Alexandria Quantum Computing Group (AleQCG)

INTERNATIONAL YEAR OF Quantum Science and Technology

Day2: Basics of Quantum Computing

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Outline

- Quantum Data – qubit
- Quantum Superposition
- Bloch Sphere
- Dirac Notations
- Linear Algebra for QC
- Entanglement.
- Measurements.
- No Cloning Theory
- Quantum Gates and Quantum Circuits

Quantum Data-qubit

A quantum bit of data is represented by a single atom that is in one of two states denoted by **|0>** and **|1>**. A single bit of this form is known as a ***qubit***

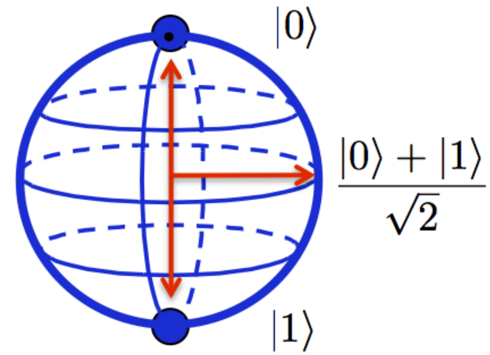
Quantum Superposition

A single qubit can be forced into a ***superposition*** of the two states denoted by the addition of the state vectors:

$$|\psi> = a\ |0> + b\ |1>$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Classical Bit



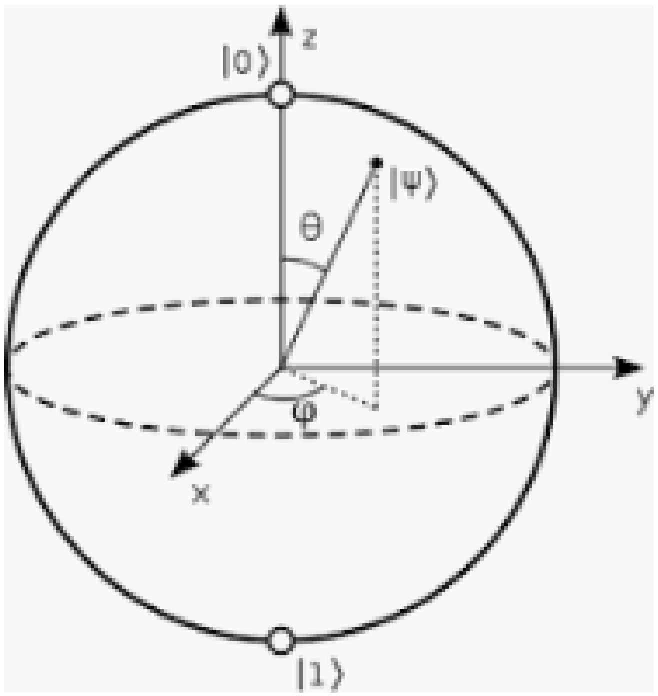
Qubit

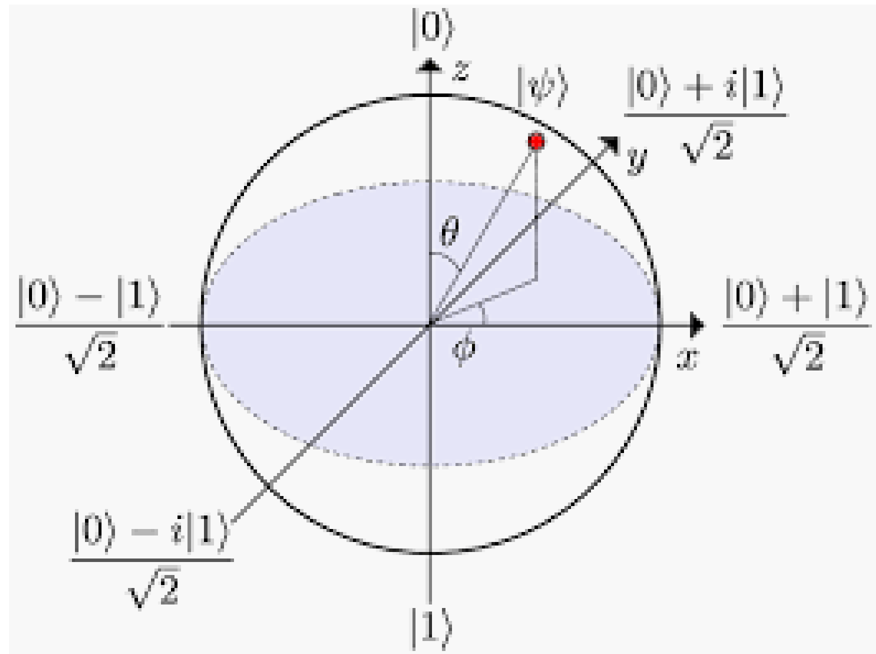
where ***a* and *b*** are complex numbers and $|a|^2 + |b|^2 = 1$

and $|a|^2 = a^*a^*$

A qubit in superposition is in both of the states |1> and |0> at the same time

Bloch Sphere





$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$$

where $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$.

Dirac Notations

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \langle 0| = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \langle 0| = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\langle \psi| = a^*\langle 0| + b^*\langle 1| = \begin{bmatrix} a^* & b^* \end{bmatrix}$$

Inner Product

- Inner product between two vectors $|\psi_1>$ and $|\psi_2>$ is defined as follows:

$$|\psi_1\rangle = a_1|0\rangle + b_1|1\rangle$$

$$|\psi_2\rangle = a_2|0\rangle + b_2|1\rangle$$

$$\langle\psi_1|\psi_2\rangle = a_1 * a_2 + b_1 * b_2 (scaler)$$

$$\langle 0|0\rangle = \langle 1|1\rangle = 1$$

$$\langle 0|1\rangle = \langle 1|0\rangle = 0$$

Outer Product

- Outer product between two vectors $|\psi_1>$ and $|\psi_2>$ is defined as follows:

$$|\psi_1\rangle = a_1|0\rangle + b_1|1\rangle$$

$$|\psi_2\rangle = a_2|0\rangle + b_2|1\rangle$$

$$|\psi_1\rangle\langle\psi_2| = a_1a_2{*}|0\rangle\langle 0| + a_1b_2{*}|0\rangle\langle 1| + b_1a_2{*}|1\rangle\langle 0| + b_1b_2{*}|1\rangle\langle 1|$$

$$= \begin{bmatrix} a_1a_2{*} & a_1b_2{*} \\ b_1a_2{*} & b_1b_2{*} \end{bmatrix}$$

$$|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, |0\rangle\langle 1| = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$|1\rangle\langle 0| = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, |1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiple Qubits

- For 2-qubit systems we have four states, so the system is described as either its individual components (if possible) or as a single system.
- Given the components of the system, we can combine the components using Tensor product.

$$
\begin{aligned}
|\psi_1\rangle &= a_1|0\rangle + b_1|1\rangle \\
|\psi_2\rangle &= a_2|0\rangle + b_2|1\rangle \\
|\psi\rangle &= |\psi_1\rangle \otimes |\psi_2\rangle \\
&= \left(a_1|0\rangle + b_1|1\rangle\right) \otimes \left(a_2|0\rangle + b_2|1\rangle\right) \\
&= a_1a_2\left(|0\rangle \otimes |0\rangle\right) + a_1b_2\left(|0\rangle \otimes |1\rangle\right) + b_1a_2\left(|1\rangle \otimes |0\rangle\right) + b_1b_2\left(|1\rangle \otimes |1\rangle\right) \\
&= \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle \\
&= \sum_{j=0}^{3} \alpha_j|j\rangle, \ \sum_{j=0}^{3} |\alpha_j|^2 = 1,
\end{aligned}
$$

