Physical Cosmology and Galaxies II
Thermal History

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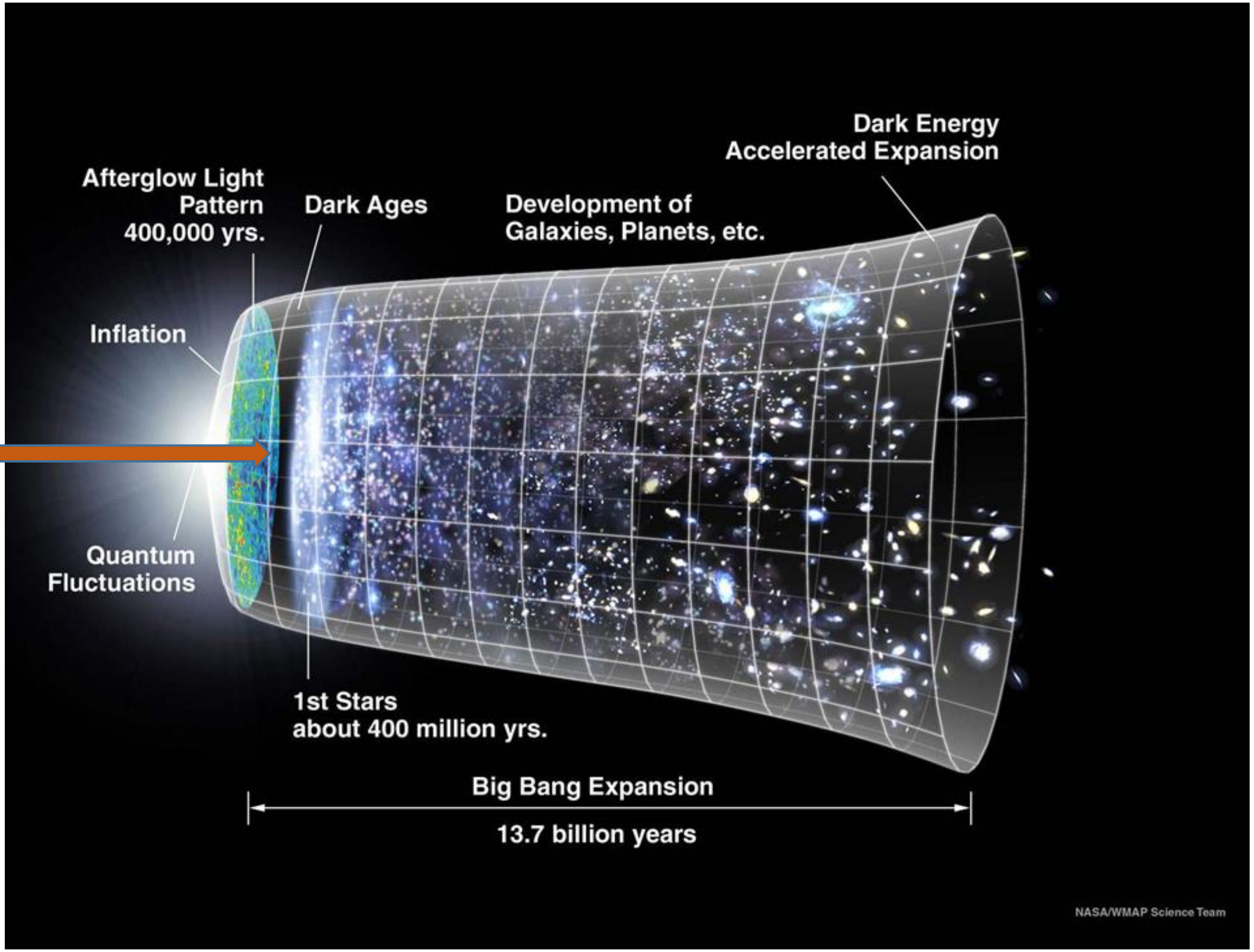
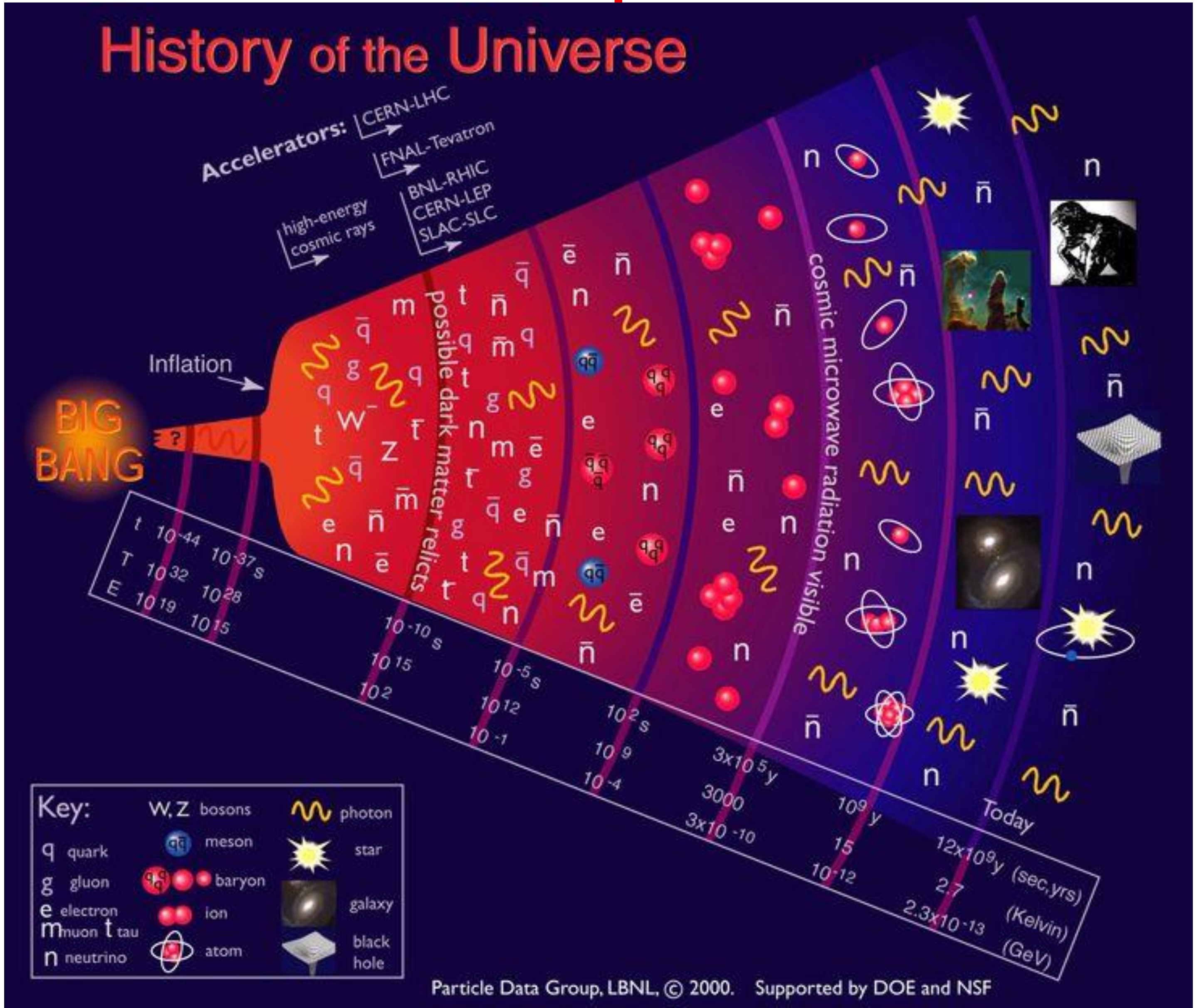
Google 'Cosmic History' → Images: Things Like

