}$, where κ is the 5D Einstein constant related to the Planck mass as $\kappa^2 \sim \frac{1}{M^3}$ and Λ is the cosmological constant.

The metric eventually takes the form

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$



Hierarchy problem

- This model solves the hierarchy problem by connecting the four-dimensional Planck scale and mass parameters to the five-dimensional scales.

- The law energy limit is approached by a weak-field background perturbation $h_{\mu\nu}(x^\mu) \ll 1$:

$$ds^2 = e^{-2kT(x^\mu)|\phi|} dx^\mu dx^\nu [\eta_{\mu\nu} + h_{\mu\nu}(x^\mu)] + T^2(x^\mu) dy^2$$

Where the modulus field $T(x)$ is stabilized at r_c being the vacuum expectation value $\langle T(x) \rangle \equiv r_c$

$$ds^2 = e^{-2kr_c|\phi|} dx^\mu dx^\nu [\eta_{\mu\nu} + h_{\mu\nu}(x^\mu)] + {r_c}^2 dy^2$$

- The effective action implies:

$$S = -M^3 \int d^5x \sqrt{g^{(5)}} (R^{(5)} + \kappa^2) \supset -M^3 \int dy e^{-2k|y|} \int d^4x \sqrt{g^{(4)}} (R^{(4)} + \kappa^2)$$

- We now compare this to the four-dimensional action $S = -M_{Pl}^2 \int d^4x \sqrt{g^{(4)}} (R^{(4)} + \kappa^2)$ to get

$$M_{pl}^2 = M^3 \int_{y=-b}^{y=b} dy e^{-2k|y|} = \frac{M^3}{k} [1 - e^{-2kb}]$$

- It is evident by now that this scenario provides a quite different approach than ADD model. The effect of the warp factor is negligible $M_{Pl} \approx M_*$

Hierarchy problem

- The Electroweak mass scale however will be modified. The matter field which, unlike gravity, is localized to one of the branes. The action for the Higgs scalar

$$S^H = \int d^4x \sqrt{g^{ind}}[g_{\mu\nu} D^\mu H D^\nu H - \lambda((H^\dagger H) - v^2)^2]$$

- The induced metric at the brane at $y = b$ is $g^{ind}_{\mu\nu} = e^{2kR}\eta_{\mu\nu}$

$$S^H = \int d^4x e^{-4kb}[e^{2kb}\eta_{\mu\nu} D^\mu H D^\nu H - \lambda((H^\dagger H) - v^2)^2]$$

- The field can be redefined as $\tilde{H} = e^{-kb}H$ to get a canonically normalized field. The action is then

$$S^H = \int d^4x[\eta_{\mu\nu}\partial^\mu \tilde{H}\partial^\nu \tilde{H} - \lambda((\tilde{H}^\dagger \tilde{H}) - (e^{-kb}v)^2)^2]$$

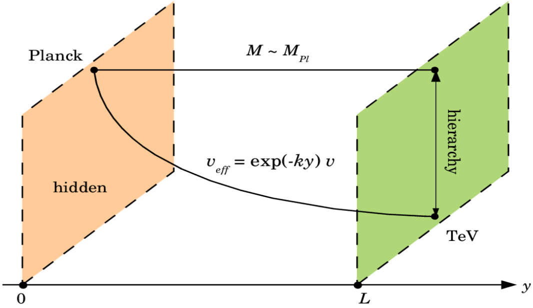
- Thus:

$$\tilde{v} = e^{-kR} v.$$

- If v is regarded as the fundamental mass scale, the warp factor can be used to generate the TeV scale of the weak scale.

- The brane at $y = R$ is referred to as the TeV brane and that at $y = 0$ is referred to as the Planck brane where the warp factor would have no effect and the mass scale parameters are of the Planck mass order.

RS1 Model



- In this model, no large dimension necessary to explain weakness of gravity
- Graviton?s interaction is exponentially suppressed away from ?Gravitybrane?
- Gravity is weak everywhere except Gravitybrane
- Mass hierarchy natural on Weakbrane!

RS1 Signatures

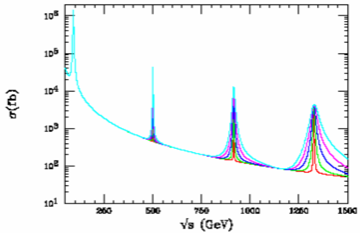
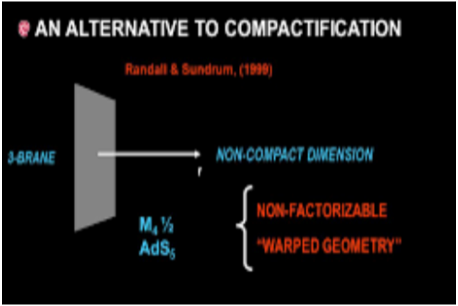


Figure 4: The cross section for $e^+e^- \to \mu^+\mu^-$ including the exchange of a KK tower of gravitons in the Randall-Sundrum model with $m_1 = 500$ GeV. The curves correspond to $k/\overline{M}_{Pl} =$ in the range $0.01 - 0.05$.