Multiple Qubits

$$\alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle = \begin{vmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{vmatrix}$$

where,

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \ |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \ |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\underbrace{|00\ldots00\rangle}_{n} \ , \ |00\ldots01\rangle \ , \ \ldots \ , \ |11\ldots10\rangle \ , \ |11\ldots11\rangle.$$

The standard way to associate column vectors corresponding to these basis vectors is as follows:

$$|00\ldots00\rangle \iff \left.\begin{pmatrix}1\\0\\0\\\vdots\\0\\0\end{pmatrix}\right\}2^n, \quad |00\ldots01\rangle \iff \begin{pmatrix}0\\1\\0\\\vdots\\0\\0\end{pmatrix}, \quad \cdots$$

$$\cdots \quad , \quad |11\ldots10\rangle \iff \begin{pmatrix}0\\0\\0\\\vdots\\1\\0\end{pmatrix}, \quad |11\ldots11\rangle \iff \begin{pmatrix}0\\0\\0\\\vdots\\0\\1\end{pmatrix}.$$

Quantum Measurement

- Quantum system can be transformed to a classical system using measurement.
- The superposition is collapsed to one it's possible states in a probabilistic way.

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

- Probability to find the 1st qubit in state |0> is $|\alpha_0|^2+|\alpha_1|^2$.
- Probability to find the 2nd qubit in state |1> is $|\alpha_1|^2+|\alpha_3|^2$.

Quantum Measurement

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

- If the 1st qubit of this system is measured and the outcome is |1>, then the system will be transformed to the following system

$$|\psi'\rangle = \frac{1}{\sqrt{|\alpha_2|^2 + |\alpha_3|^2}}\left(\alpha_2|10\rangle + \alpha_3|11\rangle\right)$$

- The amplitudes are re-normalized and the superposition of the second qubit is not affected by the measurement.

Entanglement

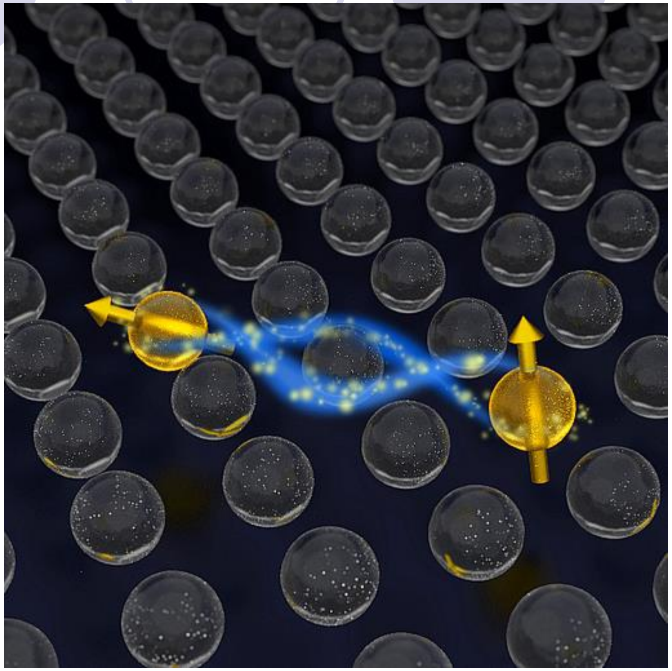
- *Entanglement* is the ability of quantum systems to exhibit correlations between states within a superposition.

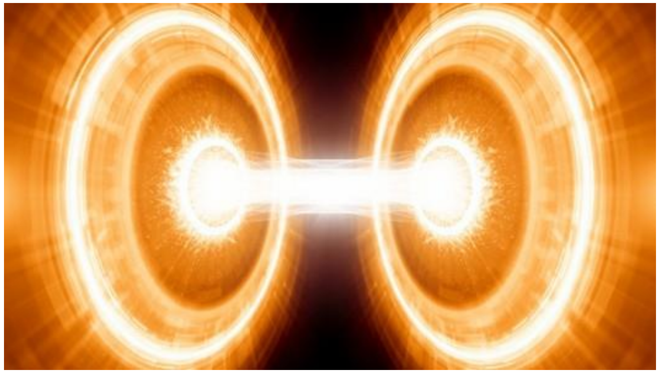
- Imagine two qubits, each in the state (a superposition of the 0 and 1.)

$$(a_1|0> + b_1|1>) \otimes (a_2|0> + b_2|1>)$$

- We can entangle the two qubits such that the measurement of one qubit is always correlated to the measurement of the other qubit.

$$a|00>+b|11>$$





Entanglement

- An arbitrary 2-qubit system can be represented as follows:

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

where α_j's can take any value as long as

$$\sum_{j=0}^{3}|\alpha_j|^2 = 1,$$

If α_0=0 and α_1=0:

$$\alpha_2|10\rangle + \alpha_3|11\rangle = |1\rangle \otimes \left(\alpha_2|0\rangle + \alpha_3|1\rangle\right)$$

If α_1=0 and α_3=0.

$$\alpha_0|00\rangle + \alpha_2|10\rangle = \left(\alpha_0|0\rangle + \alpha_2|1\rangle\right) \otimes |0\rangle$$

Entanglement

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

If α_1=0 and α_2=0:

$$\alpha_0|00\rangle + \alpha_3|11\rangle$$

If α_0=0 and α_3=0.

$$\alpha_1|01\rangle + \alpha_2|10\rangle$$

- These two systems are entangled and cannot be represented using their individual components.
- A measurement on one qubit affects the state of the other qubit.

Bell States

- Entangled states are considered as the heart for many quantum algorithms.
- For example, quantum teleportation, dense coding and quantum searching.
- Two-qubit entangled states are usually referred to as *Bell states, EPR states, EPR pairs* or *Bell basis*.

$$\frac{(|00\rangle \pm |11\rangle)}{\sqrt{2}}, \quad \frac{(|01\rangle \pm |10\rangle)}{\sqrt{2}}.$$

No Cloning Theory

- It is not possible to clone an unknown quantum state

No Cloning Assume we have a unitary operator U_{cl} and two quantum states $|\phi\rangle$ and $|\psi\rangle$ which U_{cl} copies, i.e.,

$$|\phi\rangle \otimes |0\rangle \xrightarrow{U_{cl}} |\phi\rangle \otimes |\phi\rangle$$

$$|\psi\rangle \otimes |0\rangle \xrightarrow{U_{cl}} |\psi\rangle \otimes |\psi\rangle$$

Proof : Suppose there exists a unitary operator U_{cl} that can indeed clone an unknown quantum state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$. Then

$$\begin{aligned} |\phi\rangle|0\rangle \xrightarrow{Ucl} |\phi\rangle|\phi\rangle &= (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle) \\ &= \alpha^2|00\rangle + \beta\alpha|10\rangle + \alpha\beta|01\rangle + \beta^2|11\rangle \end{aligned}$$

But now if we use U_{cl} to clone the expansion of $|\phi\rangle$, we arrive at a different state:

$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle \xrightarrow{U_{cl}} \alpha|00\rangle + \beta|11\rangle.$$

Here there are no cross terms. Thus we have a contradiction and therefore there cannot exist such a unitary operator U_{cl}.