This talk



Google some more: A Thermal Bath of Particles and Antiparticles that Leaves Relics



Tightly coupled, highly interacting, system

Couple of refs , including detailed/proper treatments

Kolb & Turner: The Early Universe (standard text)
Classic text; Chapter 3 particularly useful

Daniel Baumann Tripos lectures Chapter 3
Similar notes are now on his Amsterdam website.
(which I follow to some extent)

Subject Matter

- Universe is expanding → Should have been **‘hot’, in equilibrium,** in past

→ As ***T*** rises:

- Atoms ionize

- Nuclei disassociate → individual protons neutrons → quarks-gluons

- SM phase transitions (electroweak, QCD) expected. Others (GUT) predicted

electroweak → **mass**; QCD → **nuclei**

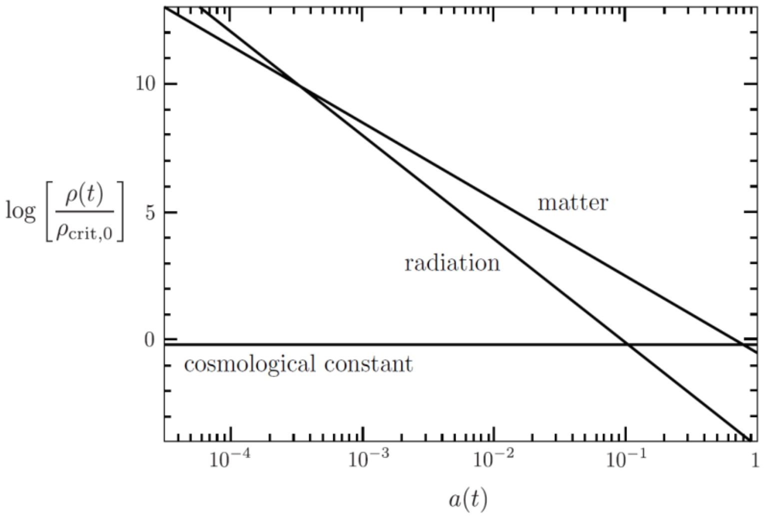
→ Universe is **testing ground for HEP (including dark matter models)**

Recall FRW Models and Eras

Today Talk → mostly Radiation Dominated <~ 50 000 yr

Our understanding is the universe went through the following phases

1- Vacuum domination and vast exponential expansion ('inflation')
2- Radiation domination
3- Matter radiation
4- 'Recent' vacuum donation (again)



Recall

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

Time evolution for flat Universe
(always true for early uni.)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

	w	$\rho(a)$	$a(t)$
RD	$\frac{1}{3}$	a^{-4}	$t^{1/2}$
MD	0	a^{-3}	$t^{2/3}$
ΛD	-1	a^{0}	e^{Ht}

$$\mathrm{d}\rho + \left(\rho + \frac{p}{c^2}\right)\frac{\mathrm{d}V}{V} = 0$$

$$p = w\rho c^2$$

Units and Estimates

- Using **'natural units': $c = \hbar = G = k_B = 1$**

- Temperature, energy, momentum and mass are in electron volts
- Length and time are in inverse electron volts

→ Radiation era expansion rate

$$H \sim T^2/M_{\rm pl}$$

Recall $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$

$$H \sim \sqrt{\rho}/M_{\rm pl}.$$

Uses **Stephan Boltzmann** law

reduced Planck mass

$$M_{Pl} = \sqrt{\frac{\hbar}{8\pi G}} = 2.43\ 10^{18}\text{GeV}$$

- **Already twiddle '~' sign reappearing!**

→we will be making mainly order of magnitude (factor ten) estimates

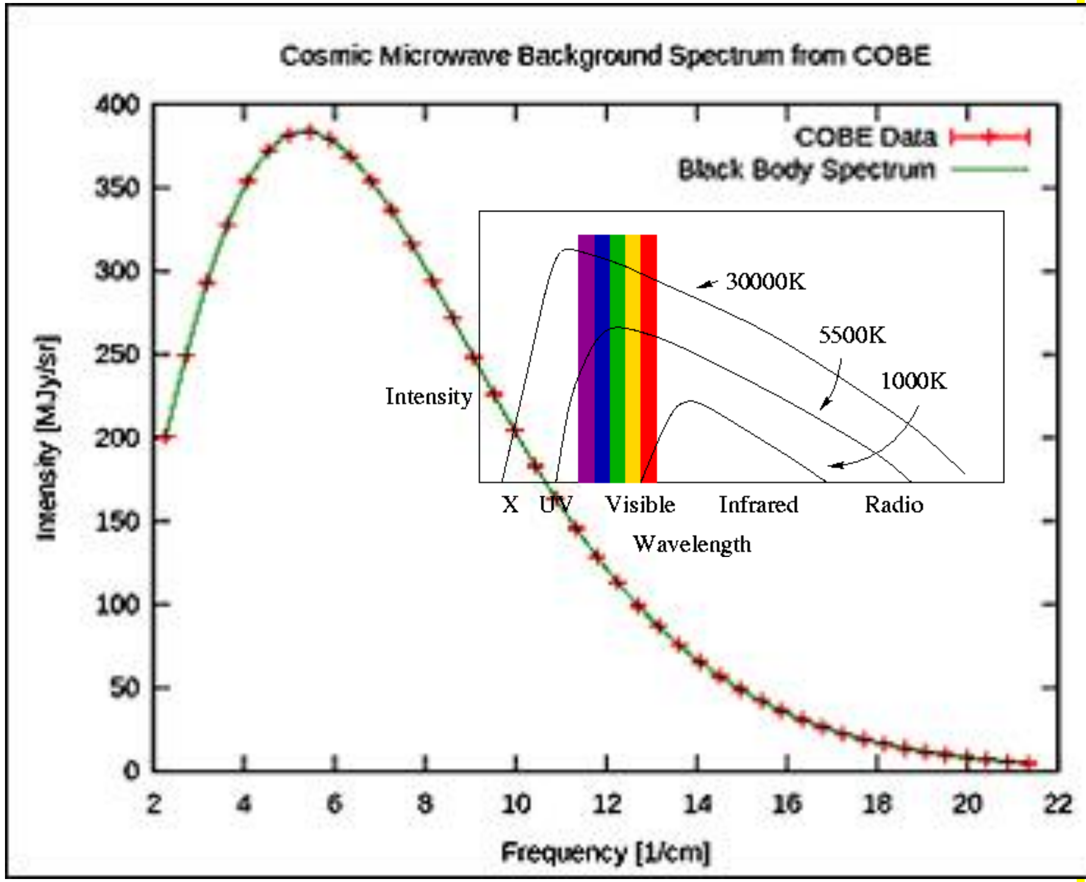
→As always, important in astrophysics/cosmology

