Physical Cosmology and Galaxies
Summer Internship Programme
July 2021

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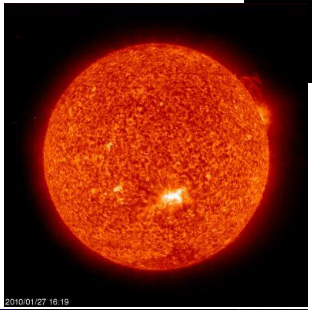
Useful refs:

- Liddle: Introduction to Cosmology (Focus on Newtonian background cosmic evolution)
-Mo, van den Bosch & White: Galaxy Formation and Evolution (a modern cosmological context)
- Ferreira: Lectures on General Relativity and Cosmology (simple intro with essentials)
http://wwwastro.physics.ox.ac.uk/~pgf/B3..pdf
- Peacock: Cosmological Physics (Newtonian + GR).
- Peebles: principles of Physical Cosmology (a classic with careful treatment; bit outdated)
- Weinberg: Cosmology (advanced, first chapter, to p 100, quite useful).
- Ryden: Introduction to cosmology (covers a lot at a reasonably simple level)

Cosmic Distance Scale

- Earth-Moon ~ 1 light second



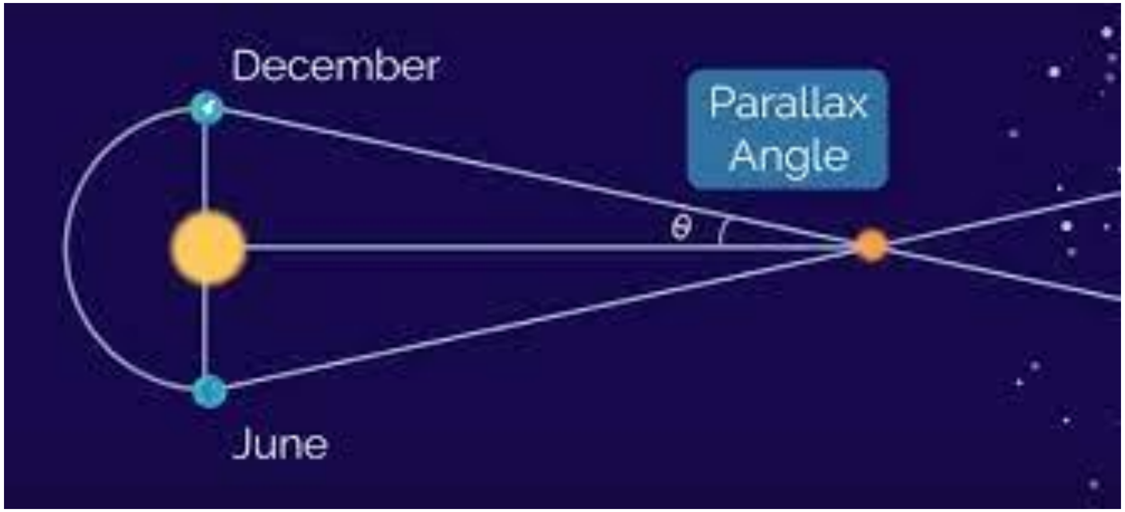
- Earth-Sun ~ 8 light minutes

- Nearest stars

>~ few light yrs ~ parsec

What is a parsec?

- Useful intro into diameter distance
- Distance at which earth orbit round sun (1 AU) subtends an angle ϑ of **1 arcsecond**



parsec = pc

kpc = 1000 pc

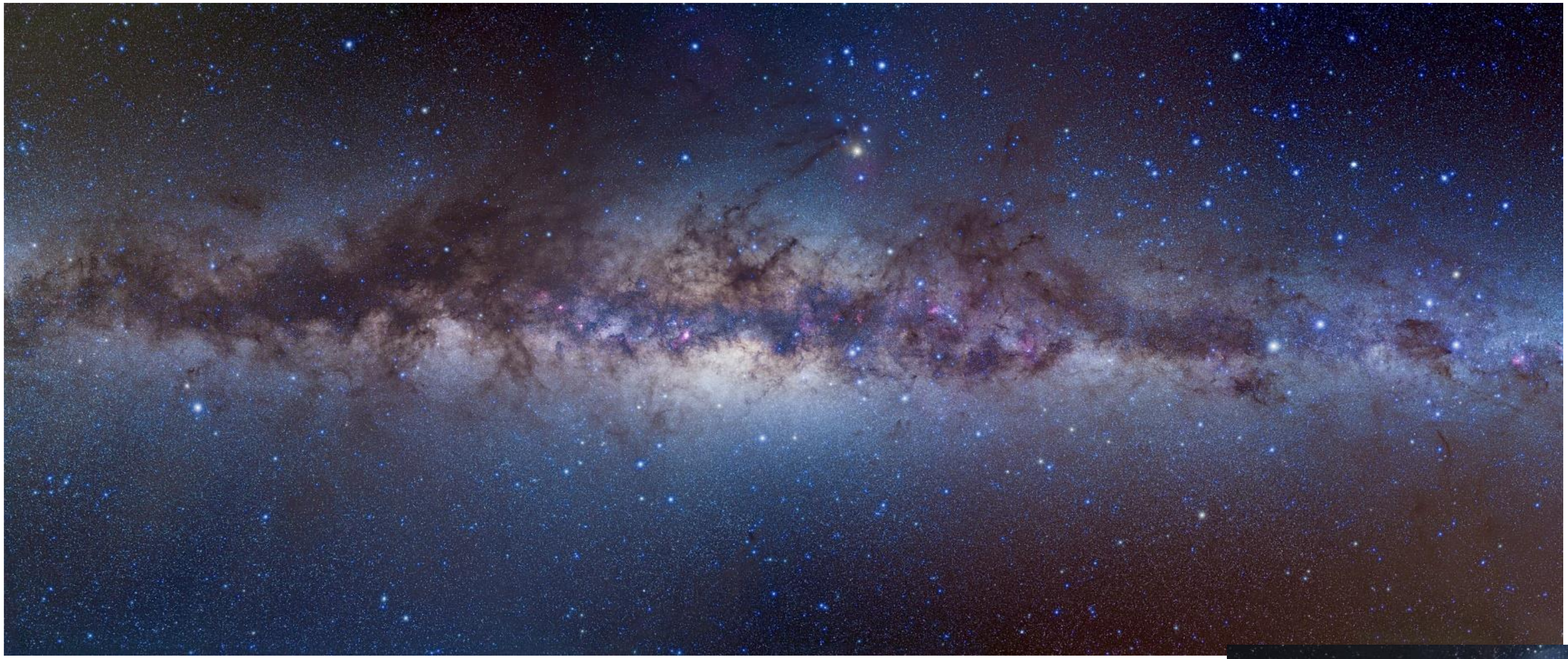
Mpc = 1000 000 pc

Gpc = 1000 000 000 pc

Exercise: Find that 1 pc = 3.26 light year

The Milky Way Galaxy

Distance from sun to centre ~ 25 000 light years (8 kpc)
Farthest individual stars seen by naked eye ~ 1000 light years



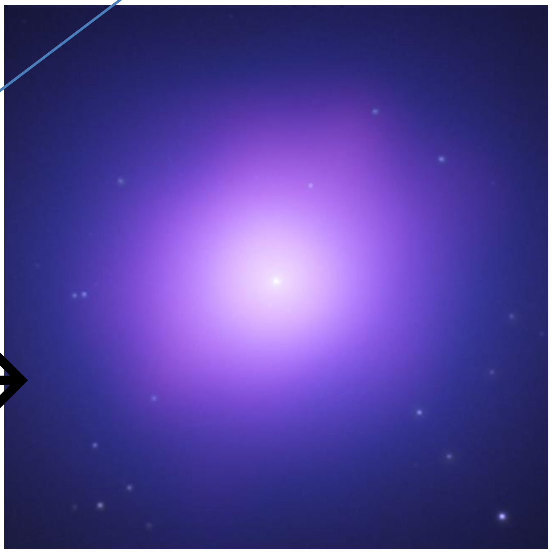
The (Local) Galaxy Population

Stars + Gas + Dust (+ Dark Matter)

- May be disk or spheroid,; contain lots of gas or little; have active star formation or not. Some have quasars.

Random , 'pressure' support

Rotational support



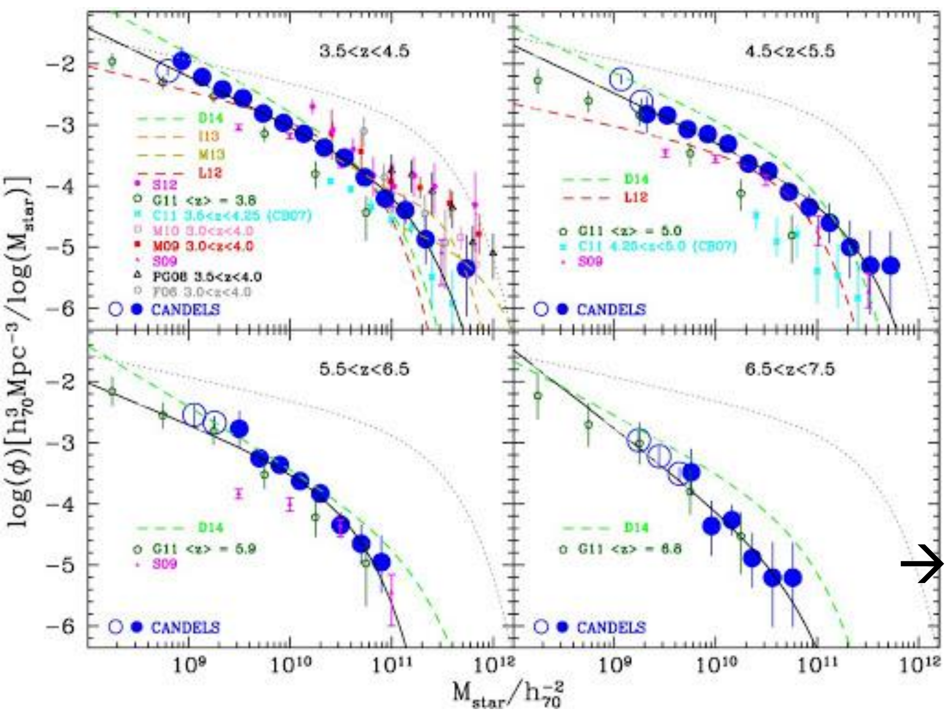
Younger stars with heavy elements

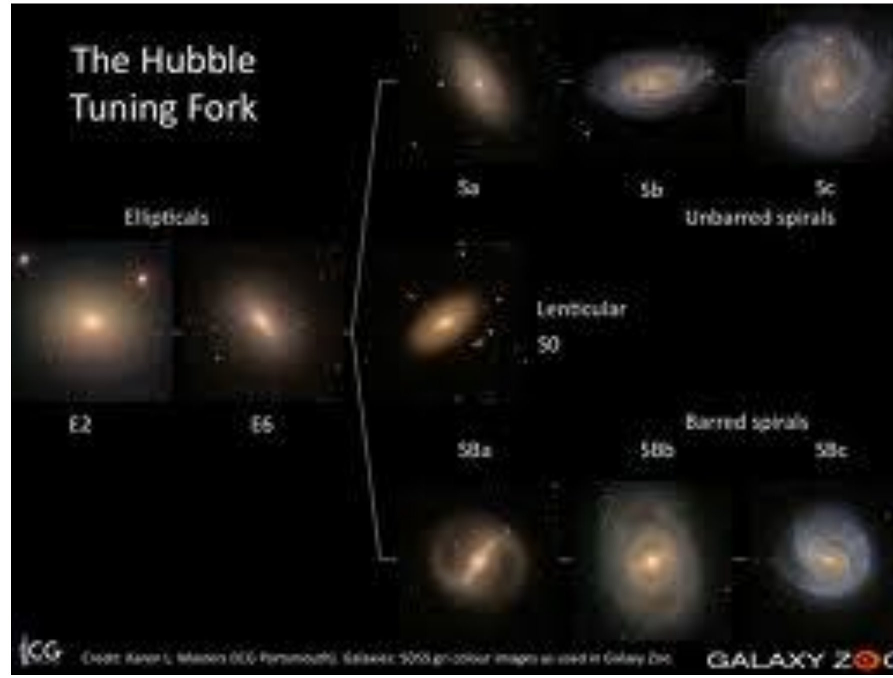
Old and poor stars

Need to Explain

- Shapes, sizes, masses of galaxies
- Distribution on sky
- Content

- evolution of above





→**Early evolution of galaxy mass function**

(Grazian et. al. 2015)

Within a cosmological model for the evolution of Universe! Using known laws of physics

GRAVITY GOVERNS

VERY WEAK → Long time scales

BUT

- ONLY ATTRACTIVE (NO POSITIVE AND NEGATIVE)
- LONG RANGE

→ WINS ON COSMIC SCALES

- Makes and holds together stars and galaxies and determines the cosmological evolution

Dynamical (virial) Equilibrium

$$\frac{GM^2}{R} \sim M\left\langle V^2 \right\rangle \sim Nk_B T$$

Note; **Negative specific heat**: **E ↓ → R ↓ → V ↑ → T ↑ (if thermal)**

Exercise: Find dynamical cal (crossing) time R/v (density)

The Peculiar force of gravity →

No standard thermal equilibrium

→ Higher entropy states → more inhomogeneous

Galaxies are not Relativistic

- By shrinking -- or increasing mass – pot. energy can become arbitrarily negative → equ implies $v_{\max} \to \infty$

- In GR → BH forms before ∞!