Quantum Gates and Circuits

Outline

- Quantum gates.
- Quantum circuit model.
- Quantum truth table.
- Boolean quantum circuits.
- Quantum Simulation

Computation with Qubits

How does the use of qubits affect computation?

Classical Computation

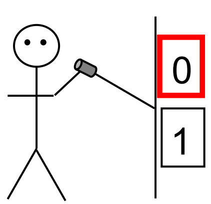
Data unit: bit

● = ‘1’ ○ = ‘0’

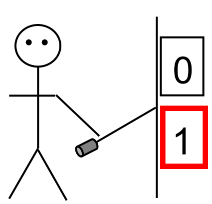
Valid states:

x = ‘0’ or ‘1’

x = 0

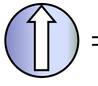
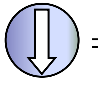


x = 1



Quantum Computation

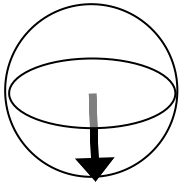
Data unit: qubit

 $=|1\rangle$  $=|0\rangle$

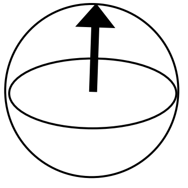
Valid states:

$|\psi\rangle = c_1|0\rangle + c_2|1\rangle$

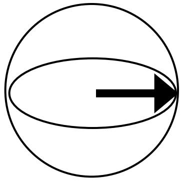
$|\psi\rangle = |0\rangle$



$|\psi\rangle = |1\rangle$



$|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$



Computation with Qubits

How does the use of qubits affect computation?

Classical Computation

Operations: logical

Valid operations:

NOT =

	in: 0	in: 1
out	1	0

1-bit

AND =

in	in: 0	in: 1
0	0	0
1	0	1

(out)

2-bit

Quantum Computation

Operations: unitary

Valid operations:

1-qubit

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

2-qubit

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Computation with Qubits

- Computation in quantum systems must be *reversible*, so that *no loss in energy* during the computation process.
- Quantum gates are represented as square matrices U that satisfy the *unitary* condition:

$$U^\dagger = U^{-1}$$

$$U^\dagger U = U U^\dagger = I.$$

Quantum Operators

- For a 1-qubit system, the quantum gate must be 2x2.
- For a 2-qubit system, the quantum gate must be 4x4.
- For a *n*-qubit system, the quantum gate must be 2^nx2^n.

Linear Transformations

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$U = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$$

$$|\psi'\rangle = U|\psi\rangle = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \begin{bmatrix} x_0 a + x_1 b \\ x_2 a + x_3 b \end{bmatrix}$$

$$= (x_0 a + x_1 b)|0\rangle + (x_2 a + x_3 b)|1\rangle$$

Single-qubit Quantum Gates

Identity Gate (I gate)

Unitary matrix representation, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$\|1\rangle$

truth table

Diagonal representation, $I = |0\rangle\langle 0| + |1\rangle\langle 1|$.

And its circuit takes the form,

$$(a|0\rangle + b|1\rangle) \; -\!\!\boxed{\;I\;}\!\!- \; (a|0\rangle + b|1\rangle)$$

NOT Gate (Pauli–X gate)

Unitary matrix representation, $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Input	Output
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

truth table

Diagonal representation, $X = |0\rangle\langle 1| + |1\rangle\langle 0|$.

And its circuit takes the form,

$$(a|0\rangle + b|1\rangle) \; -\!\!\boxed{\;X\;}\!\!- \; (a|1\rangle + b|0\rangle)$$

Pauli–Y Gate

Unitary matrix representation, $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}.$

Input	Output
$\|0\rangle$	$i\,\|1\rangle$
$\|1\rangle$	$-i\,\|0\rangle$

truth table

Diagonal representation, $Y = -i\left(|0\rangle\langle 1| - |1\rangle\langle 0|\right).$

And its circuit takes the form,

$$(a\,|0\rangle + b\,|1\rangle) \;—\; \boxed{Y} \;—\; (ai\,|1\rangle - bi\,|0\rangle)$$

Phase Shift Gate (Pauli–Z gate)

Unitary matrix representation, $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$-\|1\rangle$

truth table

Diagonal representation, $Z = |0\rangle\langle 0| - |1\rangle\langle 1|.$

And its circuit takes the form,

$$(a|0\rangle + b|1\rangle) \;—\!\!\boxed{Z}\!\!—\; (a|0\rangle - b|1\rangle)$$

Hadamard Gate (H gate)

Unitary matrix representation, $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$

Input	Output
$\vert 0\rangle$	$\frac{1}{\sqrt{2}}(\vert 0\rangle + \vert 1\rangle)$
$\vert 1\rangle$	$\frac{1}{\sqrt{2}}(\vert 0\rangle - \vert 1\rangle)$

truth table.

Diagonal representation,

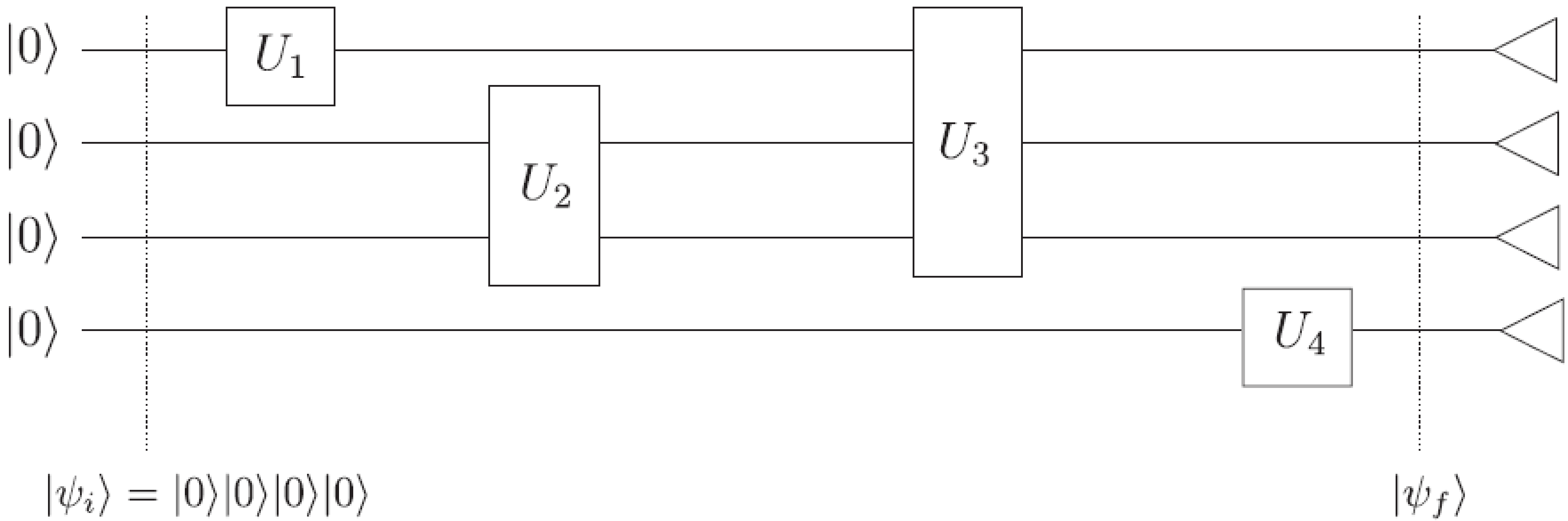
$$H = \frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right).$$