The Cosmic Microwave Background: Tells of Prior Thermal Equilibrium

- **Current temp.** of spectrum: **2.728 Kelvin ~ $2.4\ 10^{-4}\mathrm{eV}$**
- **Current energy** density of CMB:

$$(4\sigma/c)T^4 = 4.19 \times 10^{-14}\ \mathrm{J\ m^{-3}}$$

$$\rightarrow\ 2.6\ 10^{5}\mathrm{eV\ m^{-3}}$$



- **Energy of 'typical' photon** $E = h\nu \sim kT$

(since distn ~ $\nu^3 e^{-\frac{E}{kT}}$)

→ photon number density ~ $10^{9}\mathrm{m}^{-3}$ $(4\ 11\ 10^{8}\mathrm{m}^{-3})$

Compare with < one proton per cubic meter!

Photon Entropy ~ $n_\gamma a^3$ dominates. Small ratio $\frac{n_b}{n_\gamma} \equiv \eta \sim 10^{-9}$ Conserved

Ex.: use Stephan-Boltzmann law + current matter mass density, $3\ 10^{-27}\ \frac{\mathrm{kg}}{\mathrm{m}^3}$, to obtain T_{eq} of matter rad. equality ($\sim 10^4 K$)

Thermal Equilibrium and the Notion of Temperature

Off Boltzmann's tombstone $\quad S = k \ln \Omega$

Independent probabilities $\quad \Omega = p_1 p_2 \dots p_N = \prod_{1}^{N} p_i$

$$S = -k \sum_i p_i \ln p_i \quad \Rightarrow \quad dS = -k \sum_i (1 + \ln p_i) dp_i$$

Constraints

$$\sum p_i = 1 \quad \longrightarrow \quad \sum_i dp_i = 0$$

$$\sum p_i E_i = U \quad \longrightarrow \quad \sum_i E_i dp_i = 0$$

Condition

$$dS = -k \sum_i \ln p_i dp_i = 0 \quad \longrightarrow \quad \boxed{\ln p_i = \alpha - \beta E_i}$$

$$\alpha \equiv Z = \sum_{1}^{N} e^{-\beta_i E_i}$$ normalizing partition function

Allow for small change in energy

$$dS = -k \sum_i dp_i (\alpha - \beta E_i) = k\beta \sum_i dp_i E_i = k\beta dq_{rev}$$

$$dS = dq_{rev}/T \quad \Rightarrow \quad \boxed{\beta = 1/kT}$$

Number densities in Thermal Equilibrium

- Spatially homogeneous system with phase space density *f(p)* →

$$\mathrm{d}\,n = g\, f(p) dp_x dp_y dp_z \;\rightarrow\; n = 4\,\pi\, g \int f(p) p^2 dp$$

(isotropic momenta and number of internal deg. freed., e,.g. spin, *g*)

$$f(p) \sim \frac{1}{e^{\frac{E(p)}{T}} \pm 1}$$

$$n = 4\,\pi\, g \int_0^\infty \mathrm{d}p \frac{p^2}{\exp\left[\sqrt{p^2+m^2}/T\right] \pm 1}$$

Chemical equilib. → particles created – annihilated so as to keep these distn →
Non-relativistic parts → more difficult to make → lose out and suppressed

Relativistic $(m \ll T)$ $$n \sim g\, T^3$$

Non-relativistic $(m \gg T)$ $$n \sim g\,(mT)^{\frac{3}{2}}\, e^{-\frac{m}{T}}$$

→ **As T → 0 a massive particles should vanish… !**

From above, 'Normal matter'; should vanish; it's existence suggests violations of baryon number and charge parity conservation
→ ***Baryogenesis*** → baryon asymmetry (probably BSM).
Note also: We have ignored the chemical potential in above distributions. It is 0 for photons and unimportant at high T. and sums to zero for particle0anti pairs (which may annihilate to photons). We will use heuristic arguments that circumvent its use when, strictly speaking, it is

Relativistic Degrees of Freedom g_*

A thermal particle is **relativistic if:** $(m \ll T)$

A particle is in thermal **equilibrium** if:
interaction rate with thermal bath **> > expansion rate**

Relativistic particles act as 'radiation'

The total **energy density of relativistic species** is (using Stefan-Boltzmann again in natural units)