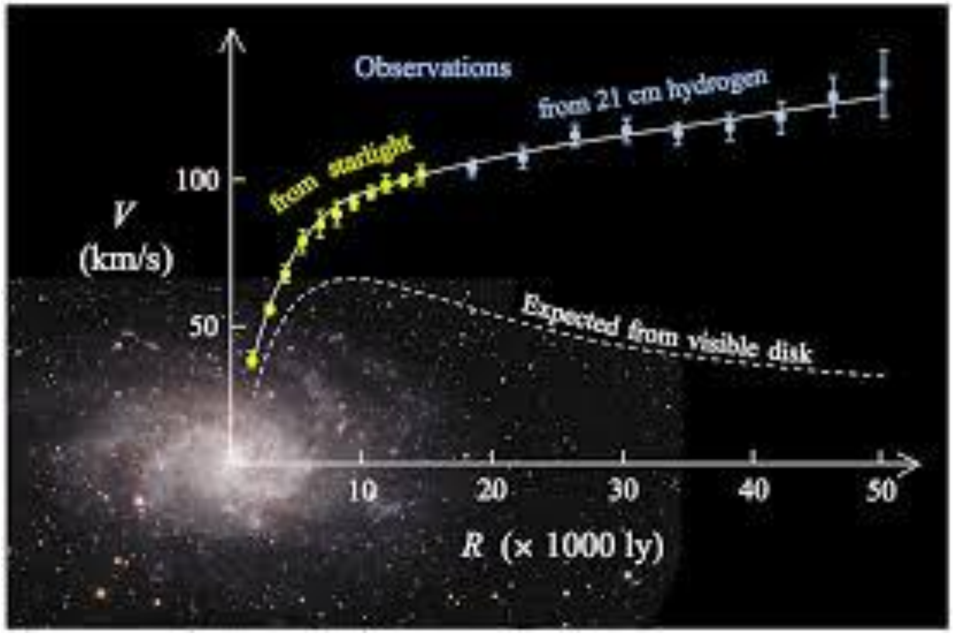
- In practice , in galaxies,

$$\frac{GM}{Rc^2} \sim \frac{P}{\rho c^2} \sim 10^{-6}$$

- Because: **System fragments into stars before cooling catastrophe complete → collisionless (no cool)**

++ Dominant non-dissipative component?

GALACTIC CHARACTERISTICS



- **Average Density** ~ 10^{-24} kg/m^3 (larger near centre)
- Compare with 5000 for Earth and 1 kg/m^3 for air
- **Time scales** ~100 million years; speeds ~10-100 km/s
- **Mass scale** ~ 10^7 to 10^{13} solar masses **(thermally governed)**
- **Most mass** (particularly in outer regions) **dark**
- **Nearest** large **galaxy** > Million light years ~ **Mpc**

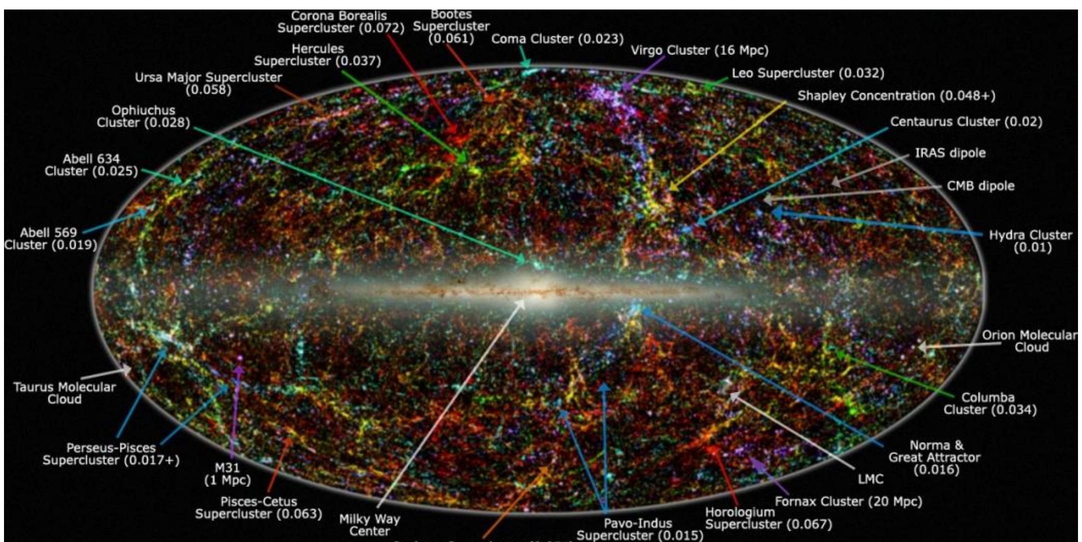
Larger scales (and back in time)

- **CLUSTERS OF GALAXIES**

1-10 Million light years ~ Mpc

- **LARGE SCALE STRUCTURE**

> few 100 Mil LY ~ 100 Mpc



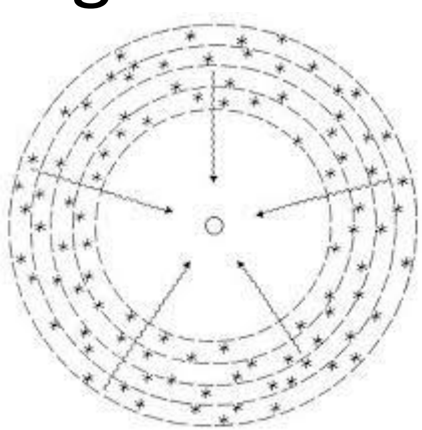
TOWARDS THE HORIZON

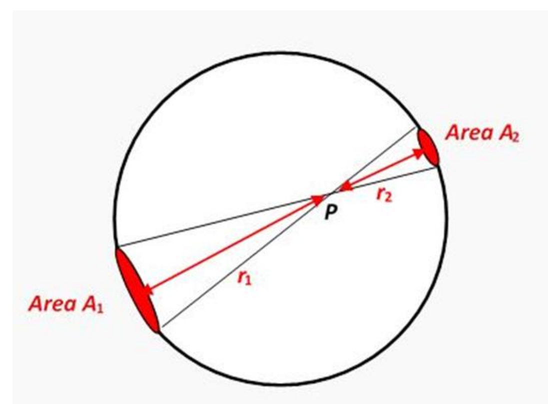
~14 Gpc and ~ 14 Gyr

Why's the Night Sky Dark?

Olber's Paradox (from Kepler to Edgar Allan Poe!)

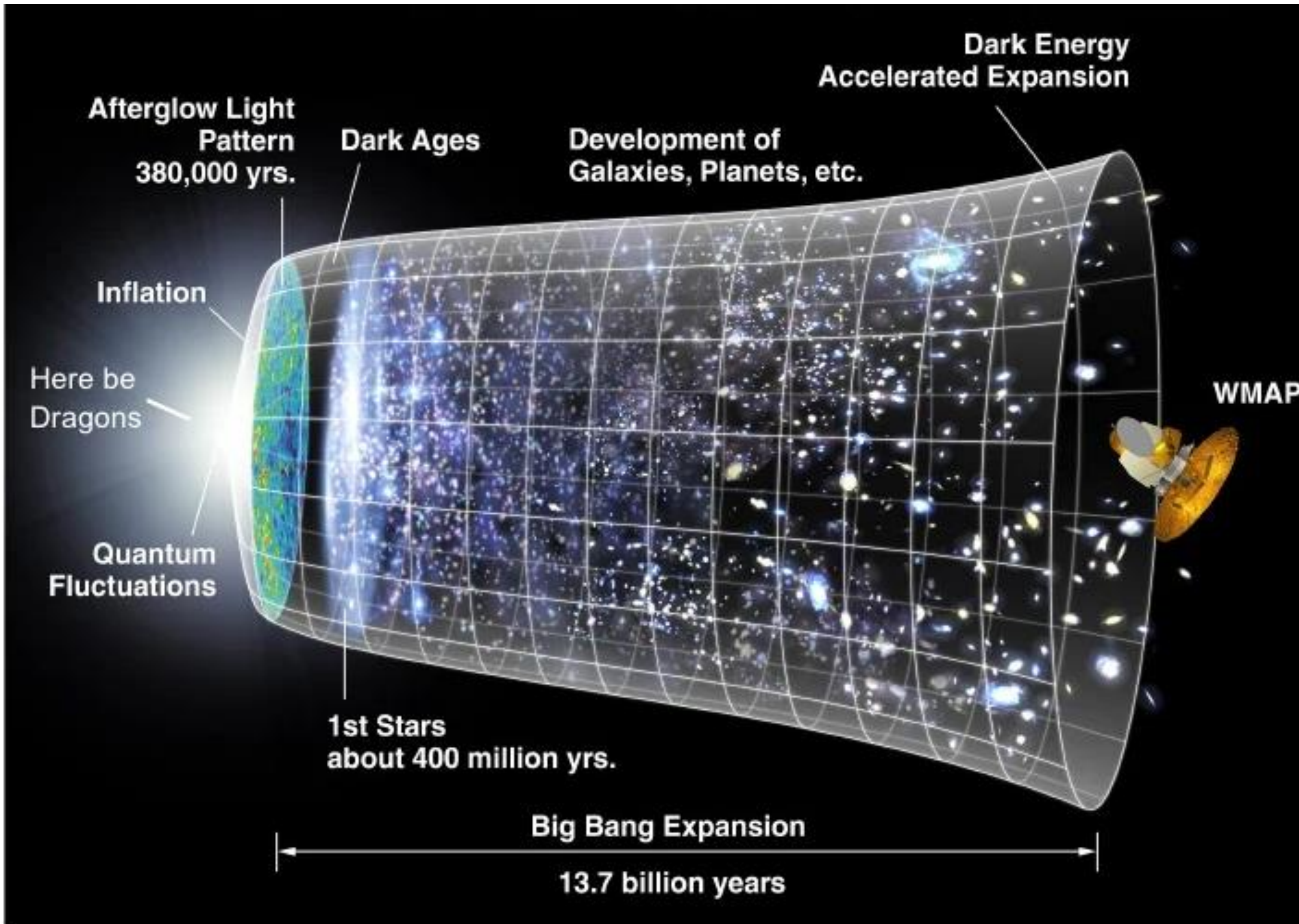
- Take any solid angle in sky
- Area subtended at distance *r* is ~ r^2
- And flux decrease goes as $\sim \frac{1}{r^2}$ → product const!
- So flux received from stacked system of stars should be huge – at least as in surface of star!





Poe → Finite age. Good but what about radiation from hot big bang?

Standard Picture of Cosmic Development



Newtonian Derivation of Cosmological Evolution Equations

- Consider universe with uniform energy density

- If scale large → need GR -- Newtonian gravity assumes instantaneous interaction (and v<<c)

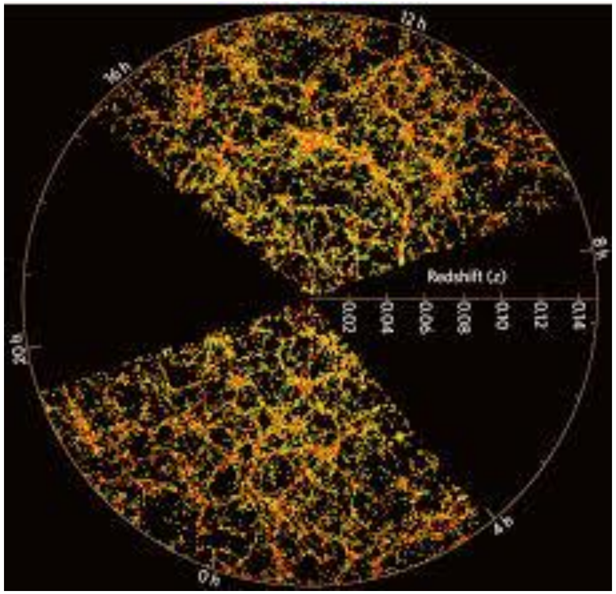
- Take instead small patch

→ fast communication ++ small speeds if homogeneously expanding/contacting.