And its circuit takes the form,

$$|x\rangle \;\text{—}\boxed{\;H\;}\text{—}\; \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^x |1\rangle\right)$$

Quantum Circuit Model

A QUANTUM MODEL OF COMPUTATION



Consider a two-qubit system $|\psi\rangle \otimes |\xi\rangle$. Applying U on $|\psi\rangle$ and V on $|\xi\rangle$ in parallel can be written as follows,

$$U \otimes V \left(|\psi\rangle \otimes |\xi\rangle\right) = U \left|\psi\right\rangle \otimes V \left|\xi\right\rangle .$$

where $U \otimes V$ can be combined in a single matrix of size 4×4 as follows,

$$\begin{aligned}
U \otimes V &= \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{bmatrix} \otimes \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} \\
&= \begin{bmatrix} u_{00} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} & u_{01} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} \\ u_{10} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} & u_{11} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} \end{bmatrix} \\
&= \begin{bmatrix} u_{00}v_{00} & u_{00}v_{01} & u_{01}v_{00} & u_{01}v_{01} \\ u_{00}v_{10} & u_{00}v_{11} & u_{01}v_{10} & u_{01}v_{11} \\ u_{10}v_{00} & u_{10}v_{01} & u_{11}v_{00} & u_{11}v_{01} \\ u_{10}v_{10} & u_{10}v_{11} & u_{11}v_{10} & u_{11}v_{11} \end{bmatrix} .
\end{aligned}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

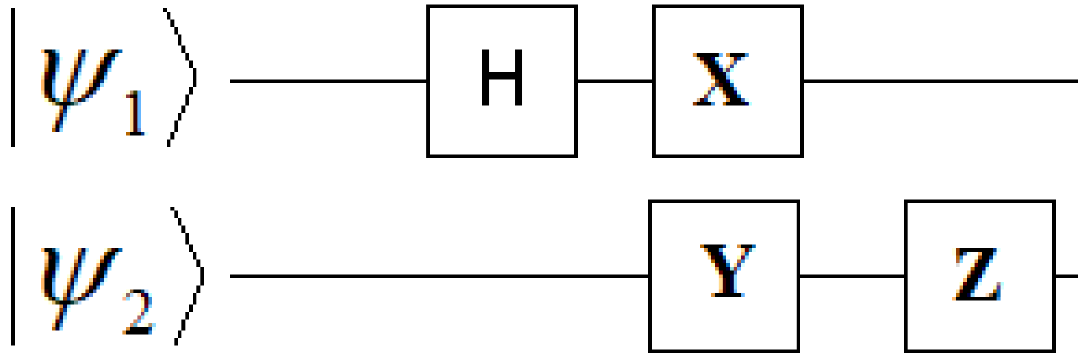
Suppose we have a 2-qubit composite system, and we apply X to the first qubit. I to the second qubit at the same time.
Thus the 2-qubit input $|\psi_1\rangle \otimes |\psi_2\rangle$ gets mapped to $X|\psi_1\rangle \otimes I|\psi_2\rangle = (X \otimes I)(|\psi_1\rangle \otimes |\psi_2\rangle)$.
That is, the linear operator describing this operation on the composite system has the matrix representation

$$X \otimes I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

1- Qubit Gate Identities

- $Y = iXZ.$
- $H = (X + Z)/\sqrt{2}.$
- $S = T^2.$
- $HXH = Z.$
- $HYH = -Y.$
- $HZH = X.$
- $XY = -YX = iZ.$
- $ZX = -XZ = iY.$
- $YZ = -ZY = iX.$
- $XX = YY = ZZ = I.$

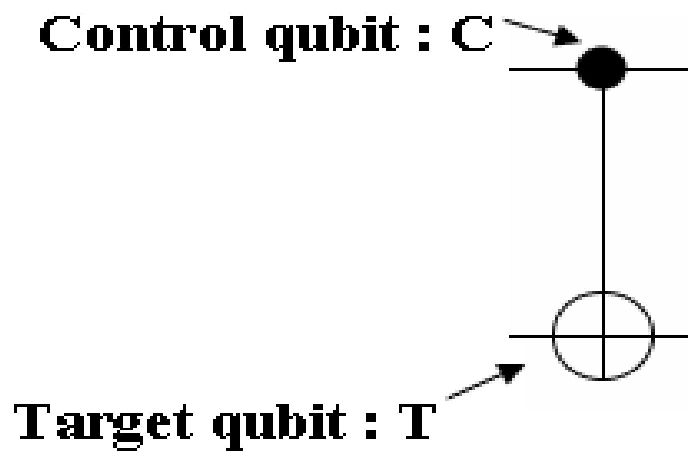
Tracing a Quantum Circuit



- What is the truth table?

Two qubit gates

- **The Controlled-NOT Gate (C_{not})**
- If C=0 then no change
- Else If C=1 then T is flipped



Diagonal representation,

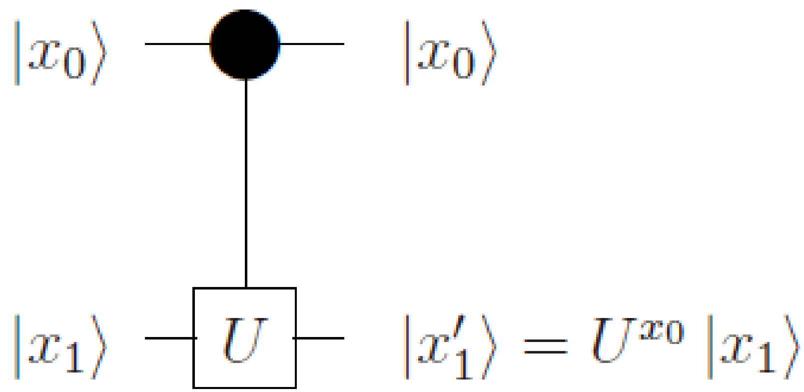
$$C_{not} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X.$$

$$C_{not} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

The C_{not} gate truth table.

The General Controlled–*U* Gate (*C*-*U* gate)



The Controlled–U gate.