- Experimental consequences very distinctive
- KK modes interact not with Planck suppressed interactions
- TeV-suppressed interactions! Means resonances produced and decay in detector
- Will look like true resonances?If we?re lucky, we can even see they are spin-2.
- Very dramatic signals if RS1 correct

RS2 model

- This model is known as an Alternative to Compactification.
- The RS2 model uses the same geometry as RS1, but there is no TeV brane.
- The particles of the standard model are presumed to be on the Planck brane.
- This model was originally of interest because it represented an infinite 5-dimensional model, which, in many respects, behaved as a 4-dimensional model.



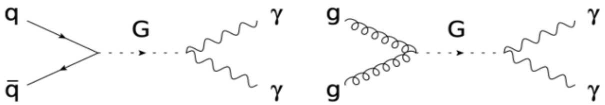
Graviton KK Modes

- 5D graviton leads to tower of KK excitations.
- Lightest KK mode $G^{(1)}$: mass $\sim$ TeV, couples to SM:

$$\frac{1}{\Lambda_\pi} \sim \frac{k}{\overline{M}_{\rm Pl}} \times \text{TeV}$$

- Parameters: $m_G, \; k/\overline{M}_{\rm Pl} \in [0.01, 0.1]$

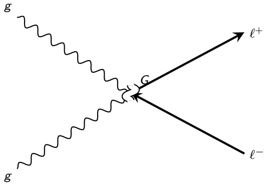
$G \to \gamma\gamma$



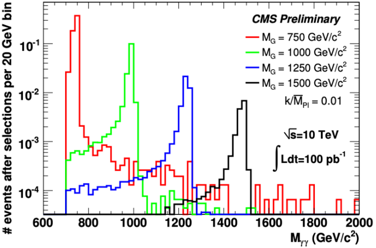
- EM objects offer a clean experimental signature with excellent mass resolution
- RS gravitons have twice the branching ratio to decay to photons as to electrons
- In the diphoton channel, there is less background because the Drell-Yan process ($Z/\gamma^* \to ll$), which dominates in the dilepton channel, is not present

Production and Decay at the LHC

- Dominant production via gluon fusion or $q\bar{q} \to G$
- Decays:
  - $\ell^+\ell^-$, $\gamma\gamma$, W^+W^-, $t\bar{t}$, jj
- Spin-2: unique angular distributions



Other decay channels: $\gamma\gamma$, WW, $t\bar{t}$, hadronic jets



Diphoton invariant mass spectra after selection is applied, scaled to 100 pb^{-1} for $M_1 = 750$, 1000, 1250, and 1500 GeV/c^2, $\tilde{k} = 0.01$ samples.

Experimental Signature

- Narrow resonance in invariant mass spectrum:
  - $pp \to G^{(1)} \to \ell^+\ell^-, \gamma\gamma$
- Clean, high-resolution final states.
- Search for peak in high mass tail.

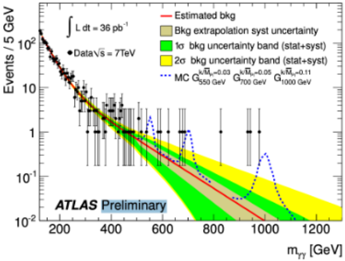
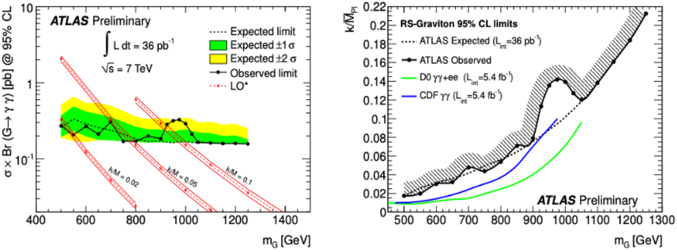


Figure 1: Reconstructed $m_{\gamma\gamma}$ distribution for data (points) and expected background (red line) [5]. Also shown are graviton signals of masses 550, 700 and 1000 GeV and couplings $k/\overline{M}_{\rm Pl}$= 0.03, 0.05 and 0.11, respectively. The signal is normalized to the number of expected events in an integrated luminosity of 36 pb^{-1}.

Current Experimental Limits

- ATLAS/CMS Run 2 ($\sqrt{s} = 13$ TeV, 139 fb^{-1}):
  - No excess observed
  - Exclusion limits on m_G as function of $k/\overline{M}_{\rm Pl}$



(left) The 95% CL limit on the production cross section times branching ratio of an RS model graviton decaying into two photons as a function of the graviton mass.

(right) 95% CL excluded region in the plane of k/M_{Pl} versus graviton mass.

Summary

- RS model offers elegant solution to hierarchy problem.
- Graviton KK modes produce striking resonant signatures at the LHC.
- Current data places strong constraints on RS parameter space.
- No evidence of a narrow resonance decaying into a pair of photons above the continuum background is observed.
- The results exclude at 95% CL RS graviton masses below 545 (920) GeV for the dimensionless RS coupling $k/M_{Pl} = 0.02(0.1)$.
- HL-LHC will enhance sensitivity to higher m_G.