- Because of homogeneity → all patches same

Newton-Birkhoff theorem

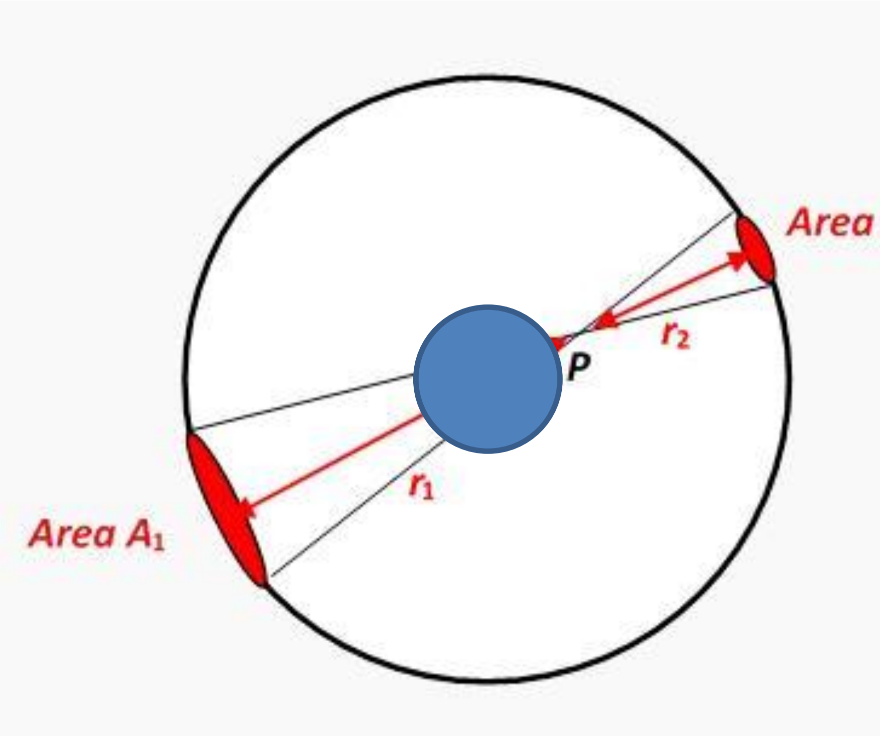
- Take said patch to be spherical (isotropy→)



- **Equation of motion**

With M = M (< r) – enclosed!

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GM}{r^2}$$



‘Energy Integral’ and interpretation

- Integrate, keeping enclosed mass constant

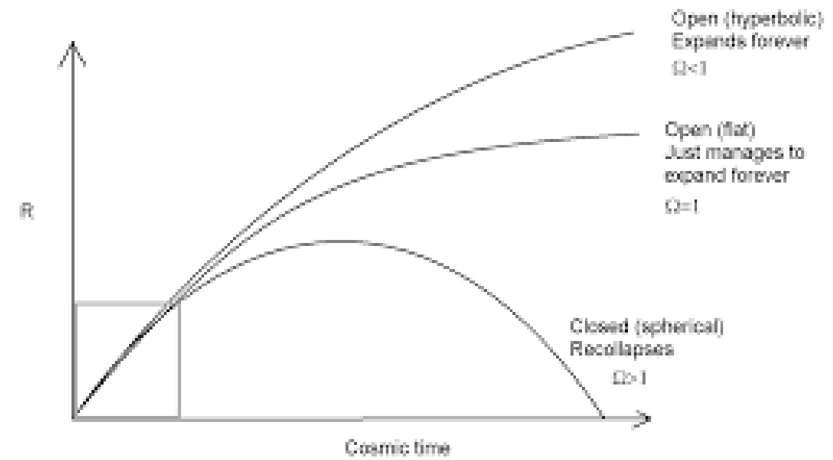
$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{G\,M\,(<r)}{r} = \mathrm{E}$$

equilibrium

- ‘Energy’ E → universe forever expands (E>0) or recontracts (E<0)

- No solutions

→ Like a ball thrown up!



Exercise: use above equation, with E =0, to derive ‘typical timescale’ of evolution (density)
What do you notice?

From Wayward Ball to **Cosmological Fluid**:

Friedmann Equations

1- Expanding radius through **scale factor**: $\boldsymbol{r = R\, a(t)}$

2- Active gravitational mass **density:** $\rho' = \rho + \frac{3p}{c^2}$

- **Both rest energy and momentum contribute**
- **Recall that hydro stress** $\sim\boldsymbol{\rho}\,\langle \boldsymbol{v_i v_j}\rangle$ ***with*** $(\boldsymbol{i, j = 1, 3})$

By Symmetry →

No cross terms (anisotropic momenta) + no streaming (mass) motions -- $\langle v_i\rangle = 0$.

→ left with isotropic pressure terms (trace of space part) + density

- **Weak field/small patch** (quasi-static)

$$\rightarrow \nabla^2\phi = 4\,\pi\, G\, \rho \;\rightarrow\; 4\,\pi\, G\, (\rho\, + 3\, p/c^2)$$

→ $M = \rho' r^3$ →

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\rho'}{3} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

Notes regarding pressure

- **Its effect is unlike hydro pressure force.**
- **The latter comes from gradient, while in Friedmann universes $\nabla p = 0$.**
- **It comes from its mass equivalence**
- **Hence attractive if +ve and repulsive when –ve**
- **As it enters through a term $\frac{P}{c^2} \sim \frac{\rho v^2}{c^2} + \rho$**

→ important only for particles if relativistic

Fundamental Friedmann Equation

Include pressure in energy conservation equation →

- Use 1st law of thermo for adiabatic expansion

$$\mathrm{d}U + p\,\mathrm{d}V = 0 \qquad U = \rho c^2 V$$

$$c^2\mathrm{d}(\rho V) + p\,\mathrm{d}V = 0 \qquad V \propto a^3(t)$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left(2\rho + a\frac{\mathrm{d}\rho}{\mathrm{d}a}\right) \Longrightarrow \boxed{\dot{a}^2 - \frac{8\pi G\rho}{3}a^2 = 2E}$$

Note: Because of homogeneity, local energy conservation holds. Not always true for gravity
Note: *E* here will have dimension of energy if scale factor is dimensional, otherwise [E] = [t^{-2}]

In General Relativity (max symmetric space)

FRW Metric $$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{d^2 r}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Metric tensor $$g_{\alpha\beta} = diag(-1, \frac{a^2}{1-kr^2}, a^2r^2, a^2r^2\sin\theta^2)$$

Field equations $$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Ricci tensor $$R_{00} = 3\frac{\ddot{a}}{a}$$

$$\tilde{R}_{ij} = -2\,k\;\tilde{g}_{ij}$$

Stress tensor $$T_{00} = \rho c^2$$
$$T_{ij} = a^2 P \tilde{g}_{ij}$$

$$\dot{a}^2 - \frac{8\pi G\rho}{3}a^2 = 2E \quad\Longrightarrow\quad \boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}}$$

Space time decomposition

- Universe appears isotropic
- 'Copernican principle': isotropic everywhere

→ Homogeneous

→ Observers synchronize clocks

→ agree on **proper time of fundamental observers** (cf. also Peebles Sec. I.4)

Defined, e.g., in terms of the homogeneous density

$$\rightarrow ds^2 = -c^2 dt^2 + a(t)\, dl^2$$

At proper time *t*, the **proper distance is $a(t)l$**

The Static (closed) Space Metric

- Choose four dim. Spherical coordinates:

$$
\begin{aligned}
X &= R \sin\chi \sin\theta \cos\phi \\
Y &= R \sin\chi \sin\theta \sin\phi \\
Z &= R \sin\chi \cos\theta \\
W &= R \cos\chi
\end{aligned}
$$

***R* = const defines surface of 3-sphere**
***R* sin χ defines radius of 2-subsphere**

- Line element on surface of sphere

→ $$X^2 + Y^2 + Z^2 + W^2 = R^2$$

→

Exercise
++ -ve curvature

$$d\,l^2 = dX^2 + dY^2 + dZ^2 + dW^2 = R^2 \left[d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

Thus one can write, or

$$d\,l^2 = R^2\left[d\chi^2 + \sin^2\chi\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

- If $r = R\sin\chi \rightarrow$

$$d\,l^2 = \left[\frac{d^2r}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

$$|k| = 1/R^2$$

Or, more compactly

$$d\,l^2 = R^2\, d\,\chi^2 + r^2 d\,\Omega$$

Note:

1- $R\,\chi$ is a geodesic distance

2- $r\,\vartheta$ is an arclength on sub-sphere (circle on fig.)

3- $r^2 d\,\Omega$ is an area on sub-sphere (circle * d phi)

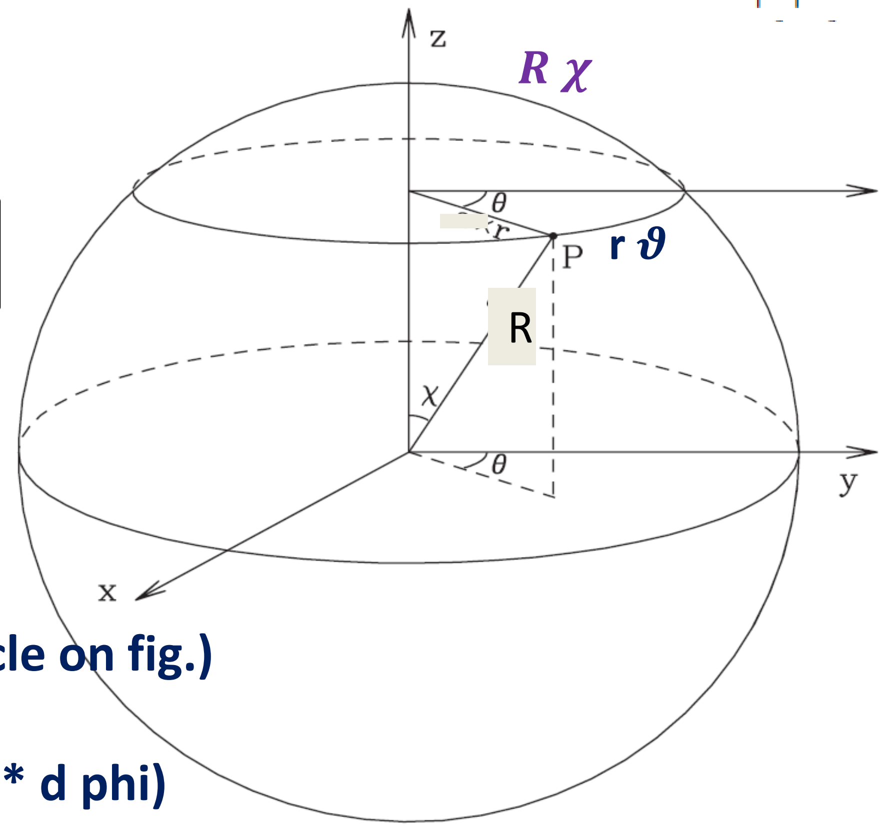


Fig. 3.1. The φ = constant section of a Robertson–Walker metric with $K = 1$, showing the g
meanings of various coordinates.

After Mo, van den Bosch & White

Adding Expansion and Time

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{d^2r}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$$d\,s^2 = -c^2 d\,t^2 + a^2(t)\ \left[R^2\,d\,\chi^2 + r^2 d\,\Omega\right]$$

$R\,a$ is radius of curvature
→ Ricci scalar ~ $(R\,a)^{-2}$

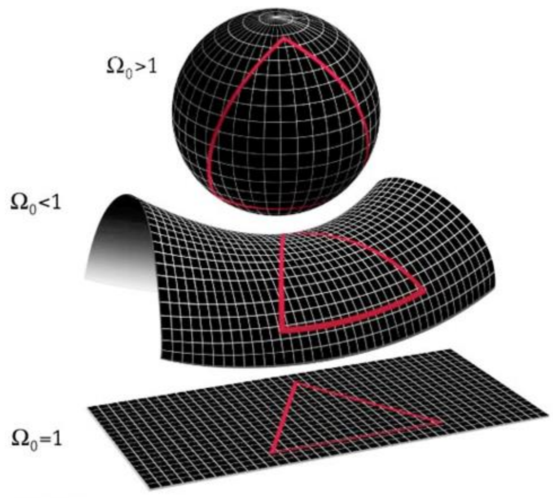
Conventions: i) $k = -\frac{1}{R^2}, 0, +\frac{1}{R^2}$ → r has length unit → a is dimensionless (with $a_0 = 1$)