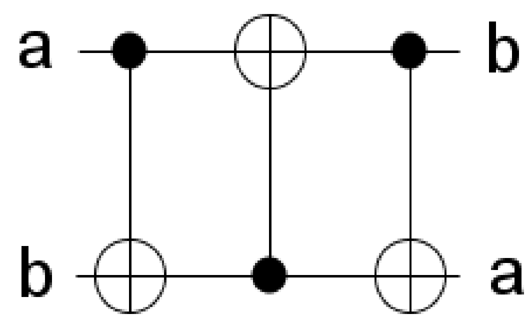
It works as follows: U will be applied on the target qubit $|x_1\rangle$ if and only if the control qubit $|x_0\rangle$ is set to $|1\rangle$

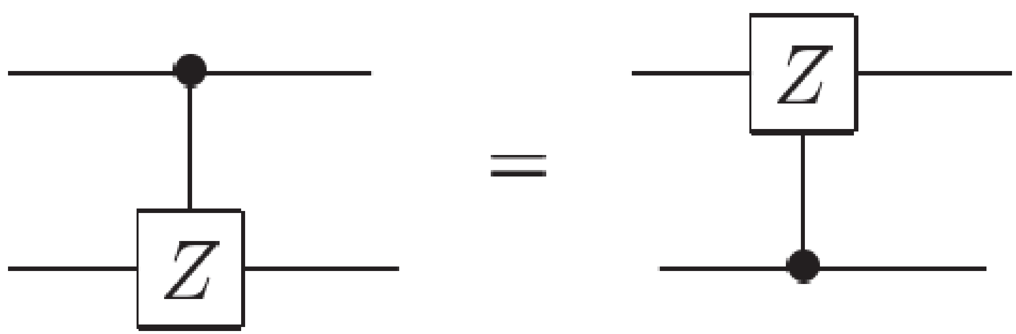
$$C - U = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U.$$

Examples

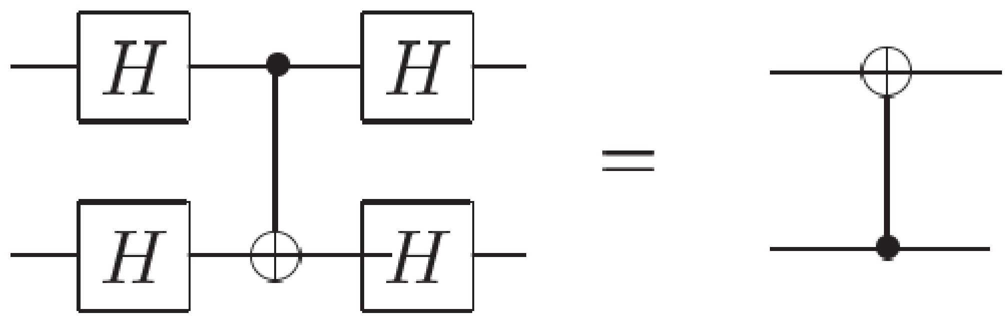
Swap Circuit:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





Examples



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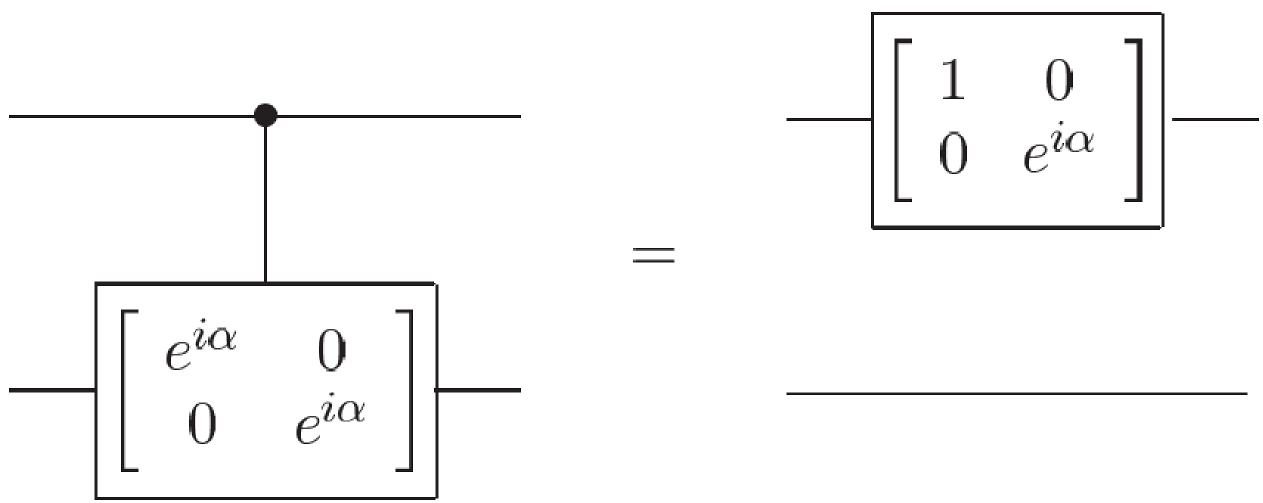
H
=



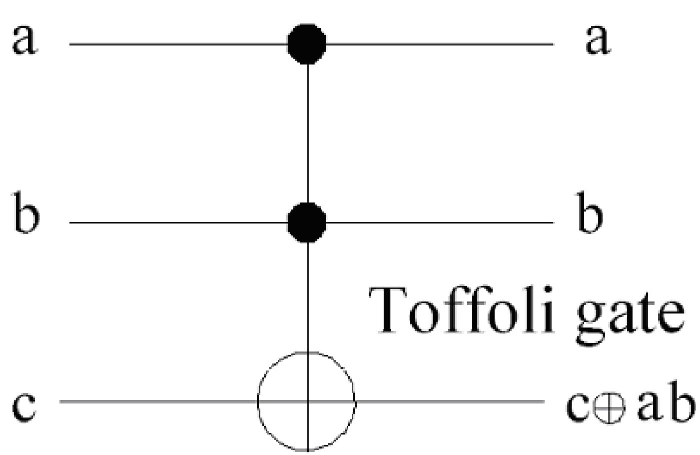
$\begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha} \end{bmatrix}$
=
$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{bmatrix}$

Three qubit gates

Toffoli gate :

Is considered to be universal…

Setting C=1 will convert it to classical *NAND* gate which is universal from classical point of view.

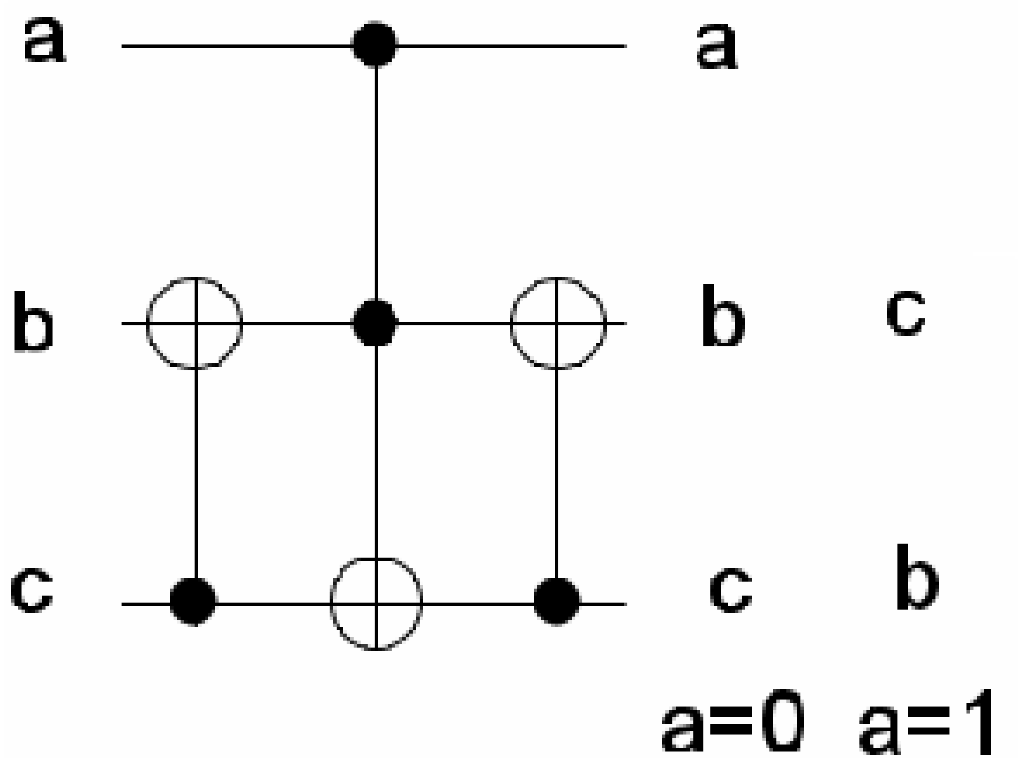


Input	Output
$\|000\rangle$	$\|000\rangle$
$\|001\rangle$	$\|001\rangle$
$\|010\rangle$	$\|010\rangle$
$\|011\rangle$	$\|011\rangle$
$\|100\rangle$	$\|100\rangle$
$\|101\rangle$	$\|101\rangle$
$\|110\rangle$	$\|111\rangle$
$\|111\rangle$	$\|110\rangle$