$$\rho_r = \sum_i \rho_i = \frac{\pi^2}{30} g_\star(T) T^4$$

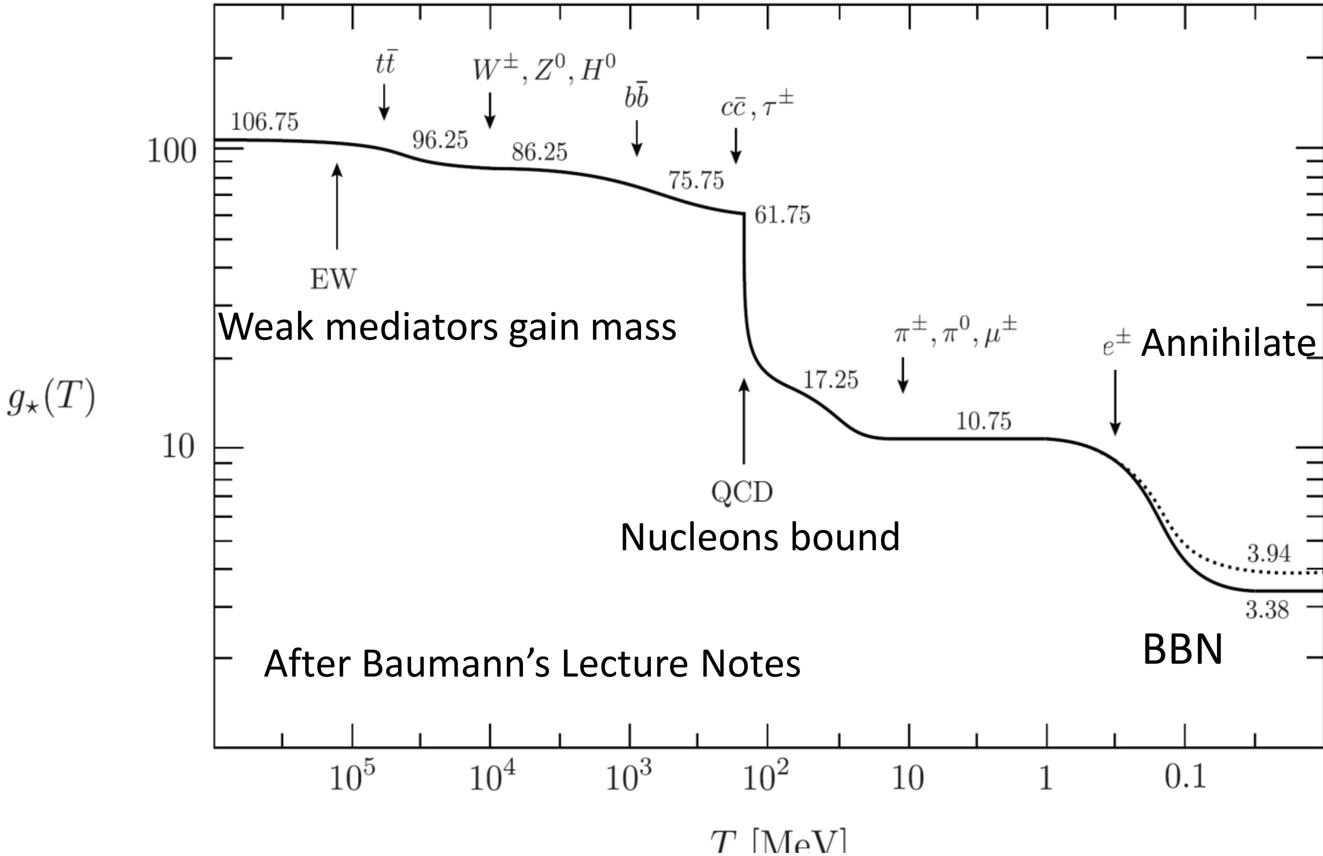
$$s = \sum_i \frac{\rho_i + P_i}{T_i} \equiv \frac{2\pi^2}{45} g_{\star S}(T) T^3$$

Expansion influenced by number of relativistic degrees of freedom (essentially number of species and their internal degrees of freedom; as spin)

$$\frac{T}{1\,\mathrm{MeV}} \simeq 1.5\, g_\star^{-1/4} \left(\frac{1\,\mathrm{sec}}{t}\right)^{1/2}$$

Number density of photons $n \sim \frac{1}{a^3} \sim T^3 \rightarrow T \sim \frac{1}{a}$

Including all relativistic d.o.f.: $T \propto g_{\star S}^{-1/3} a^{-1}$



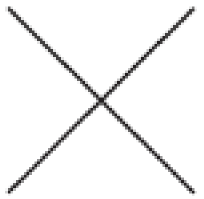
Annihilation → states transferred to photon bath → entropy Conserved

Neutrino Decoupling

- Neutrinos are **coupled to electrons through** **weak interactions** $G_F \sim \alpha / M_W^2 \sim 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$

Below electroweak scale (~100 GeV) ++ relativistic crossection
('four Fermion' interaction)

$\sigma \sim G_F^2\, E^2$

$\sigma \sim \Big|$  $\Big|^2 \sim G_F^2 T^2$

$n\, \sigma\, v_{rel}$ $\qquad\qquad$ H

Rule of thumb: decoupling → decouple when **interaction rate ~< expansion rate:**

time >~ time

(**recall** ***H ~ T²*** ***in rad era*** **and assumed** ***relativistic*** $n_e \sim T^3$) $\dfrac{\Gamma}{H} \sim \dfrac{\alpha^2 M_{\mathrm{pl}} T^3}{M_W^4} \sim \left(\dfrac{T}{1\ \mathrm{MeV}}\right)^3$

→ **When scales ~ 3 billion times smaller than today ~ 1 s after start of expansion**

Cosmic Plasma Coupling -- electromagnetic >> weak

- **Gas fully ionized** (and non-relativistic) → interacts with photons by **Thompson scattering**:

- Electron placed in EM field → $m_e \frac{d^2 z}{dt^2} = -e\, E_0 \sin(\omega\, t)$, → oscillates → radiates back

Crossection ≡ power radiated / mean incident energy flux **~ (classical electron radius)^2**

$$r_e = \frac{e^2}{4\pi\, \epsilon_0\, m_e\, c^2} = 2.82 \times 10^{-15}\,\mathrm{m}$$

$$\sigma_T \approx 2 \times 10^{-3}\,\mathrm{MeV}^{-2}$$

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

Photon
Interaction rate $= n_e\ \sigma_T\ v_{rel}$

(note relative vely *c = 1* here!)

Coupling in Ionised System

Again... **Interaction rate > expansion rate** → **Thermal Coupling**

(interaction time < age of universe)

++ $n_e \sim n_b \sim 10^{-9}\, n_\gamma \sim 10^{-9} T^3$ →

$$10^{-9} T^3 * 2\, 10^{-15}\, \mathrm{eV}^{-2} \sim \frac{T^2}{M_{\mathrm{pl}}}$$

$$\boldsymbol{n_e} \qquad \boldsymbol{\sigma_T} \qquad \sim \qquad \boldsymbol{H}$$

→ $T_{\mathrm{coupling}} \sim 10^{-3} \mathrm{eV} \sim 10\ \mathrm{K}\,!$ →

This lies well within matter domination

$$T_{eq} \simeq 1\ eV$$

→ Should use appropriate H-scaling; but would change main conclusion little!