ii) k = -1, 0, + 1 → r is dimensionless → R = 1 → a has length unit

Positive Curvature → $r = R\sin\chi$

Negative curvature → $r = R\sinh\chi$

Flat (zero curvature) $r = R\ \chi$

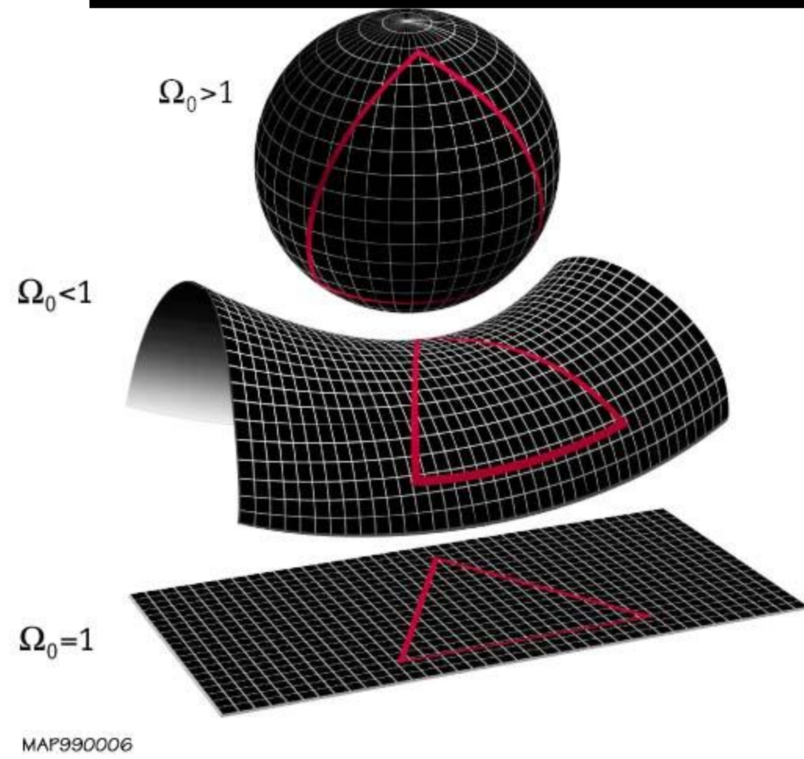


Geometry and Density

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

If *E= k = 0* flat space

$$\rho_c(t) \equiv \frac{3\dot{a}^2}{8\pi G a^2} = \frac{3H^2(t)}{8\pi G}$$



In general

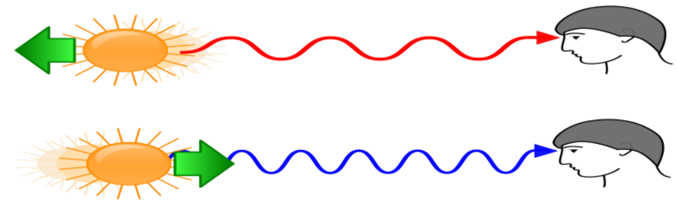
$$\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = \frac{8\pi G\rho(t)}{3H^2(t)}$$

$$\Omega^{-1} - 1 = \frac{3E}{4\pi G\rho a^2} = -\frac{3kc^2}{8\pi G\rho a^2}$$

Note:
What starts flat stays flat
Same for open (-ve k)
and closed (+ ve k)

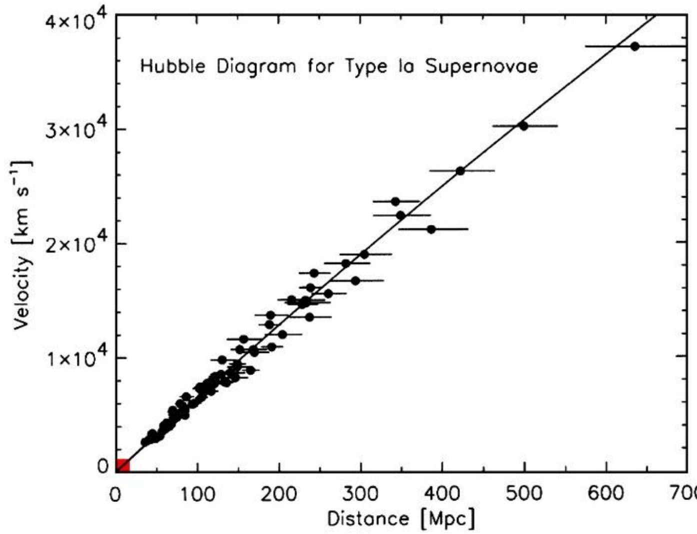
To begin **Fixing our Model:**

Expansion→ Redshift



$$t = t_1 \rightarrow\rightarrow\rightarrow \quad t_0$$

$$\lambda_0 = \frac{a(t_0)}{a(t_1)} \lambda_1$$

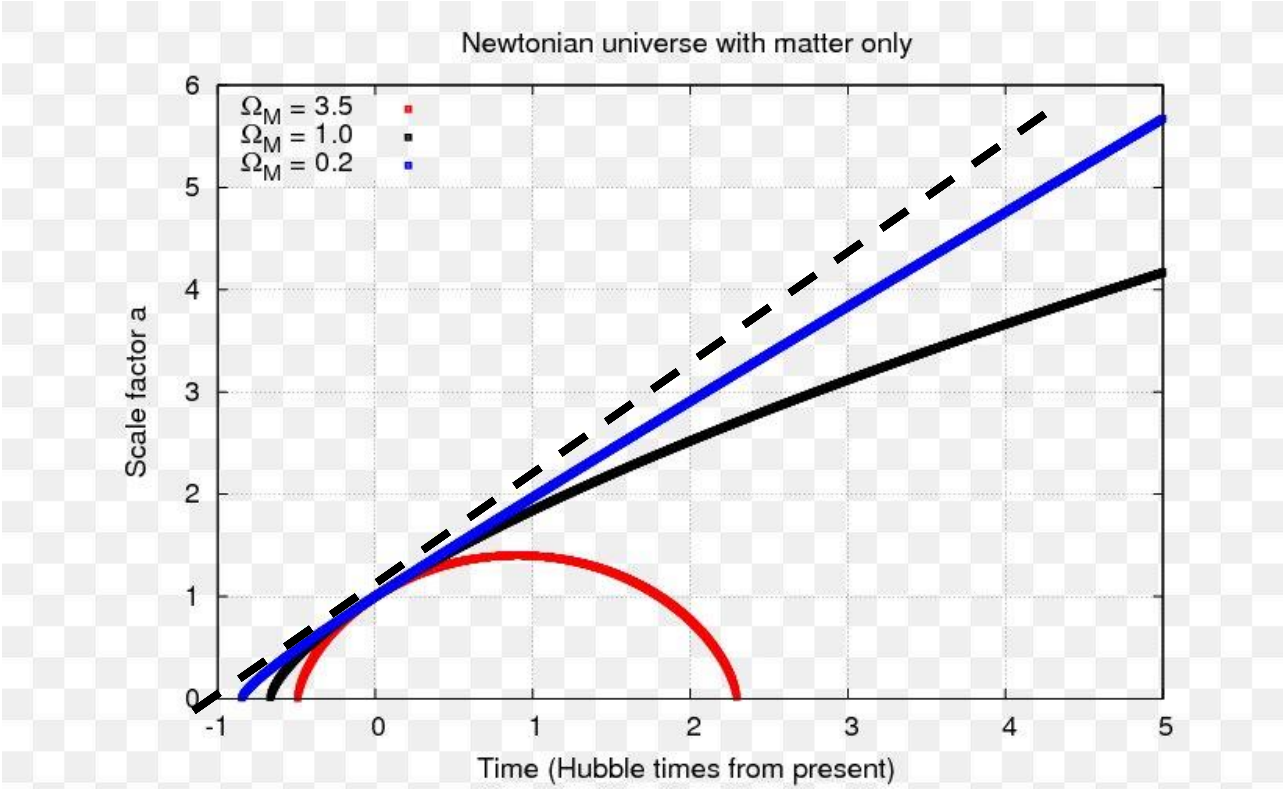


$$z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} \qquad 1 + z = \frac{a(t_0)}{a(t_1)}$$

$$z = H_0(t_0 - t_1) + \cdots \qquad z \simeq H_0 d/c$$

Hubble Constant usually expressed in km/s/Mpc

Hubble Time $t_H = H_0^{-1}$



Exercise: Check that for Hubble const of 71 km/s/Mpc, the Hubble time is a bit less than 14 Gyr

Hubble sphere radius → $D_H = c\, H_0^{-1}$

d $\ll D_H$ → (communication time faster than expansion time)

Conformal Time and Comoving Distance

- In terms of **conformal time**

$$\tau(t) = \int_0^t \frac{c\,\mathrm{d}t'}{a(t')}$$

- Metric becomes

$$ds^2 = a^2(\tau)\ [-d\tau^2 + R^2\, d\chi^2 + r^2 d\Omega]$$

- **Comoving distance**

between 'us' (at $t = t_0$) and *light (ds = 0)* emitted at time t:

→

$$D_C = R\,\chi(r) = \tau(t_0) - \tau(t)$$

$$D_C = c\int_t^{t_0} \frac{dt'}{a(t')}$$

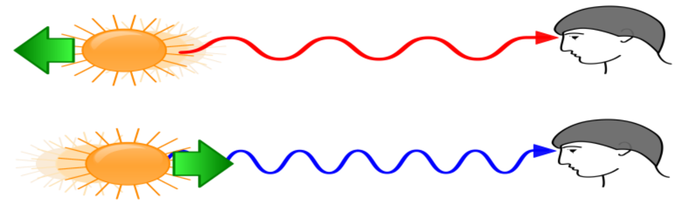
Is a geodesic distance; not measurable

In "today's currency" -- measure

Fixing our Model:

Expansion → Redshift

$$\tau\,(t_0) \;-\; \tau\,(t_1) = D_C \;\rightarrow\; \text{(invariant)}$$



$$t = t_1 \;\rightarrow\rightarrow\rightarrow\; t_0$$

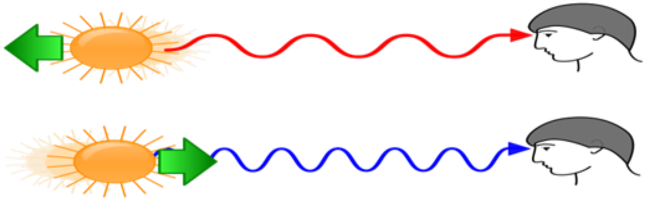
$$\rightarrow\; \delta\,[\tau\,(t_0)] - \delta\,[\tau\,(t_1)] = 0$$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)}$$

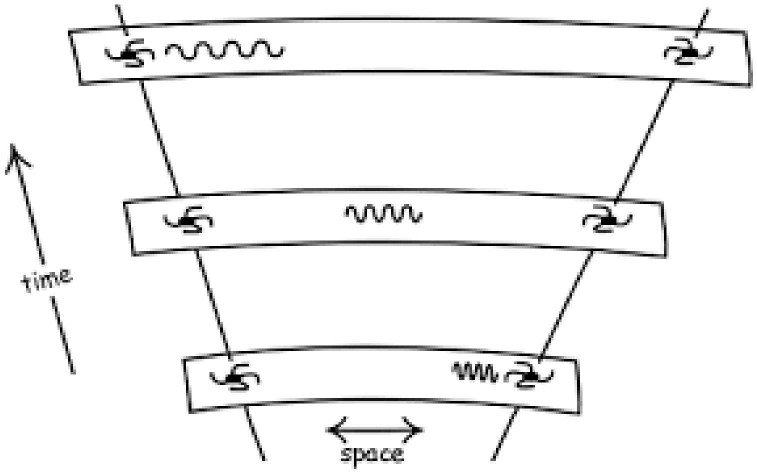
$$\tau(t) = \int_0^t \frac{c\,\mathrm{d}t'}{a(t')}$$

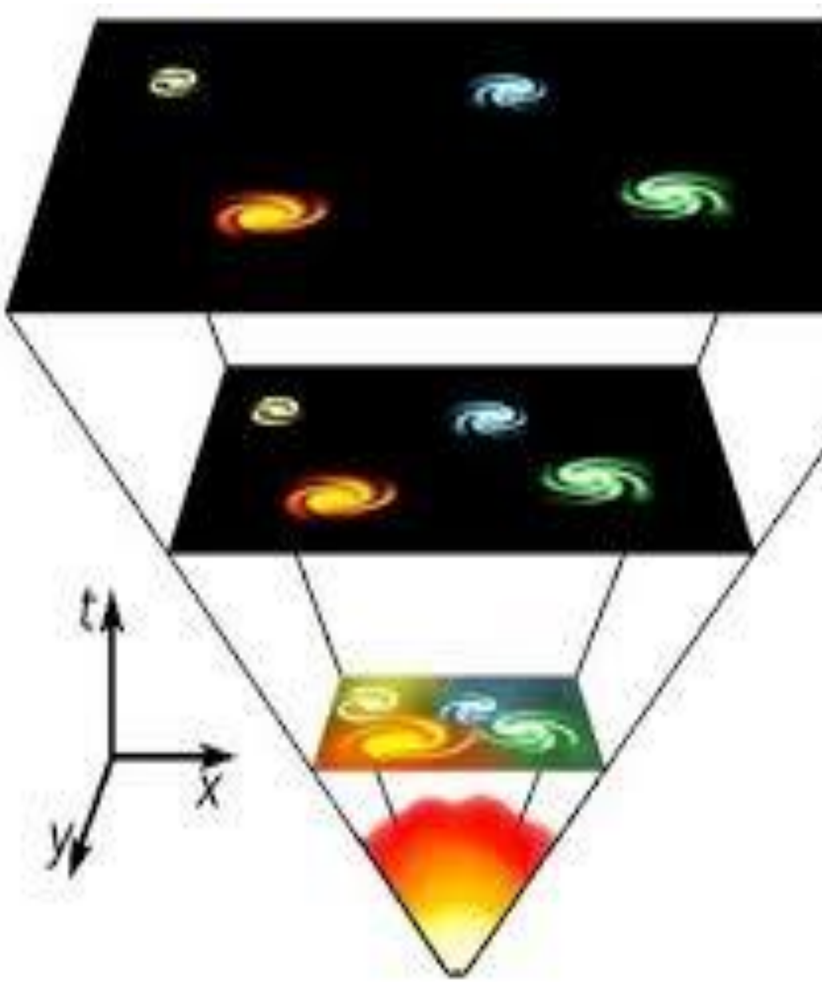
Frequencies proportional to time intervals →