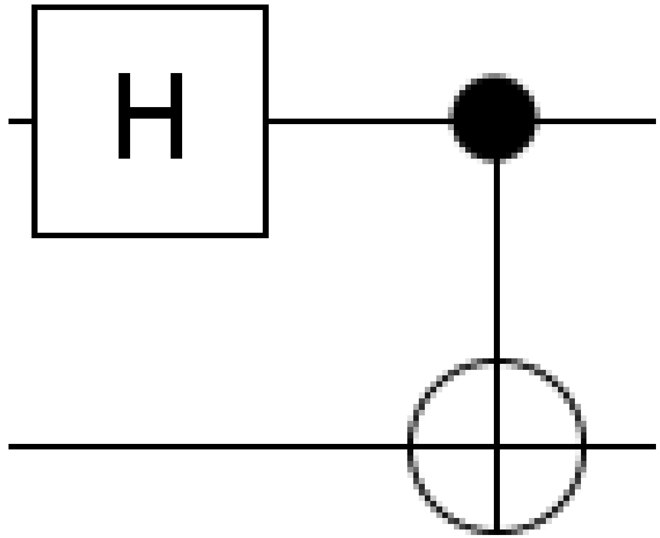
truth table.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

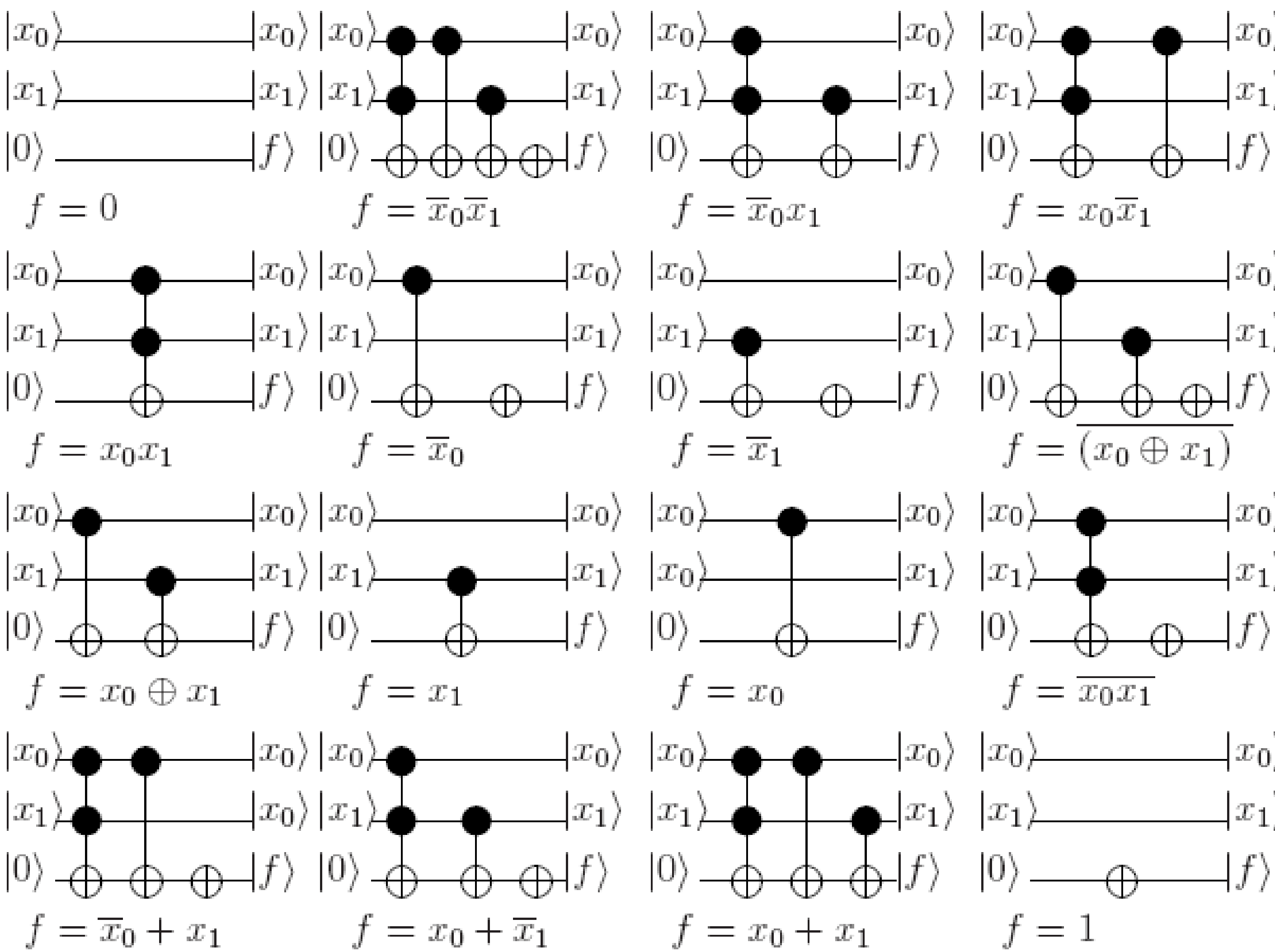
Controlled Swap Circuit (Fredkin Gate)



???