Recombination: Era of Tightly Coupled Plasma Ends

- **But When?**

Boltzmann factor $e^{-\frac{B_H}{T}}$ $\left(B_H = 13.6\ \mathrm{eV}:\ \text{Hydrogen's binding energy}\right)$

→**Probability of electron meeting ionizing photon**

→ Ionising Rate vs Expansion rate

$$n_\gamma \sigma_T\, e^{-\frac{B_H}{T}} \sim H \rightarrow T^3\, \sigma_T\, e^{-\frac{B_H}{T}} \sim \frac{T^2}{M_{\mathrm{pl}}} * \left(\frac{T_{\mathrm{eq}}}{T}\right)^{\frac{1}{2}}$$

→ $T\, e^{-\frac{13.6\ \mathrm{eV}}{T}} \sim 2\ 10^{-13} \left(\frac{\mathrm{T_{eq}}}{\mathrm{T}}\right)^{\frac{1}{2}}\ \mathrm{eV}$ → $T_{\mathrm{rec}} \simeq 0.47\ \mathrm{eV}$

Proper calculation → 0.3 eV (e.g., Bauman's lecture notes)

(0.3 eV)

3600 K→ a (rec) =1/ 1300 → z (rec.) = 1300 → t (rec) ~ 300 000 yr for $a(t) = (t/t_0)^{2/3}$

Cosmological Element Production (BBN)

- **Elements beyond hydrogen need neutrons**, which are in **equilibrium with protons until weak scale freeze out:**

$$n + \nu_e \leftrightarrow p^+ + e^-$$
$$n + e^+ \leftrightarrow p^+ + \bar{\nu}_e$$

→ $$\left(\frac{n_n}{n_p}\right)_{\rm eq} = e^{-Q/T} \qquad Q \equiv m_n - m_p = 1.30 \text{ MeV}.$$

****At weak freeze out** (~ 1 MeV, as we saw) **neutron fraction ~ 1/6**

**** Elements cannot form until Boltzmann suppression ~ $10^9 e^{-\frac{B_D}{T}}$ overcome at ~ 0.1 MeV** (**as in CMB recombination at ~ 0.3 eV)**

→ **Neutrons decay till binding energy (B_D) bottleneck passed** → $\frac{n_n}{n_p}$ **-> ~ 1/8**

** ~ all neutrons go to (energetically favoured) Helium (once *'D bottleneck'* overcome – only two body interactions possible))

→ **abundance of Helium nuclei (2 neutrons each)** ~ $\frac{n_{\rm He}}{n_{\rm H}} \sim \frac{1}{16}$ → **by mass 1/4**

****Heavier elements absent due to 'delay' → low densities (process ends after ~ three min… estimate it!)**

Of BBN and BSM (earliest empirical relic yet)

**Dependence on baryon dens. →

Non-Baryonic Dark Matter dominant

** Dependence on expansion rate → number of relativistic species (with m << T)**

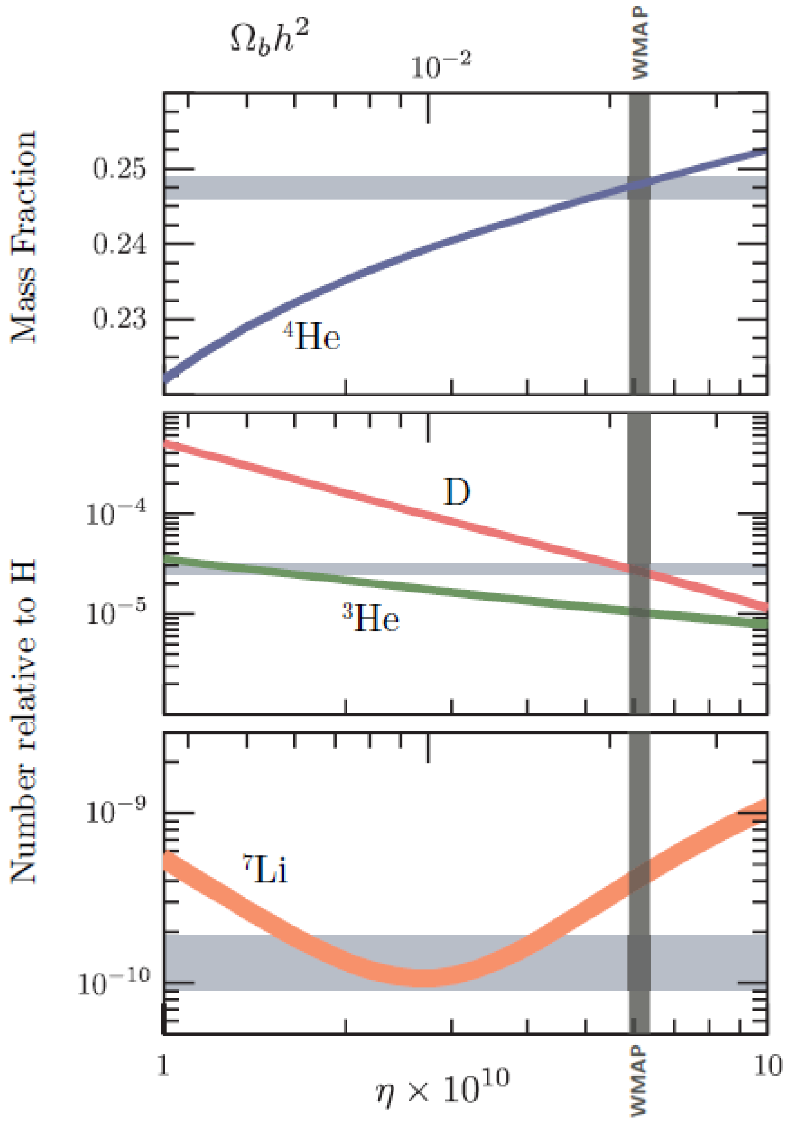
(Recall the expansion rate $H^2 \sim \rho \sim g_*$)

→puts bounds on neutrino species

(and any other relativistic species prior to T~MeV)

**Places constraints on G and G_F at early times

++ Constraints on non-standard cosmology



Vertical line Baryon fraction ~ 5 %

What is the DM: A Thermal WIMP Miracle?

Assume DM is composed of weakly interacting particles

Freeze out interaction rate ~ expansion rate → $\frac{\Gamma}{H} \sim \frac{\alpha^2 M_{\rm pl} T^3}{M_W^4} \sim \left(\frac{T}{1\ \rm MeV}\right)^3$ for relativistic particle --- m_{DM} < Mev

→'Freeze out' abundance ~ photons → mass density relative to protons $\sim \frac{10^9 m_{DM}}{1\ \rm Gev}$ -> Huge, unless *m tiny*

- Number Density *that matches measured* $f_{DM} = \frac{\Omega_{DM}}{\Omega_b} \simeq 5$ is $n_{DM} \sim f_{DM} \frac{1\ \rm GeV}{m_{DM}} 10^{-9}\, T^3$

Non relativistic limit: *Boltzmann suppression* σ → *constant crossection*

→ Decoupling Condition $n_{DM}\, \sigma\, v \sim f_{DM} 10^{-9}\, T^3 v_{rel}\, \sigma \sim \frac{T^2}{M_{pl}}$ with $v_{rel} \sim \left(\frac{T}{m_{DM}}\right)^{\frac{1}{2}}$

$$\frac{m_{DM}}{T_{dec}} \sim \left(10^{-9}\, f_{DM}\ \ \sigma\ \ M_{\rm pl}\right)^{\frac{2}{3}} \sim \left(10^{-9}\ \frac{f_{DM}}{5}\ \frac{\sigma}{10^{-8} GeV}\ \ M_{\rm pl}\right)^{\frac{2}{3}} \sim 24$$

The 'Miracle': Non-relativistic equilibrium $\frac{n_{DM}}{n_\gamma} \sim \left(\frac{m_{DM}}{T}\right)^{\frac{3}{2}} e^{-\frac{m_{DM}}{T}}$

For $\frac{m_{DM}}{T_{dec}}$ as above → right abundance if m_{DM} ~ 100 GeV → weak scale!