$$\lambda_0 = \frac{a(t_0)}{a(t_1)}\,\lambda_1$$

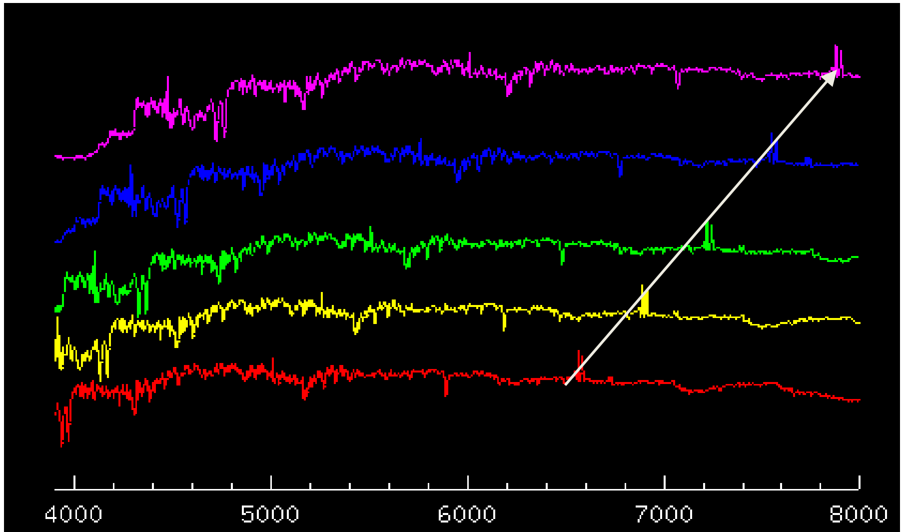


Measuring Redshifts





Light from galaxies redshifted



Nobel 2011 → acceleration!

Distances: Angular and Luminosity

- In flat space: Object of physical size D has angular extension $\delta\vartheta$ if it is at distance D_A:

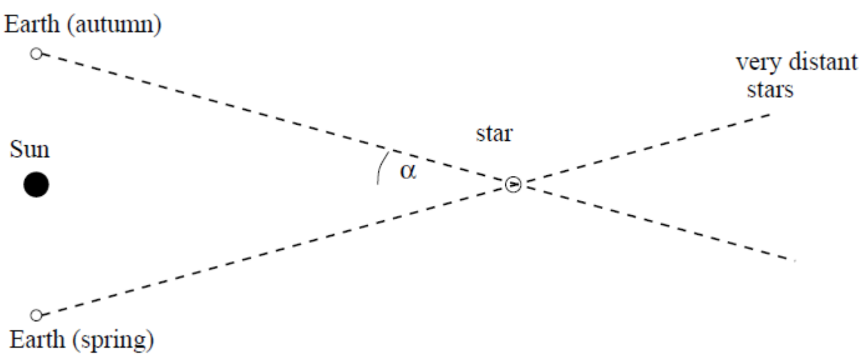
$$D = D_A\,\delta\vartheta$$

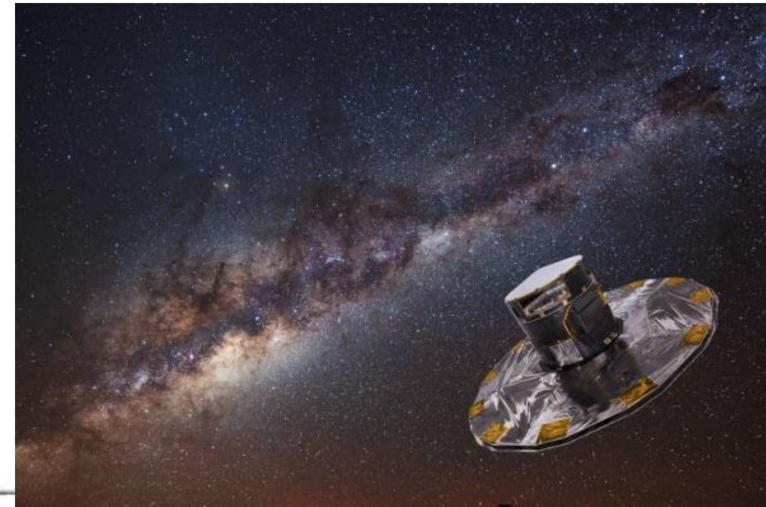
- In flat space luminosity of object follows inverse square law. The luminosity distance is defined in terms of the intrinsic luminosity L (energy emitted per second) as,

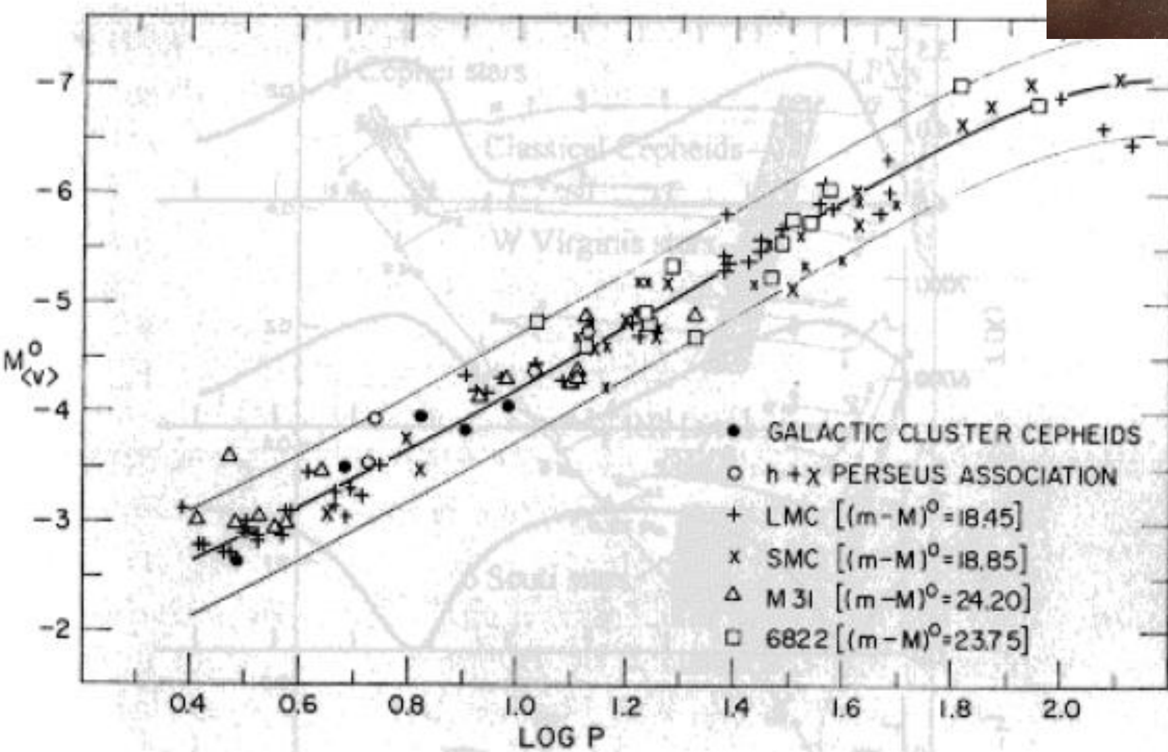
$$F = \frac{L}{4\pi D_L^2}$$

- D_L here is just the regular Euclidean distance
- F the observed flux (energy reconceived per second)
- Assumed transparent medium

Calibrating distances: Parallaxes and Cepheids







Magnitudes: from Ptolemy of Alexandria to the Distance Modulus

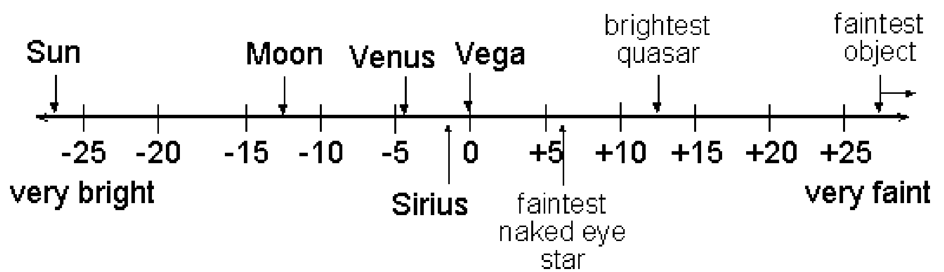
- **Classified stars from (apparent) brightest 1st to least bright 6th**

→ Magnitude gets smaller for brighter objects

- Defines a logscale (eye response to light logarithmic)
- For historical reasons (see Weinberg and refs there.. Or Wiki!)
- $$\boldsymbol{M}\,(\mathbf{Object}) - \boldsymbol{M}\,(\mathbf{Sun}) = -2.5\,\mathbf{log}_{10}\,\frac{\boldsymbol{L}(\mathbf{object})}{\boldsymbol{L}(\mathbf{Sun})}$$

Apparent bolometric magnitudes m and **absolute** M (above; def. at 10 pc) are related by

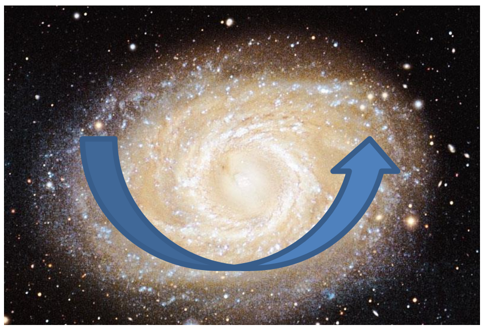
$$D_L = 10^{1+(m-M)/5}\,\mathrm{pc}$$

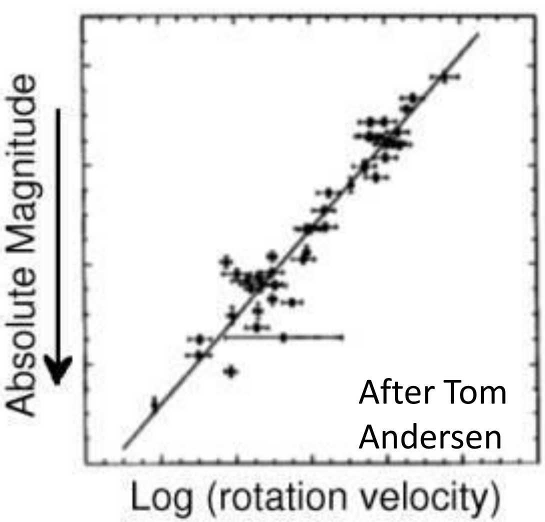


Apparent brightnesses of some objects in the magnitude system

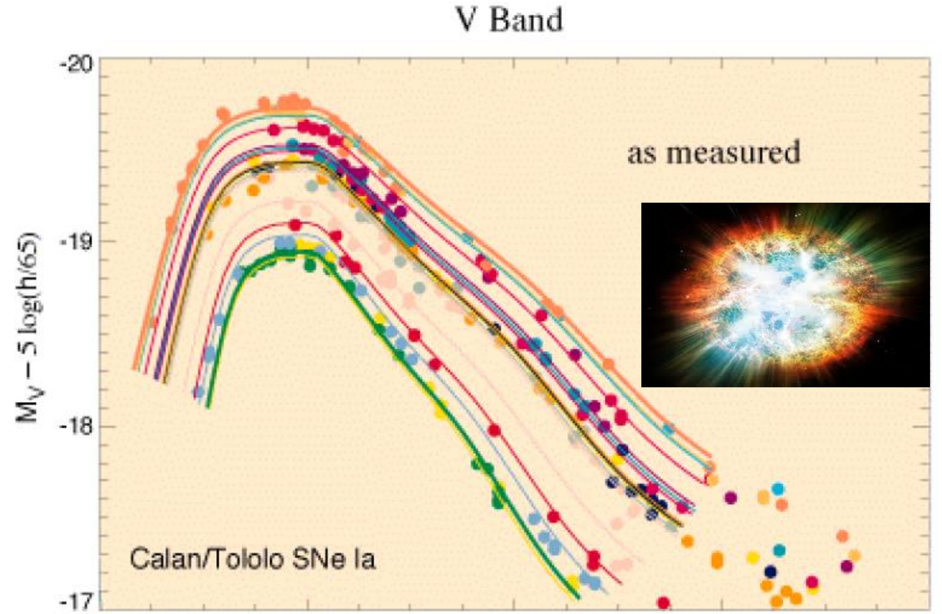
Fixing Intrinsic Luminosity at > Gpc: Supernovae 1A

Galaxy Rotation (Tully-Fisher Relation)