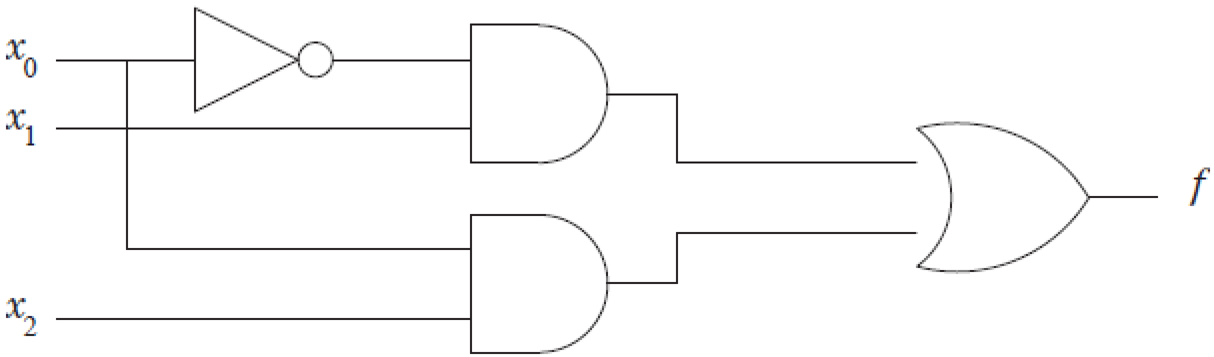

H

Two-qubits Boolean Circuits

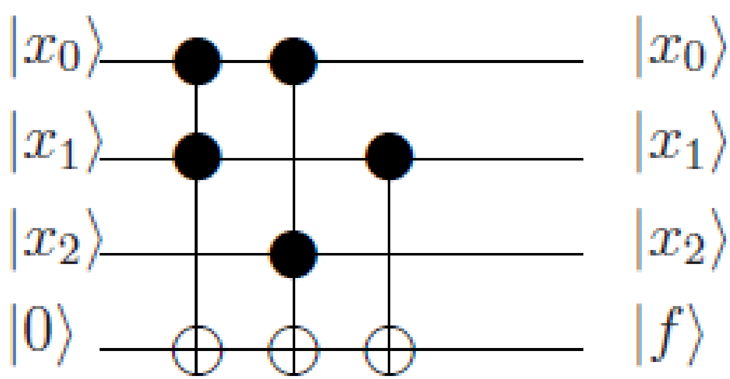


Boolean Quantum Circuits

$$f = \overline{x_0}x_1 + x_0x_2$$



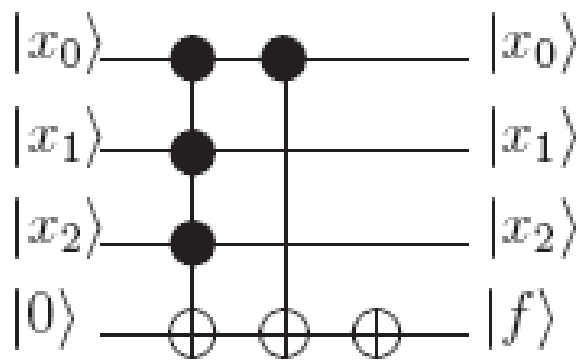
Digital circuit



Quantum circuit

x_0	x_1	x_2	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

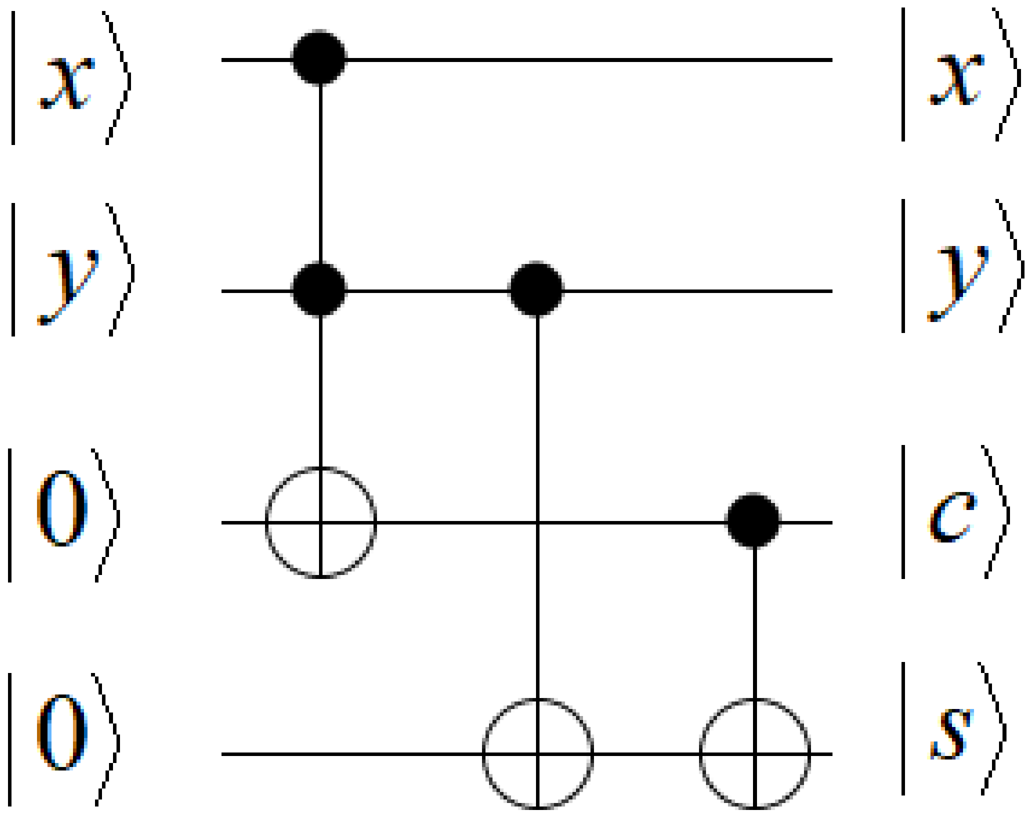
Boolean Quantum Circuits



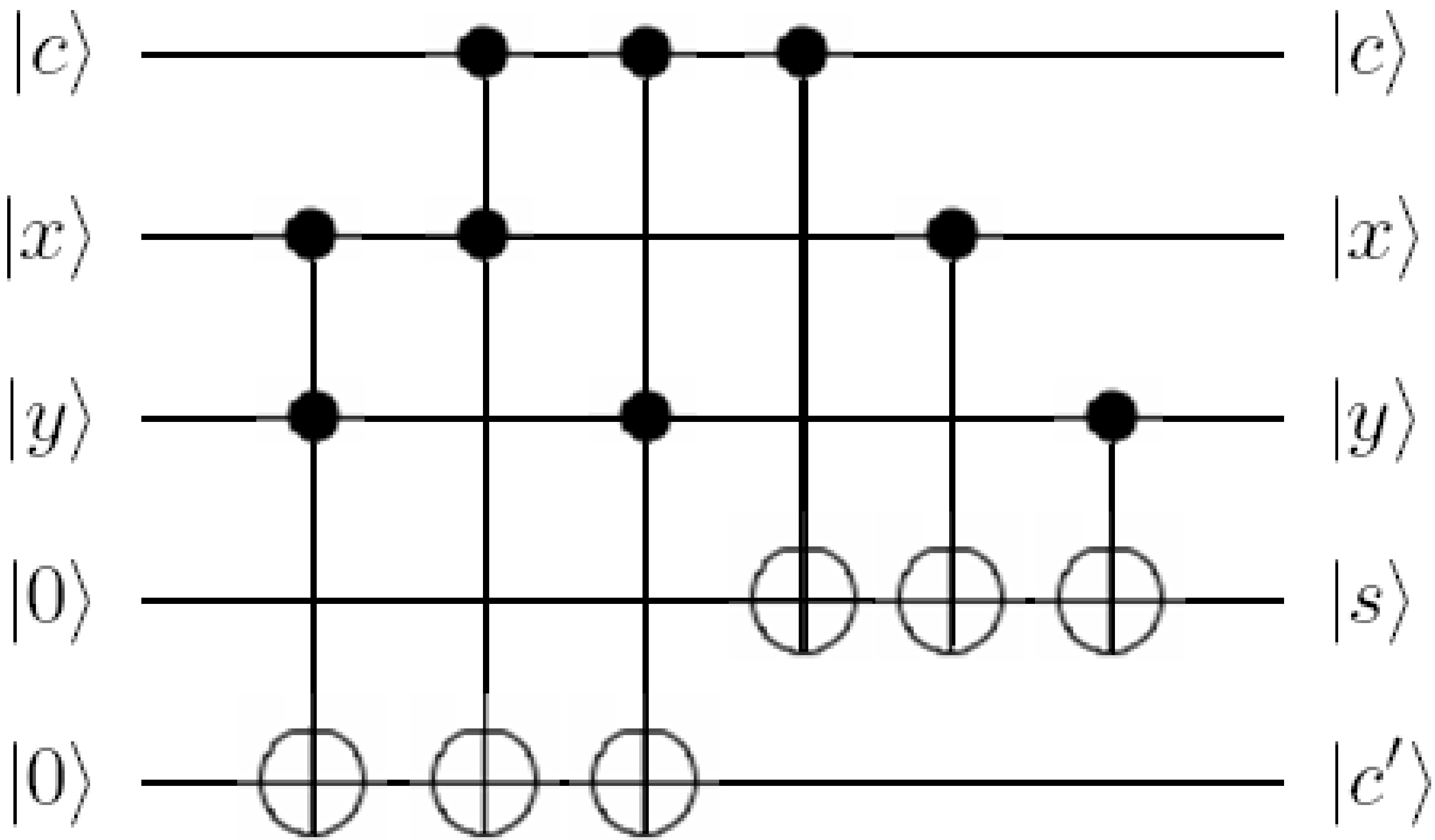
Quantum circuit implementation for $f(x_0, x_1, x_2) = \overline{x_0} + x_1 x_2$.