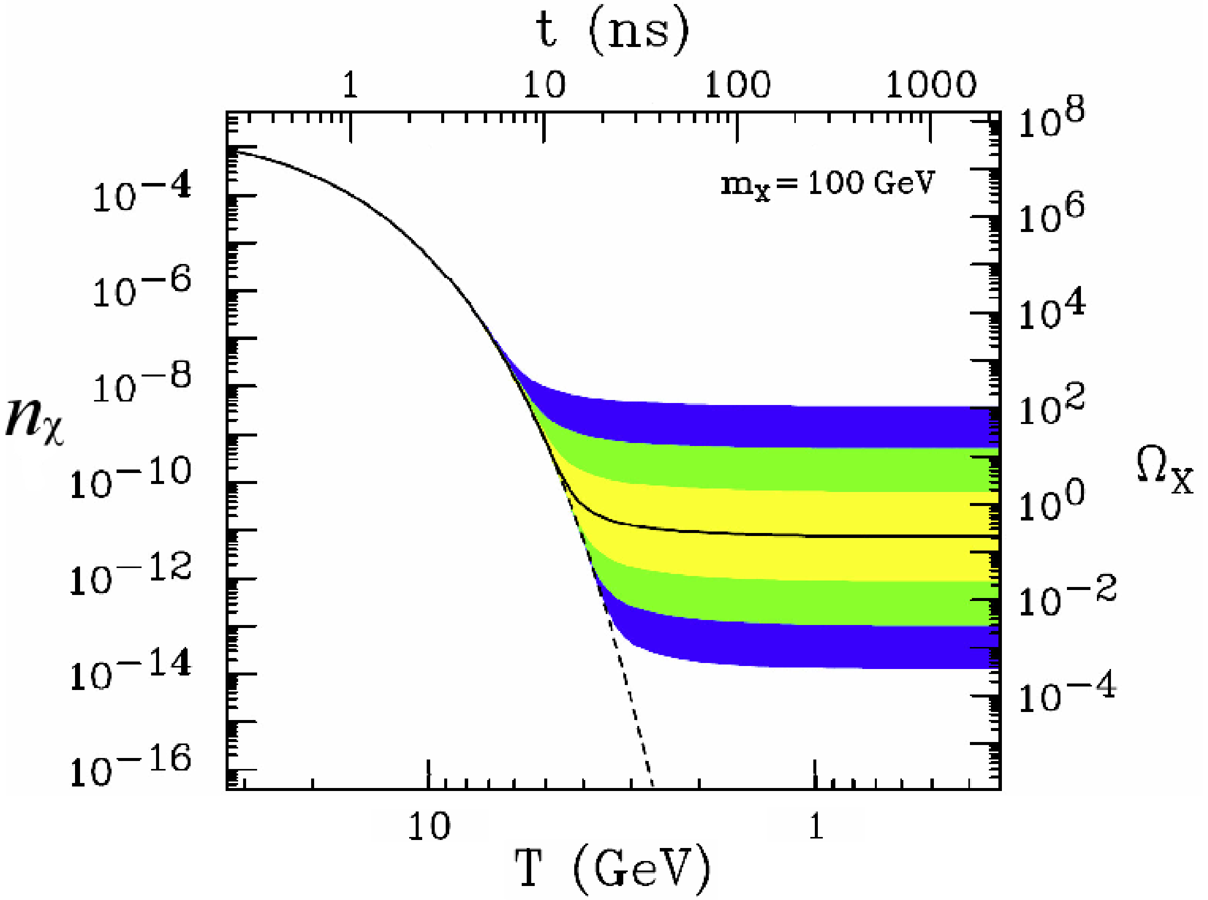
- More sophisticated?
- **Use Boltzmann equation**

for *comoving* number density of DM candidate X

$$\frac{dN_X}{dt} = -s\langle\sigma v\rangle\left[N_X^2 - (N_X^{eq})^2\right]$$

(s ~$1/a^3$)