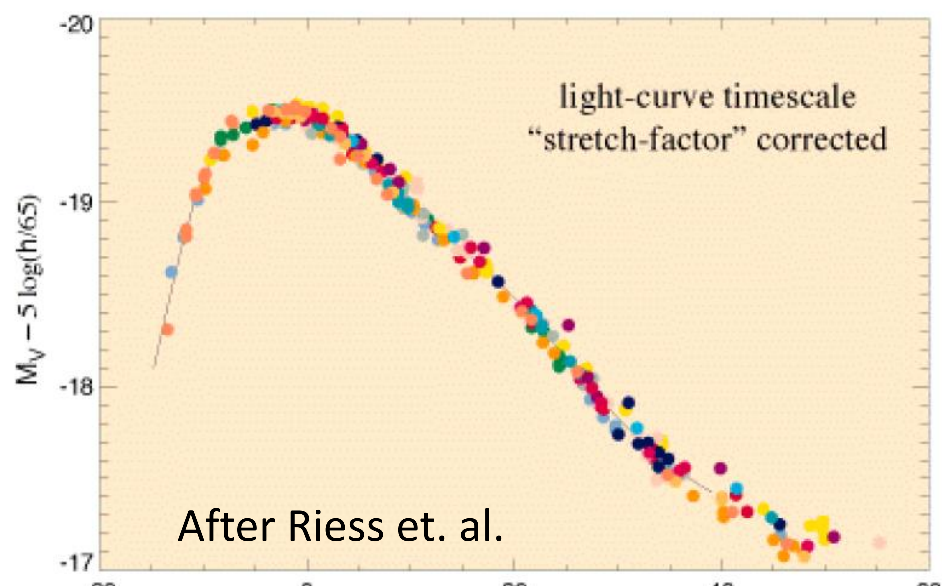




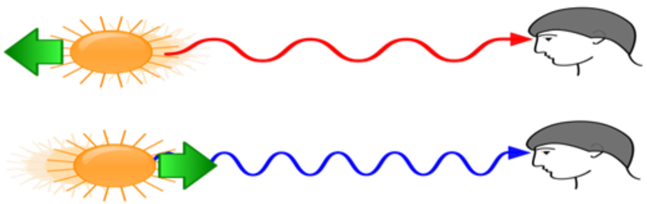
Ongoing progress at high z…

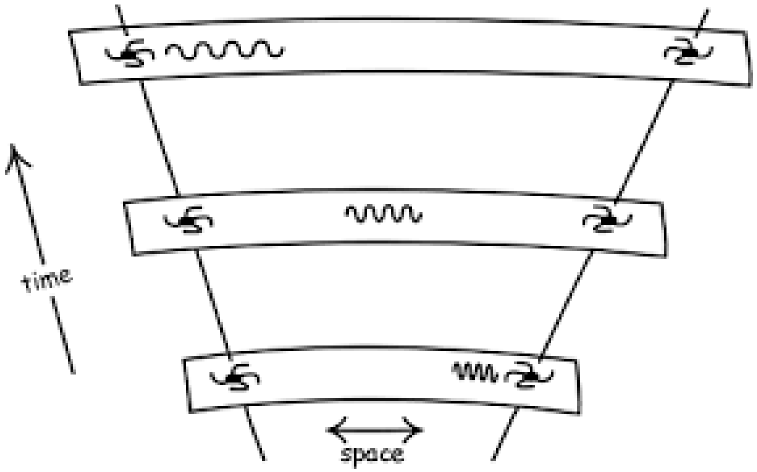


M (visual, Sun)) ~ 5

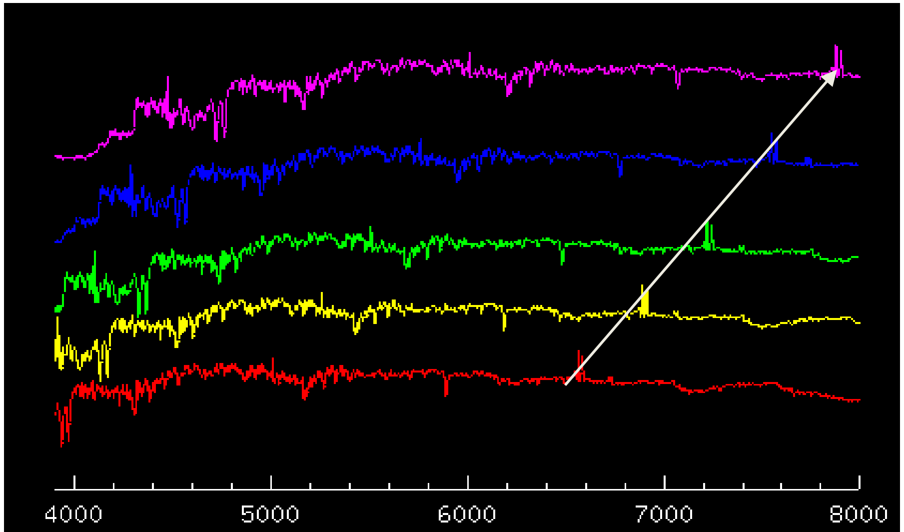


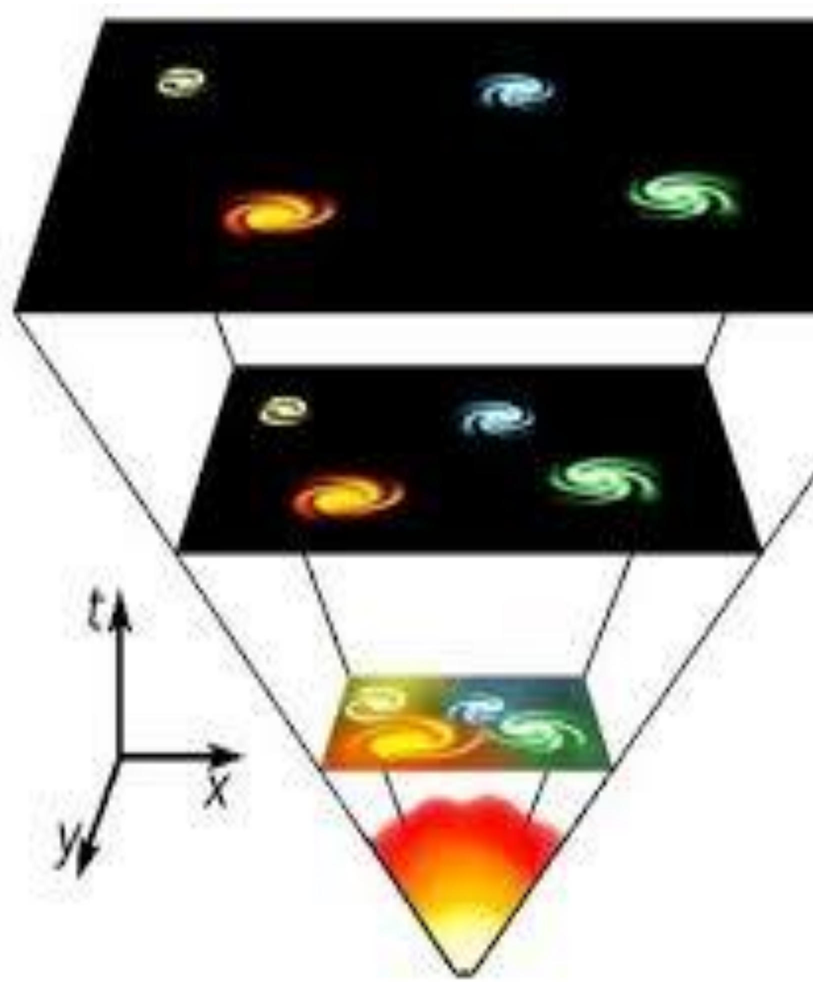
Measuring Redshifts





Light from galaxies redshifted





Nobel 2011 → acceleration!

Distances: Angular and Luminosity

- In flat space: Object of physical size *D* has angular extension $\delta\vartheta$ if it is at distance D_A:

$$D = D_A \, \delta\vartheta$$

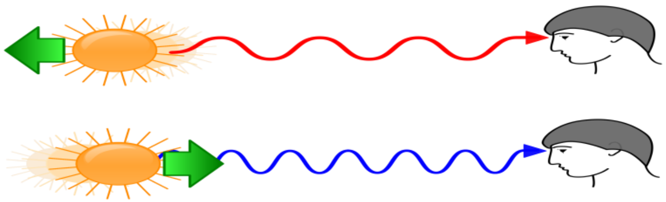
- In flat space luminosity of object follows inverse square law. The luminosity distance is defined in terms of the intrinsic luminosity L (energy emitted per second) as,

$$F = \frac{L}{4\pi D_L^2}$$

- D_L here is just the regular Euclidean distance
- F the observed flux (energy reconceived per second)
- Assumed transparent medium

To begin **Fixing our Model:**

Expansion→ Redshift

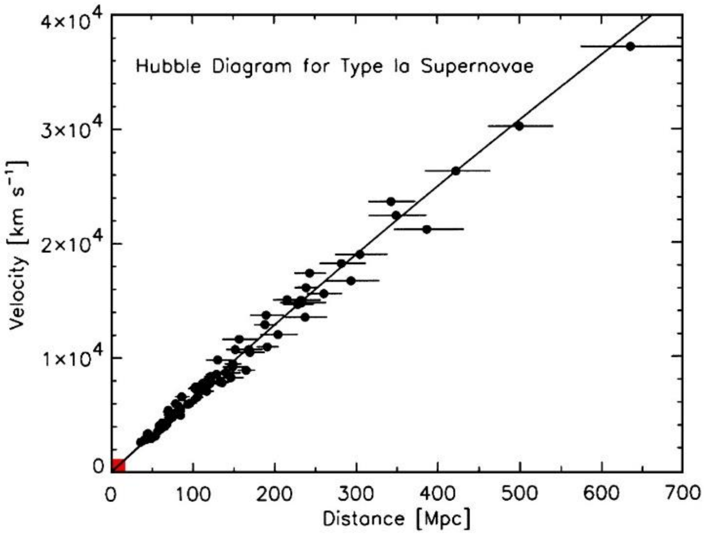


$$t = t_1 \;\rightarrow\rightarrow\rightarrow\; t_0$$

$$\lambda_0 = \frac{a(t_0)}{a(t_1)}\lambda_1$$

$$z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} \qquad 1 + z = \frac{a(t_0)}{a(t_1)}$$

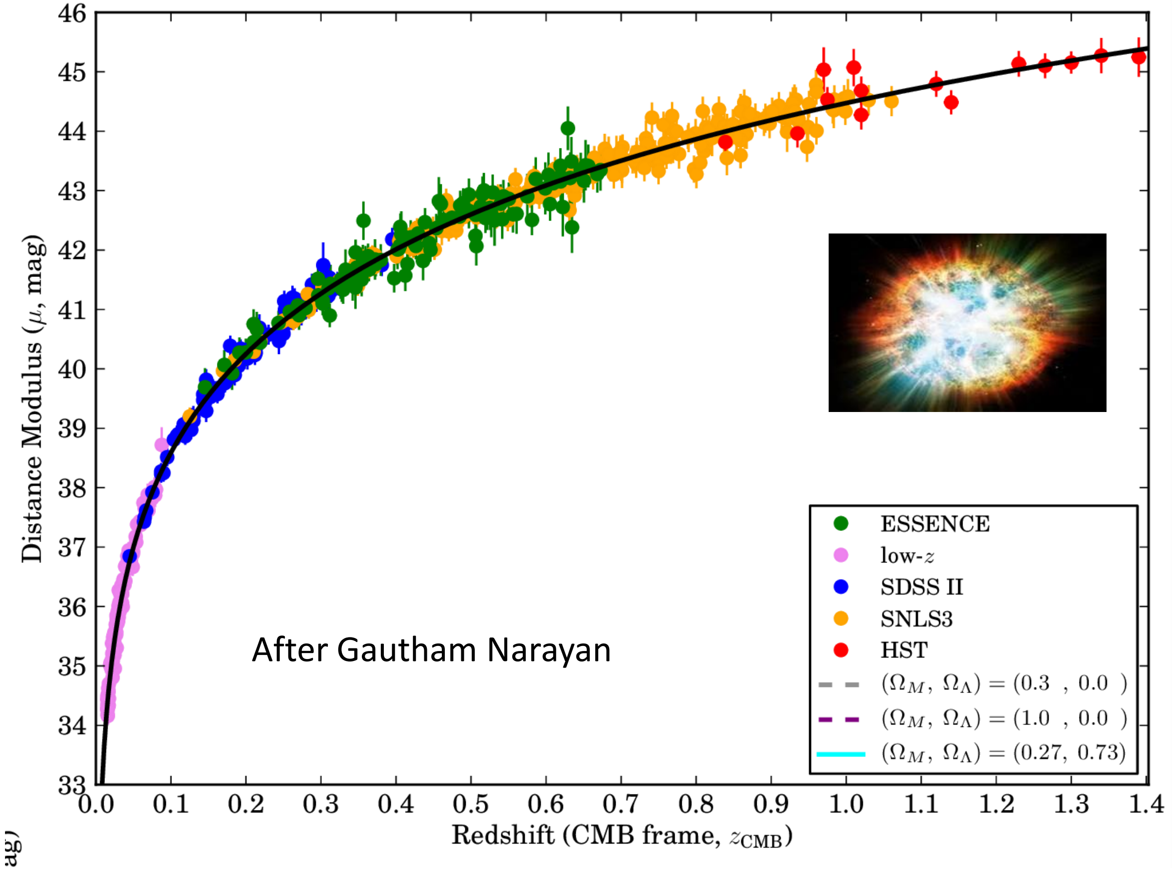
$$z = H_0(t_0 - t_1) + \cdots \qquad z \simeq H_0 d/c$$



Hubble Constant usually expressed in km/s/Mpc

A higher z Hubble diagram

- Contents → Dynamical History → Which model?