$$f(x_0, x_1, x_2) = \overline{x_0} + x_1 x_2 = x_0 x_1 x_2 \oplus x_0 \oplus 1$$

1-bit Half Adder



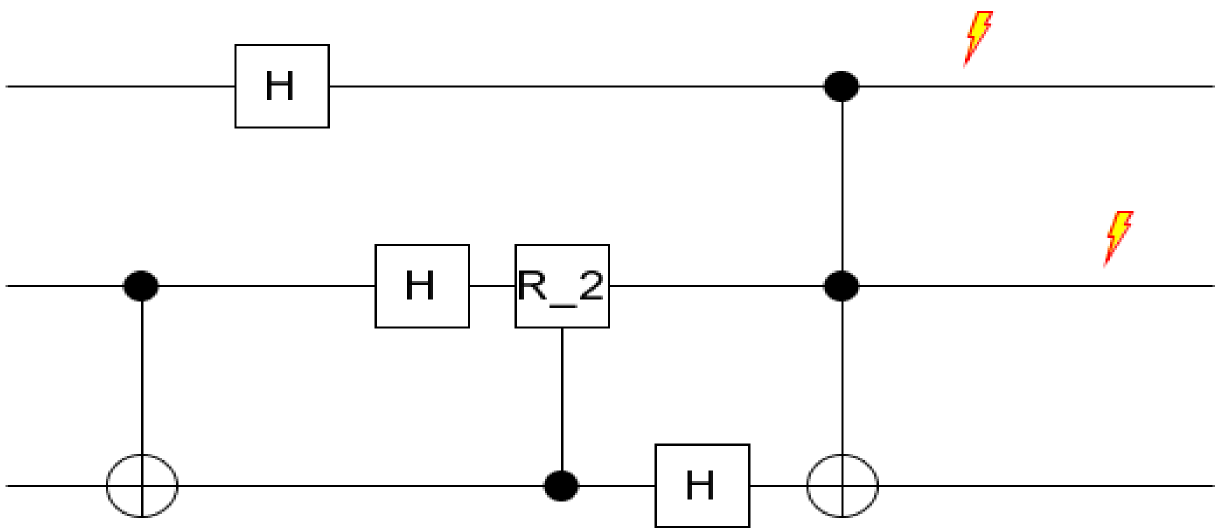
1-bit Full Adder



Let |c> = |1>, |x> = |0>, |y> = |1>
Then |s> = |0>, |c'> = |1>

Quantum Computation

- Quantum computation can be summarised as applying a sequence of transformations, called *quantum gates*, followed by a measurement.

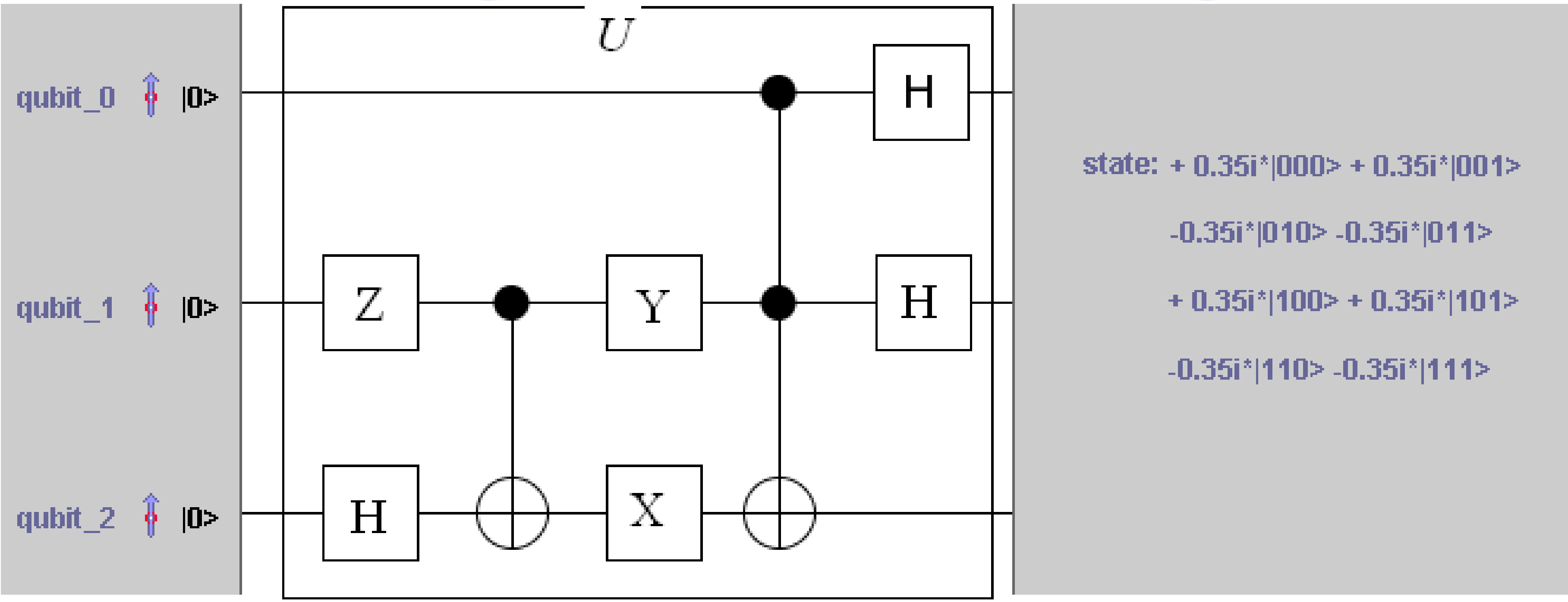


Input

Output

Set of Transformations (Gates)

Quantum Computation (cont.)



$$U = \left(H \otimes H \otimes I\right) T \left(I \otimes Y \otimes X\right)\left(I \otimes CNOT\right)\left(I \otimes Z \otimes H\right)$$

Thank you