- **Abundance suppressed right way →**

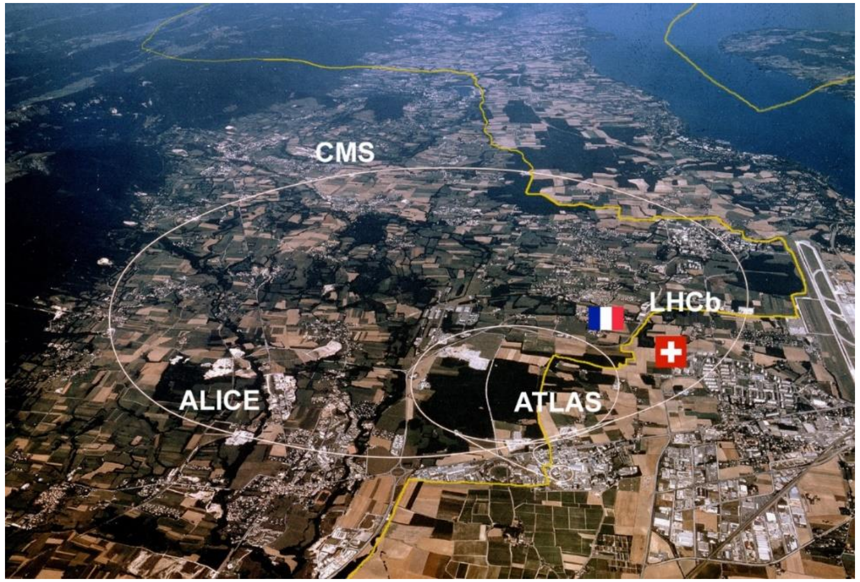


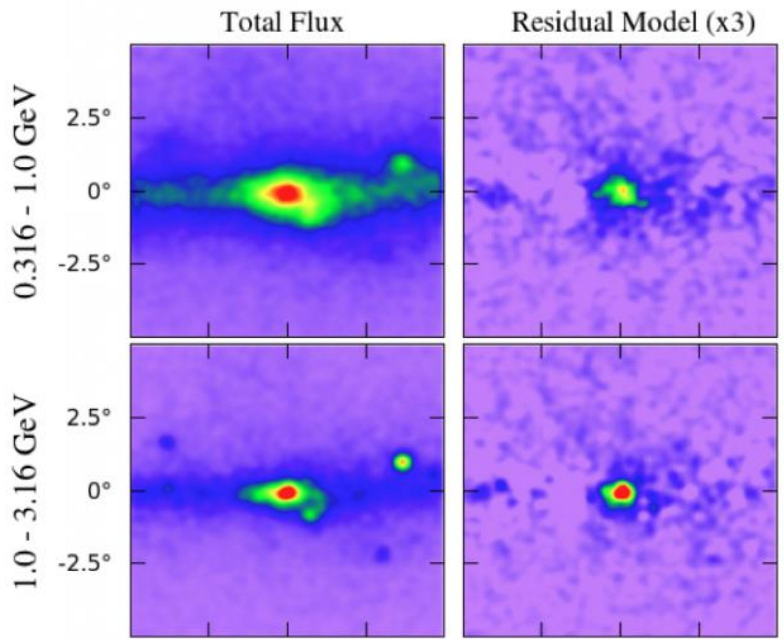
....

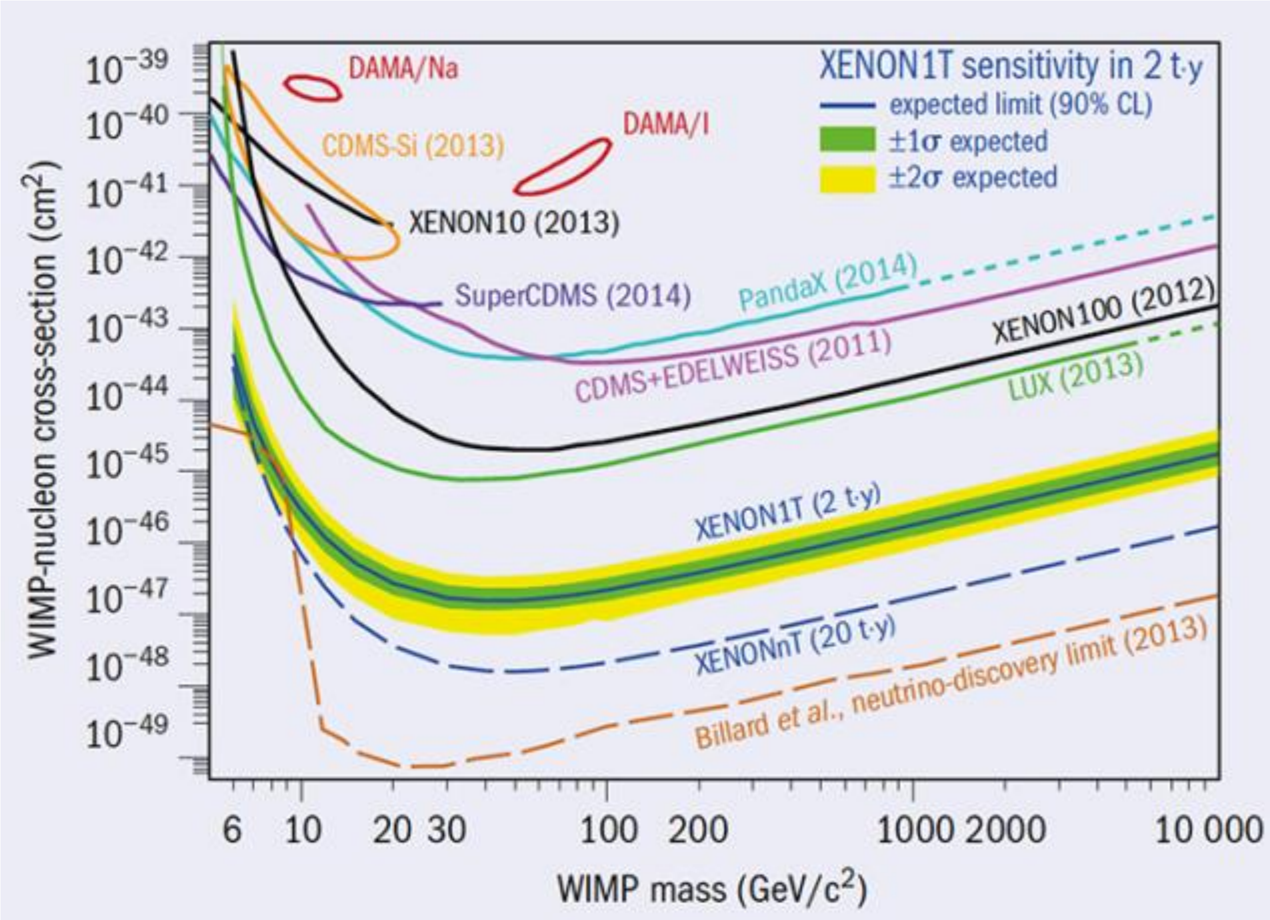
Note: 1) $\langle\sigma v\rangle$ 'thermally averaged' over a Maxwellian; $\langle\sigma\, v\rangle^{\frac{1}{2}} \sim 0.1\, G_F$ -> characteristic of weak scale.

Searching for WIMPS

- **Direct Detection** experiments (DM in the room!)
- **LHC** (at CERN)
- **Annihilation Signals** (in the sky)







Direct detection constraint (CERN Courier)

$Cm^2 \sim 4 * 10^{-28}\ GeV^{-2}$

Experimental constraints → WIMP miracle: waning and withering?

(Also appears withering at LHC…)

Some Alternatives

- **Sterile neutrinos** (can be produced from oscillations with regular ones)

→'**Warm dark matter**' in keV range

- **Axions** (introduced to solve CP violation problem in QCD and also in string theory --- currently topical 'fuzzy dark matter')

→Tiny mass and different production mechanism --- can lead to quantum wave effects