Actually… what distances we measure?

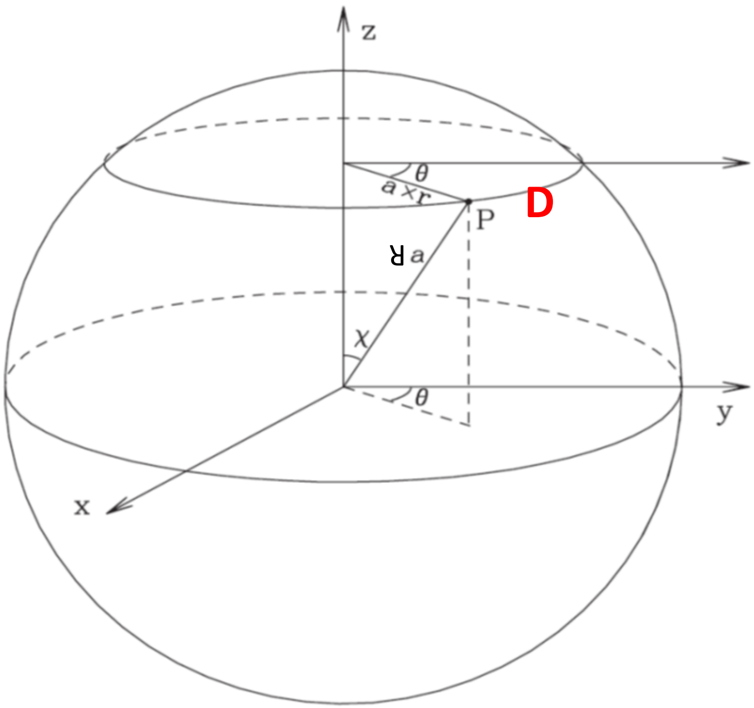
Angular dist. in cosmological context

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{d^2r}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Objects of proper length D

Emitting at a(t)
at angular scale $d\theta$

Such that



$$ds = D = a(t)r\,d\theta = D_A\,d\theta$$

$$\rightarrow D_A = r/(1+z)$$

Known object size + angle on sky → r +++ can also find r from cosmological model

Cosmological Luminosity Distance

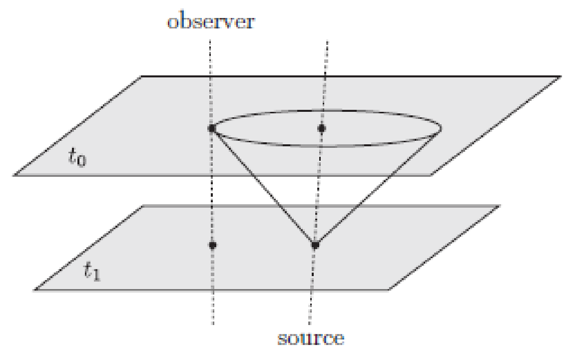
- Recall $$d\,s^2 = -c^2 d\,t^2 + a^2(t)\ \left[R^2\,d\,\chi^2 + r^2 d\,\Omega\right]$$

- Distance multiplying solid angle, $a(t)\,r^2 d\,\Omega$, → area element

- **At time t_0 light reaches us ($a_0 = 1$) proper area of sphere drawn around object $4\,\pi\,r^2$** (same as area centered here and touching object)

- Again $$F = \frac{L}{4\pi D_L^2}$$

- But two f[illegible]



- i) Photons are redshifted (less energetic)
- Ii) Rate at which they arrive is smaller
- Both by factors 1/ (1+z)

→ $$D_L = \ r\,(1+z) \ = \ D_A (1+z)^2$$

(if convention with dimensional scale factor a is used there's an extra factor of $a\,(t_0) = a_0$)

To begin *Looking for r* (from last time)

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{d^2r}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$$d\,s^2 = -c^2 d\,t^2 + a^2(t)\ \ [R^2\,d\,\chi^2 + r^2 d\,\Omega]$$

$R\,a$ is radius of curvature
→ Ricci scalar ~ $(R\,a)^{-2}$

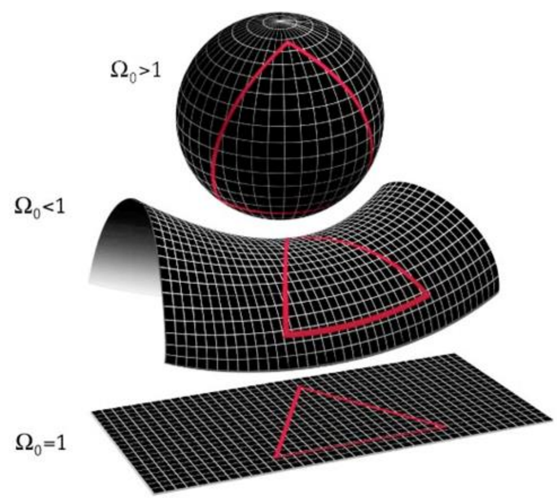
Conventions: i) $k = -\frac{1}{R^2}, 0, +\frac{1}{R^2}$ → r has length unit → a is dimensionless (with $a_0 = 1$)

ii) k = -1, 0, + 1 → r is dimensionless → R = 1 → a has length unit

Positive Curvature → $r = R\sin\chi$

Negative curvature → $r = R\sinh\chi$

Flat (zero curvature) $r = R\ \chi$



Geometry and Density

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

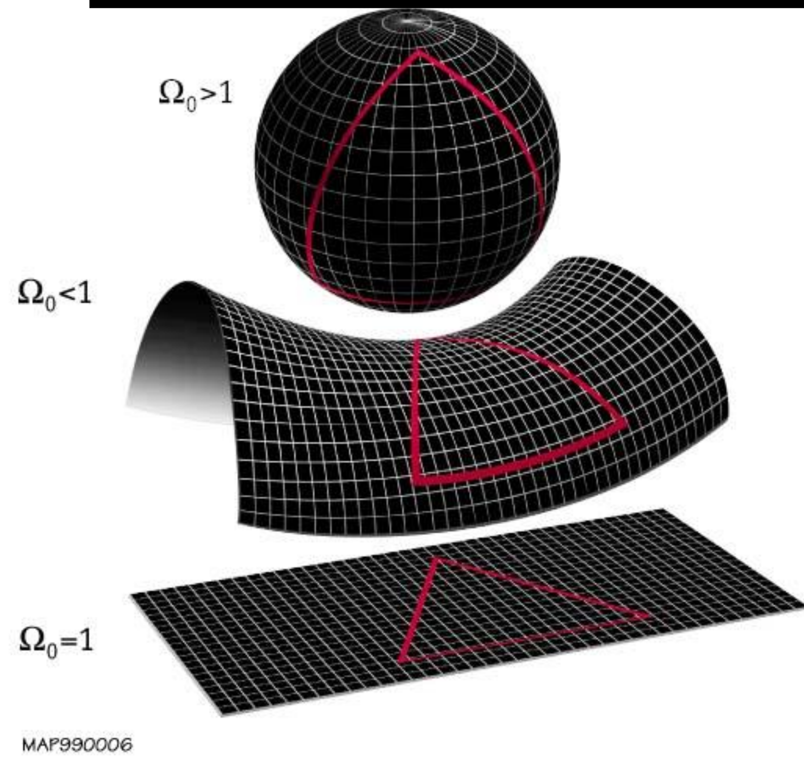
If *E= k = 0* flat space

$$\rho_c(t) \equiv \frac{3\dot{a}^2}{8\pi G a^2} = \frac{3H^2(t)}{8\pi G}$$

In general

$$\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = \frac{8\pi G\rho(t)}{3H^2(t)}$$

$$\Omega - 1 = \frac{kc^2}{a^2 H^2}$$



Note:
What starts flat stays flat
Same for open (-ve k)
and closed (+ ve k)

Fixing comoving scale

- $\chi = \frac{D_C}{R}$ and $D_C = c\int_t^{t_0} \frac{dt'}{a(t')}$

→ **Known with *a (t) and R!***

- **For R:** let scale factor **dimensionless.** $a_0 = 1$, $\boldsymbol{k = \pm 1/R^2}$

- Using $\Omega - 1 = \frac{kc^2}{a^2 H^2}$ and **Fixing at 'now':**

→ $\boldsymbol{k = \frac{H_0^2}{c^2}\,(\Omega_0 - 1)}$ → **need *a (t)* then!**

Note*:* *r* is sometimes referred to as 'proper motion distance' or 'transverse commoving distance', D_M

Evolution of Friedmann Universes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

- **To solve equation** → **need $\rho = \rho\,(a)$.**

- **'Heuristically', we have:**

Matter → $\rho_M \sim a^{-3}$

Radiation → $\rho_R \sim a^{-4}$

Cosmological constant → $\rho_\Lambda = \text{const.}$

Using conservation law

Recall

$$\dot{a}^2 - \frac{8\pi G\rho}{3}a^2 = 2E$$

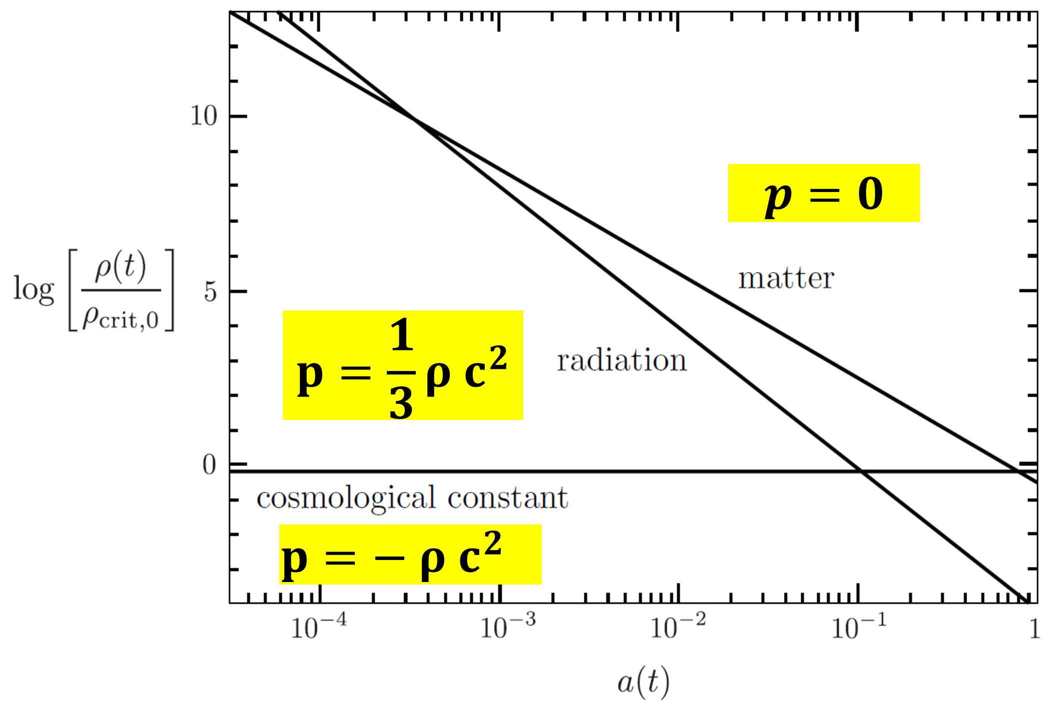
- Assume

$$p = w\rho c^2$$

- Then

$$c^2 \mathrm{d}(\rho V) + p\,\mathrm{d}V = 0 \Longrightarrow \rho \propto V^{-1-w} \propto a^{-3(1+w)}$$

$$\rho \propto \begin{cases} a^{-3} & \text{matter} \\ a^{-4} & \text{radiation} \\ a^{0} & \text{vacuum} \end{cases}$$



Our understanding → universe went through the following phases

1- Vacuum domination and vast exponential expansion ('inflation')
2- Radiation domination
3- Matter radiation
4- 'Recent' vacuum donation (again)

And that it is quite flat…

Note: early universe → nearly flat anyway!

$$\Omega^{-1} - 1 = \frac{3E}{4\pi G\rho a^2} = -\frac{3kc^2}{8\pi G\rho a^2}$$

Evolution in spatially flat Universe

$p = w\rho c^2$	w	$\rho(a)$	$a(t)$
RD	$\frac{1}{3}$	a^{-4}	$t^{1/2}$
MD	0	a^{-3}	$t^{2/3}$
ΛD	-1	a^{0}	e^{Ht}

Horizons

- Hubble Sphere $D_H = c\, H_0^{-1}$

- Light emmited at time past and probed at future time t_f, covers commoving distance:

$$D_C = c \int_{t_e}^{t_f} \frac{dt'}{a(t')}$$

- Converges at lower limit → some past events can't be observed at t_f particle horizon→ $t_e \to 0$, and $t_f = t_0$

→ Farthest we can see… ~ 14 Gpc comoving.

- Converges at upper limit current events cannot be seen in future → 'event horizon' with $t_e = t_0$ and $t_f \to \infty$

Distances: with (almost) everything in it

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$ and $$\rho_{c0} = \frac{3H_0^2}{8\pi G};\ \Omega_{M0} = \frac{\rho_{M0}}{\rho_{c0}}$$ etc ...

$$H^2(a) = H_0^2 \left[\frac{\Omega_{M0}}{a^3} + \frac{\Omega_{R0}}{a^4} + \frac{\Omega_{K0}}{a^2} + \Omega_\Lambda\right]$$ (exercise)

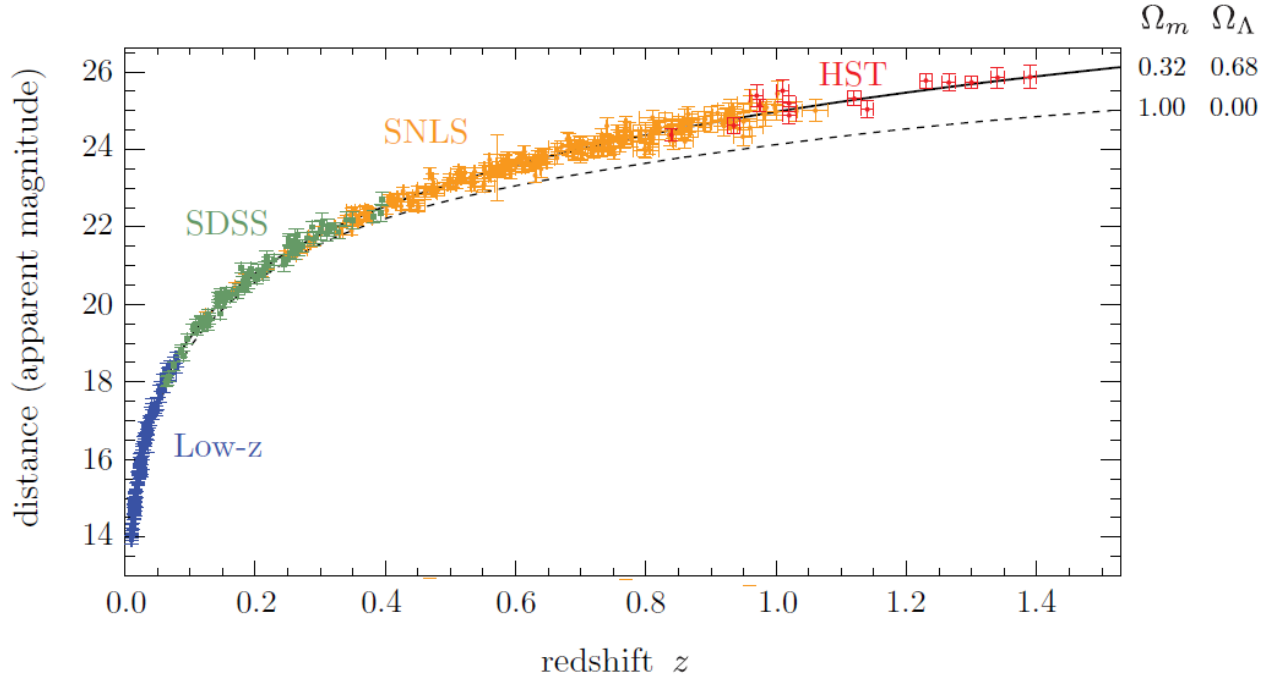
$$D_C = \int_t^{t_0} \frac{c\,dt'}{a(t')} = c\int_a^1 \frac{da}{a^2 H(a)} = D_H \int_a^1 \frac{da}{a^2\sqrt{\Omega_M/a^3 + \Omega_R/a^4 + \Omega_k/a^2 + \Omega_\Lambda}}$$

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{da}{a\sqrt{\Omega_M/a^3 + \Omega_R/a^4 + \Omega_k/a^2 + \Omega_\Lambda}}$$ Exercise

$$\Omega_k = -\frac{k\,c^2}{a^2H^2} = \pm\frac{c^2}{R^2a^2H^2} = \pm\frac{D_H^2}{R^2a^2}$$

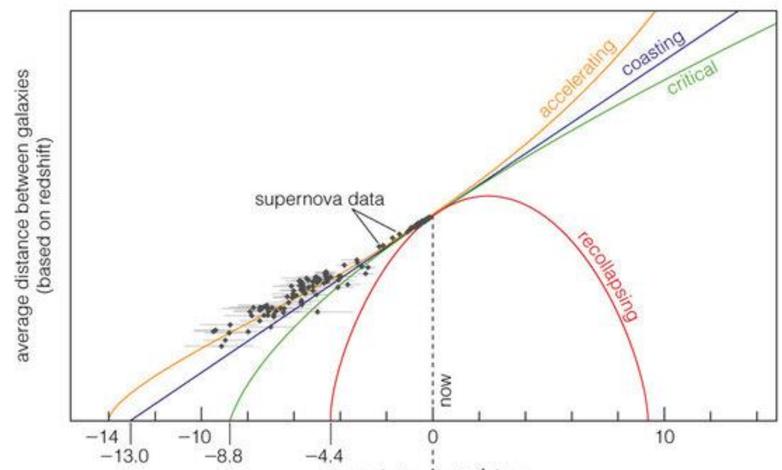
→ Now compare with observations; e.g. supernovae

Expansion and its Acceleration: Dark Energy and Dark Matter



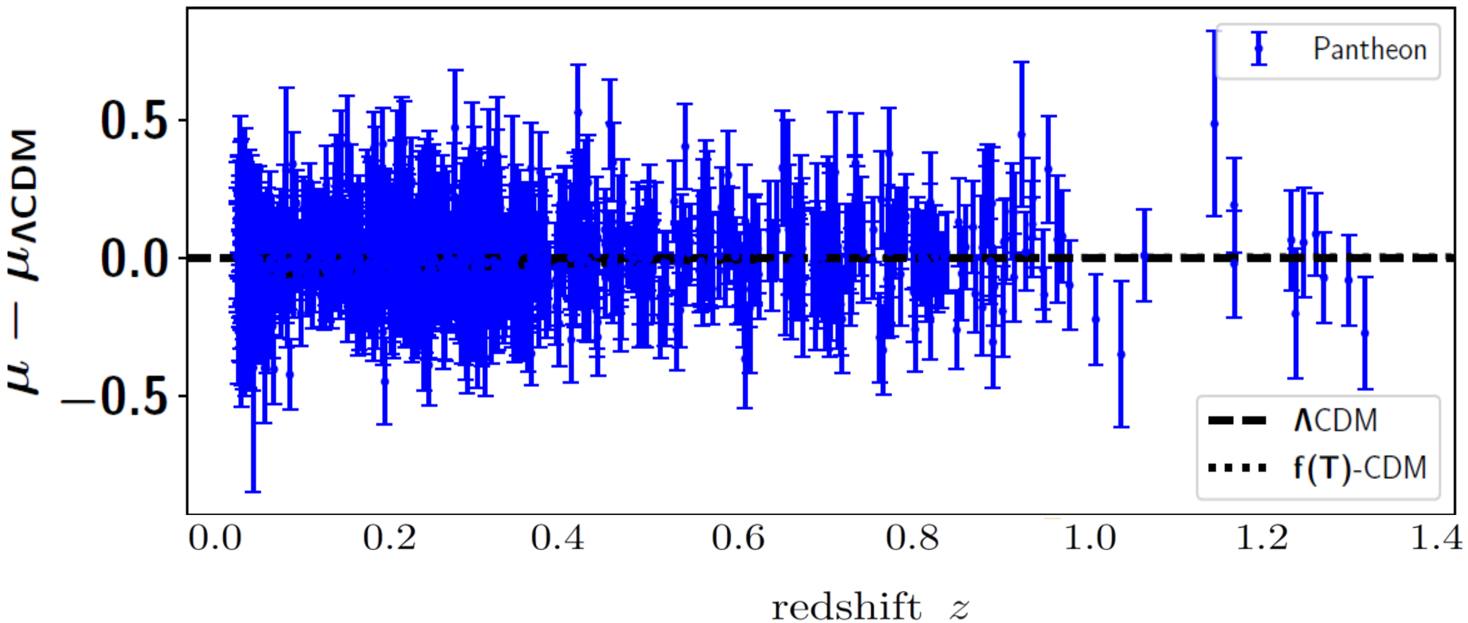
<u>Current acceleration →</u>
<u>Dark energy</u>

<u>Past deceleration rate →</u>
<u>Dark matter</u>



Lambda-CDM

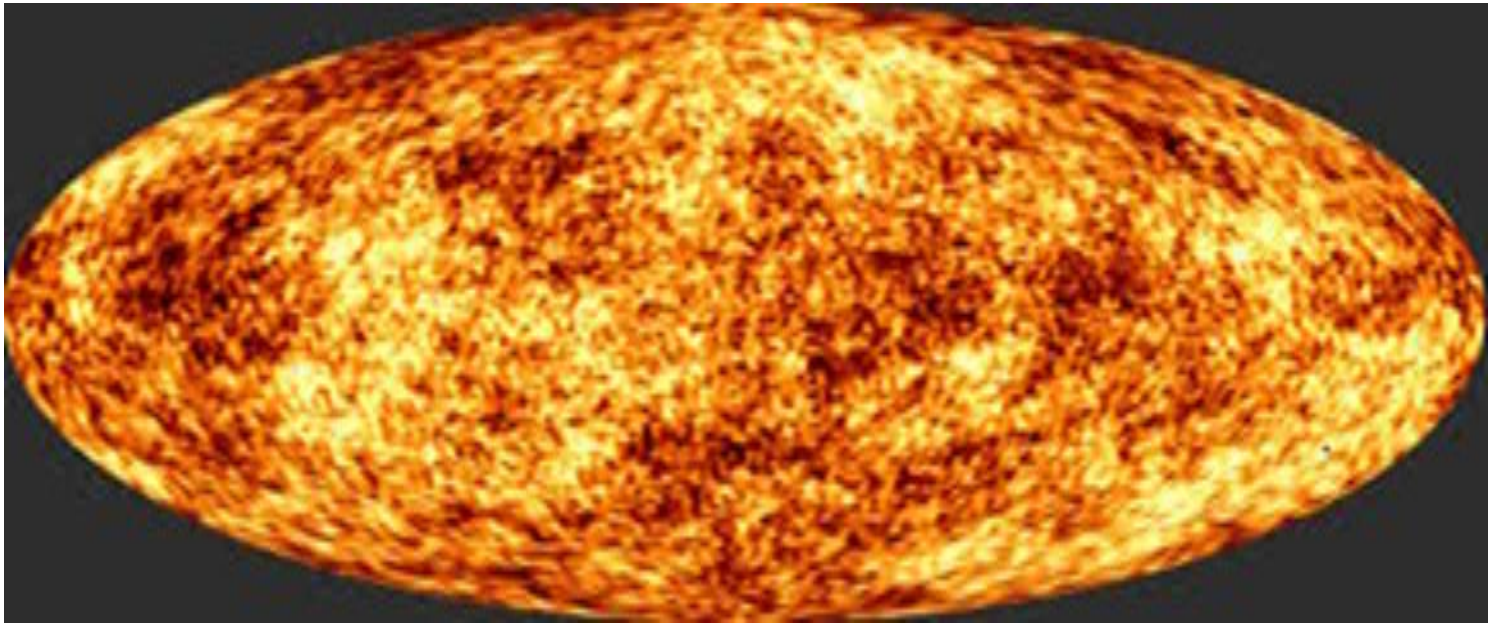
- **Measurements → vanishing spatial curvature, ~ 70 % dark energy, 25 % dark matter**



Hashim et. al. (2021)

- **Parameters in agreement with CMB and LSS (coming lectures).**
- **But measurements of Hubble parameter in tension with these.**

Fluctuations in the CMB → seeding structure



Coming talks →

Local Form of Coordinate Distance

- **Expand** scale factor around local value:

$$a(t) = a_0 \left[1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots \right]$$

- 'Deceleration' parameter $$q_0 \equiv -\frac{\ddot{a}_0 a_0}{\dot{a}_0^2}$$

- Expand r to get (note the flat space approximation emerges; technically should divide both sides of eq. by 1/R then expand, then multiply again. Assume $a_0 = 1$)

$$c\,(t - t_0) + c\,H_0 \,\frac{(t_0 - t)^2}{2} + \ \ldots = r + \ \ldots$$

- Then (exercise) → $$r \approx \frac{c}{H_0} \left[z - \frac{1}{2} z^2 (1 + q_0) + \ldots \right]$$