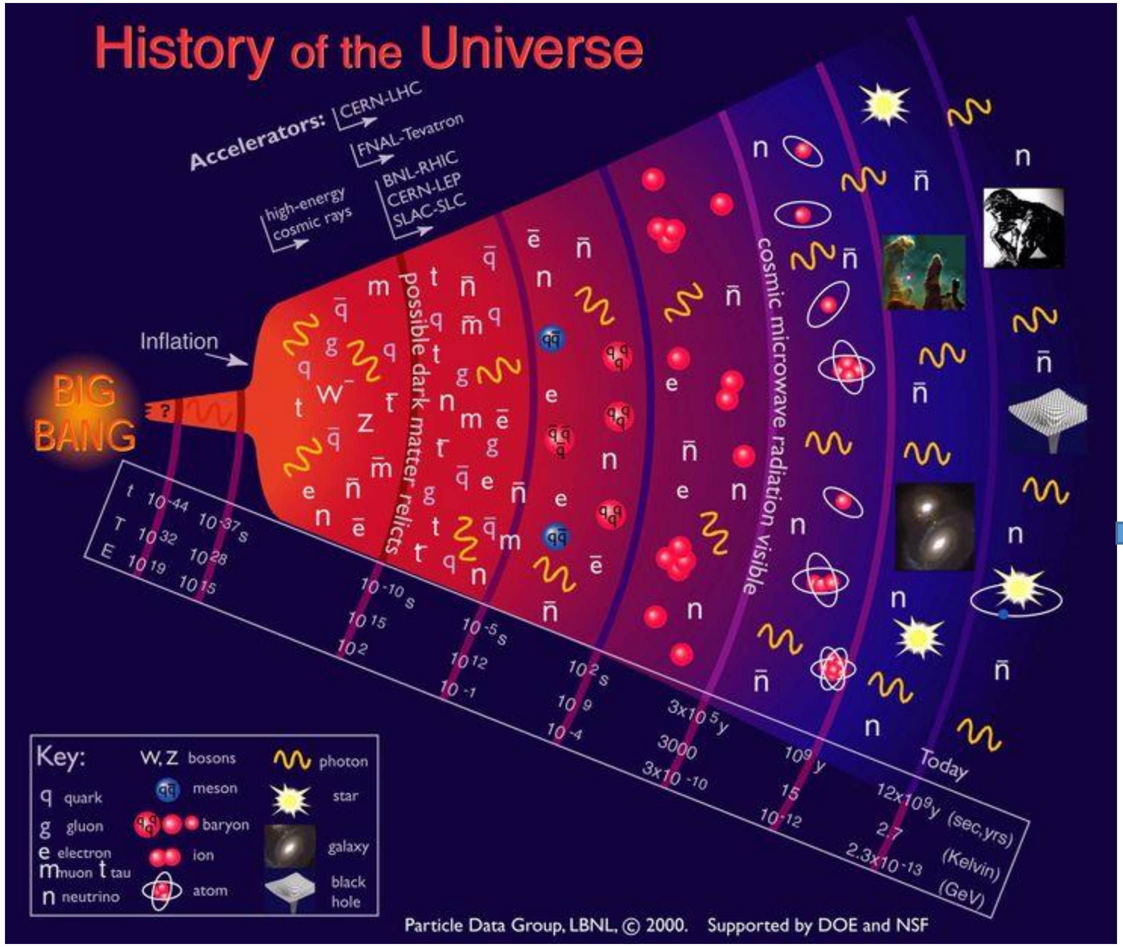
- **Non-thermal production of WIMPS or WDM**

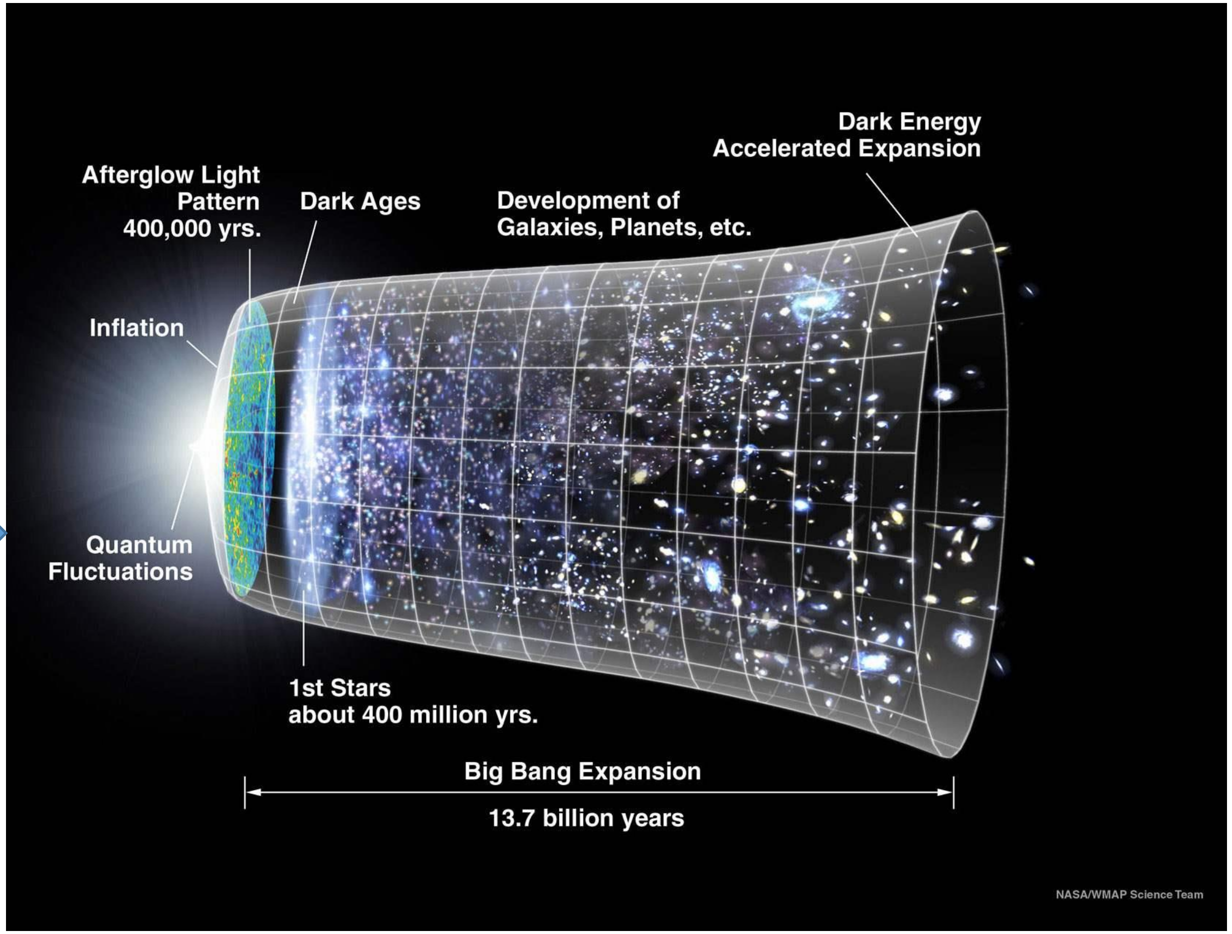
e.g., from direct decay of Inflaton like field → escapes thermal constrains.

Normally combined with 'entropy production' (decay of field into relativistic particles) which can adjust expansion rate and thus the DM abundance (diluting it)

Constrained by **BBN and CMB**

→++ **Large large scale distribution and statistical characteristics of galaxies and clusters**


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relics
cosmic microwave radiation visible
Today
(sec,yrs)
(Kelvin)
(GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Overview of Evolution

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

From lecture notes by Daniel Baumann